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Sam E. Valdez, Karen Bergeron, John G. Sclater

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THE COEFFICIENT OF FRICTION BETWEEN VARIOUS CYLINDRICAL, SPHERICAL AND FLAT SURFACES IN CONTACT

by

Sam E Valdez — Institute for Geophysics, The University of Texas at Austin, Austin TX 78713

Karen Bergeron — The Department of geological Sciences and the Institute for Geophysics, The University of Texas at Austin, Austin TX 78713

John G Sclater* — The Department of geological Sciences and the Institute for Geophysics, The University of Texas at Austin, Austin TX 78713.

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*Now at Geological Research Division, The Scripps Institution of Oceanography, University of California, San Diego CA 92093

Institute for Geophysics, The University of Texas at Austin

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INTRODUCTION

Steel balls and cylinders have long been used as analogues for understanding the behavior of aggregates under extension. In their attempts to understand the behavior of sand in sand boxes, at the grain by grain level, Boerner and Sclater (1992) carried out a series of experiments on painted steel balls. They found it necessary to know the coefficient of friction in order to interpret their results. In addition, they found that on a critical thought experiment, (Figure 14 ; Boerner and Sclater 1992) the behavior of three cylinders in contact, the behavior would differ depending upon the coefficient of static friction between the cylinders. As we wished to examine this behavior in more detail (Valdez et al., 1992) we decided to measure the coefficient of friction of a variety of cylinders. We made a series of measurements using the standard block on inclined plane method. We derived the coefficient of static friction between covered and uncovered cylinders of steel, wood and Plexiglass in contact and painted steel balls in contact and then the coefficient of friction of each of these materials in contact with a rubber sheet.

APPARATUS

We measured the coefficient of friction by using an inclined plane on a flat table. We attached the plane to the table by a hinge at one end. We used a motor and a pulley and string to raise the incline (Figure 1). We raised the pulley sufficiently slowly that we could ignore momentum forces. By trial and error we settled at a rate of 0.125 cm/s.

THEORY AND PROCEDURE

The measurement of the static coefficient of friction involves raising an inclined plane until a block resting on that plane begins to slide. The angle, ϕ, at which motion occurs can be found from the hypotenuse, d, and the height, h, of the right triangle that is formed between the plane and the table at the time the block begins to move (Figure 1). The relationship governing, ϕ, d and h is

$$\sin \phi = h/d \qquad (1)$$

The coefficient of static friction, μ, is defined as

$$\mu = \tan \phi \tag{2}$$

The average value of μ, $\bar{\mu}$ is given by

$$\bar{\mu} = \Sigma \frac{\mu}{n} \tag{3}$$

where n is the number of values and the standard deviation, σ, is given by

$$\sigma = \left[\frac{\Sigma\mu^2 - \left(\Sigma \frac{\mu^2}{n}\right)}{n - 1} \right]^{1/2} \tag{4}$$

To carry out the experiments we temporarily affixed two cylinders together to form a block. We took care to ensure that the cylinders did not role. Then, we stacked two blocks on top of each other, with the bottom set fixed to the inclined plane. We used a motor to raise the incline. The angle at which the top block begins to slide determines the coefficient of static friction. We followed the same procedure for the steel balls except we affixed four balls together.

To measure the coefficient of friction between the blocks and the rubber sheet we placed a block on top of an inclined plane to which we had temporarily attached a rubber sheet of the same type of material as we had been used in the experiments reported by Boerner and Sclater (1992) and Valdez et al., (1992). The angle at which the blocks slipped determined the coefficient of static friction.

DISCUSSION OF RESULTS

We performed a series of experiments to determine the static coefficient of friction between (a) various cylinders and steel balls, and (b) these cylinders and balls and a rubber sheet (Table 1 and Table 2 a-i) We obtained the coefficient of friction between seven different types of cylinders. We duplicated the measurements at two different times and only the values for steel on steel showed any significant difference. We are not sure why the two values for steel are so different (Table 2 b and Table 2 c). They lie within the range for greased to ungreased steel given in the Chemical Rubber Company Handbook(1967) of 0.08 to 0.40. However, the difference is surprising even though we used two different types of steel and the measurements were made about six months apart. One possible explanation is that before the first run the cylinders had been well handled and hand grease lowered the coefficient of friction. In contrast for the second run, we reduced the handling of the new cylinders that we obtained as much as possible. Also, the two cylinders had different weights and may not have had the same composition. The slickness of the Plexiglass on Plexiglass impressed us (Table 2 f) as did the high coefficient of friction when we painted these cylinders (Table 2 g). Why the painted Plexiglass cylinders should have values so much higher than the painted steel balls (Table 2 i) was also a puzzle.

