REPORT TO CRAY RESEARCH, INC.

VELOCITY ANALYSIS AND PRE-STACK REFLECTION MIGRATION IN THE τ–p DOMAIN

by

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ABSTRACT

Seismic data can be transformed to the τ-p domain, i.e., decomposed into plane wave components. The resulting data can be quickly imaged directly to depth using a reflection model. As part of this process, the velocity, thickness and dip of the subsurface geologic units must be known. Conversely, different plane wave components will image to the same depth if the velocity, thickness and dip are correct. We have developed a series of programs to accomplish both the velocity analysis and the imaging using a reflection model and plane wave data.

The first program which was written and optimized for use on the Cray performs the plane superposition and decomposition, i.e., computes the τ-p forward and inverse transform. Both cartesian and cylindrical transforms were optimized for the Cray based on an existing program. (It is critical that this program be efficient as it is the most computationally intensive part of the procedure.) The second program applies a 2D NMO correction to depth or vertical time to the plane wave data based on a velocity, thickness and dip model. NMO corrected τ-p plane wave data or stacks of the imaged plane wave seismogram can be output. A third program was developed for the Cray which develops synthetic τ-p reflection data for a 1D or 2D earth model. This program was written primarily to generate data suitable to test the first 2 programs.

INTRODUCTION

The τ-p domain has been used in various ways to filter and interpret seismic data. Stoffa et al. (1981), exploited the properties of the τ-p domain for wide angle reflection and refraction data to derive interval velocities. Tatham et al. (1984) and Tatham (1984) used the τ-p domain to separate seismic arrivals, e.g., reflections and ground roll, and compressional and shear events. The removal of multiples was also successfully performed in the τ-p domain by Brysk et al. (1986). Treital et al. (1982) used the τ-p domain (actually angle of incidence) to image multi-fold reflection data. Recently, Pan et al. (1988) exploited the τ-p domain for the direct inversion of seismic waveform data.

In most cases, the τ-p domain has been used to either filter seismic data or as a way of deriving seismic interval velocities. Depending on the application, it may not be necessary to preserve seismic waveform amplitude and phase relations and the τ-p transform can be done by simple slant stacking, (Stoffa et al., 1981). In other cases, when the correct amplitude and phase are required, the cylindrical slant stack described by Brysk and McCowan (1986) should be used. Recently Wang and McCowan (1989), have shown that for a 1D earth the proper plane wave decomposition implicitly corrects for spherical spreading. Consequently, true amplitude processing is easily accomplished in the τ-p domain if the original data are adequately sampled in the offset domain.

In this paper we review the problem of seismic velocity analysis for a 1D earth structure in the τ-p domain. We then present methods of velocity analysis for planar dipping layers in both fixed source/receiver and common mid-point coordinates. We concentrate on the 2D case, because the required seismic data are routinely acquired. The methods can be readily extended to 3D if data with sufficient inline and crossline aperture are collected.

The analysis methods described are based on a reflection model of the subsurface not a diffraction model. This makes it possible to easily solve the velocity analysis problem in terms of reflection vertical delay times. The τ-p domain serves as a convenient domain for this analysis, because the problem can be formulated in a way that the required ray tracing is quickly accomplished. This makes it possible to implement the methods described in a workstation environment.

τ–p TRAVEL-TIMES

The vertical delay time, τ, can be constructed geometrically as the time intercept at zero offset of a tangent to a seismic travel time trajectory in the source-receiver offset domain. Diebold and Stoffa (1981), and more recently Diebold (1987), developed the vertical delay time equations for planar dipping layers in 2D and 3D respectively. For a fixed source (or receiver) at position A on the surface of the earth and the receiver (or source) at position B, the 3D travel time equation is:

$$T_n = \vec{p}_b \bullet \vec{X}_n + \tau_n(\vec{p}_b) \quad , \tag{1}$$

where $\vec{p}_b = (p_{b_x}, p_{b_y})$, $\vec{X} = (x,y)$ and $\tau_n(\vec{p}_b)$ is the total vertical delay time.

For a 2D earth model, we can ignore the y dependance and write the vertical delay time contribution as

$$\tau_n(p_b) = \sum_{j=1}^{n} \Delta z_j \, (q_{a_j} + q_{b_j}) \quad . \tag{2}$$

where $q_{a_j} = \cos a_j / v_j$ and $q_{b_j} = \cos b_j / v_j$ are vertical slowness for the downgoing and upgoing ray paths, a_j and b_j are the angles of the down and upgoing rays with respect to the vertical, and $p_b = p_{b_x}$.

For a 1D earth structure the source vertical slowness contributions, q_{a_j}, and the receiver vertical slowness contributions, q_{b_j} are equal and equation 2 reduces to:

$$\tau_n(p) = \sum_{j=1}^{n} 2\Delta z_j q_j \, , \tag{3}$$

where $q_j = q_{a_j} = q_{b_j}$.

The seismic velocity analysis methods we will discuss are based on using the τ-p vertical delay time equations (2 or 3), to derive the thickness and interval velocity of each layer. The methods can be classified based on the seismic events used. For example, if only post critical reflection and refractions are used, the τ-sum method of Diebold and Stoffa (1981), can be used for a 1D earth structure. (An analog also exists for 2D and 3D earth structure, assuming the required up dip and down dip post critical reflections and refractions are observed.) Alternatively, pre- and post-critical reflections or just pre-critical reflections can be used. These are the methods described here.

In the analysis of reflection vertical delay times the methods can be either exact or approximate. The geometry used for the data acquisition, that is whether the data are common source/receiver or common mid-point, as well as the ray parameters used in the analysis, determine whether it is possible to use the approximate methods, or justify the use of the exact ones. For the reflection vertical delay times, the exact methods require that the overlying structure either be known explicitly or otherwise taken into account. That is, the vertical delay time