We completed the experiments by measuring the coefficient of static friction between the various cylinders and steel balls and a rubber sheet (Table 3 and Table 4 a-f). In all cases except for the case of the painted Plexiglass the coefficient of friction between the materials themselves was less than that between the material and the rubber sheet. We do not understand why the painted Plexiglass is different. We suspect that wet paint caused excessive adhesion in the first experiment with Plexiglass but that by the second experiment the paint had dried. This would account for the drop in the coefficient of friction.

CONCLUSIONS

The coefficients of friction which we measured were close to those that we had expected from consultation with the Chemical Rubber Company Handbook (1967). In addition, Valdez et al., (1992) found very good agreement between the measured point at which three cylinders under extension would start to roll apart and that predicted by the theory using the above measurements. This agreement gave us considerable confidence in our measurements.

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TABLE 2 e

RUBBER COVERED STEEL CYLINDER ON RUBBER COVERED STEEL CYLINDER

Run	d (cm)	h (cm)	h / d	φ (Deg.)	μ*
1	30.6	16.7	0.55	33.1	0.65
2	30.7	16.3	0.53	32.1	0.62
3	30.6	17.7	0.58	35.3	0.71
4	30.5	18.1	0.59	36.4	0.74
5	30.5	18,0	0.59	36.4	0.74
6	30.4	18.7	0.62	38.0	0.78
7	30.5	17.9	0.59	35.9	0.72
8	30.5	17.9	0.59	35.9	0.72
9	30.5	18.0	0.59	36.1	0.73
10	30.4	18.8	0.62	38.2	0.79

*Mean = 0.72 ± 0.05

TABLE 2 f

PLEXIGLASS CYLINDER ON PLEXIGLASS CYLINDER

Run	d (cm)	h (cm)	h / d	φ (Deg.)	μ*
1	32.7	8.6	0.26	15.2	0.27
2	32.6	8.8	0.27	15.7	0.28
3	32.7	8.6	0.26	15.2	0.27
4	32.7	8.6	0.26	15.2	0.27
5	33.4	7.4	0.22	12.8	0.23
6	32.5	9.1	O.28	16.3	0.29
7	32.7	8.7	0.27	15.4	0.28
8	32.6	8.9	0.27	15.8	O.28
9	32.8	8.4	0.26	14.8	0.26
10	32.2	9.7	0.30	17.5	0.31

*Mean = 0.27 ± 0.02

TABLE 2 g

PAINTED PLEXIGLASS CYLINDER ON PAINTED PLEXIGLASS CYLINDER

Run	d (cm)	h (cm)	h / d	φ (Deg.)	μ*
1	30.7	16.9	0.55	33.4	0.66
2	30.3	20.1	0.66	41.6	0.89
3	30.9	14.9	0.48	28.8	0.55
4	30.5	18.5	0.61	37.3	0.76
5	30.9	15.4	0.50	29.9	0.57

*Mean = 0.69 ± 0.14

TABLE 2 h

PLEXIGLASS CYLINDER ON PAINTED PLEXIGLASS CYLINDER

Run	d (cm)	h (cm)	h / d	φ (Deg.)	μ*
1	31.3	12.9	0.41	24.3	0.45
2	31.2	13.3	0.43	25.2	0.47
3	31.2	13.0	0.42	24.6	0.46
4	30.9	14.8	0.48	28.6	0.55
5	30.9	15.2	0.49	29.5	0.56

*Mean = 0.50 ± 0.05

TABLE 2 i

PAINTED STEEL BALLS ON PAINTED STEEL BALLS

Run	d (cm)	h (cm)	h / d	φ (Deg.)	μ*
1	30.6	15.3	0.50	30.0	0.58
2	30.9	15.2	0.49	29.5	0.57
3	31.1	13.7	0.44	26.1	0.49
4	31.1	13.4	0.43	25.5	0.48
5	31.0	14.2	0.46	27.3	0.52
6	30.9	14.5	0.47	28.0	0.53

*Mean = 0.53 ± 0.04

TABLE 3

COEFFICIENT OF FRICTION BETWEEN VARIOUS CYLINDERS AND BALLS AND A FLAT RUBBER SHEET

Material	Coefficient of friction	Standard Deviation
Wooden dowel	0.47	0.03
Steel cylinder	0.40	0.04
Rubber covered steel cylinder	0.92	0.03
Plexiglass cylinder	0.46	0.05
Painted Plexiglass cylinder	0.53	0.05
Painted steel balls	0.77	0.04

TABLE 4 a

WOODEN DOWEL ON RUBBER SHEET

Run	d (cm)	h (cm)	h / d	φ (Deg.)	μ*
1	31.1	13.7	0.44	26.1	0.49
2	31.1	14.3	0.46	27.4	0.52
3	31.3	12.8	0.41	24.1	0.45
4	31.2	13.7	0.44	26.0	0.49
5	31.3	12.7	0.41	23.9	0.44
6	31.2	13.2	0.42	25.0	0.47
7	31.2	13.4	0.43	25.4	0.48
8	31.4	12.4	0.39	23.3	0.43
9	31.4	12.4	0.39	23.3	0.43
10	31.2	13.5	0.43	25.6	0.48

*Mean = 0.47 ± 0.03

TABLE 4 b

STEEL CYLINDER ON RUBBER SHEET

Run	d (cm)	h (cm)	h / d	φ (Deg.)	μ*
1	31.3	12.9	0.41	24.3	0.45
2	31.5	12.1	0.38	22.6	0.41
3	31.8	10.6	0.33	19.5	0.35
4	31.9	10.3	0.32	18.8	0.34
5	31.8	11.0	0.35	20.2	0.37
6	31.3	12.9	0.41	24.3	0.45
7	31.5	12.1	0.38	22.6	0.41
8	31.7	11.1	0.35	20.5	0.37
9	31.6	11.2	0.35	20.8	0.38
10	31.4	12.3	0.39	23.6	0.43

*Mean = 0.40 ± 0.04

TABLE 4 c

RUBBER COVERED STEEL CYLINDER ON RUBBER SHEET

Run	d (cm)	h (cm)	h / d	φ (Deg.)	μ*
1	30.3	20.4	0.67	42.3	0.91
2	30.3	20.9	0.69	43.6	0.95
3	30.3	19.8	0.65	40.8	0.86
4	30.3	20.3	0.67	42.1	0.92
5	30.3	20.4	0.68	43.1	0.94
6	30.3	20.4	0.67	42.3	0.91
7	30.3	20.4	0.67	42.3	0.91
8	30.3	20.9	0.69	43.6	0.95
9	30.3	20.4	0.67	42.3	0.91
10	30.3	20.4	0.67	42.3	0.91

*Mean = 0.91 ± 0.03

TABLE 4 d

PLEXIGLASS CYLINDER ON RUBBER SHEET

Run	d (cm)	h (cm)	h / d	φ (Deg.)	μ*
1	31.0	14.6	0.47	28.1	0.53
2	31.4	12.5	0.39	23.5	0.43
3	31.3	13.1	0.42	24.7	0.46
4	31.5	11.9	0.38	22.2	0.41
5	31.7	11.3	0.36	20.9	0.38
6	31.1	14.0	0.45	26.7	0.54
7	31.2	13.4	0.43	25.4	0.48
8	31.1	14.0	0.45	26.8	0.50
9	31.4	12.4	0.39	23.3	0.43
10	31.2	13.2	0.42	25.0	0.47

*Mean = 0.46 + 0.05

TABLE 4 e

PAINTED PLEXIGLASS CYLINDER ON RUBBER SHEET

Run	d (cm)	h (cm)	h / d	φ (Deg.)	μ*
1	31.0	14.4	0.46	27.7	0.52
2	31.2	13.2	0.42	25.0	0.47
3	31.0	14.9	0.48	28.7	0.55
4	31.1	13.6	0.44	25.9	0.49
5	31.2	13.1	0.42	24.8	0.46
6	30.8	15.4	0.50	30.0	0.58
7	31.0	14.4	0.46	27.7	0.52
8	31.1	14.2	0.46	27.2	0.51
9	30.8	15.8	0.51	30.9	0.60
10	30.8	15.5	0.50	30.2	0.58

Mean = 0.53 ± 0.05

TABLE 4 f

PAINTED STEEL BALLS ON RUBBER SHEET

Run	d (cm)	h (cm)	h / d	φ (Deg.)	μ*
1	30.5	18.8	0.62	38.1	0.78
2	30.6	17.5	0.57	34.9	0.70
3	30.6	17.6	0.58	35.1	0.70
4	30.4	19.1	0.63	38.9	0.81
5	30.4	18.8	0.62	38.4	0.79
6	30.4	18.9	0.62	38.4	0.79
7	30.5	18.4	0.60	37.1	0.76
8	30.4	18.8	0.62	38.2	0.79

*Mean = 0.77 ± 0.04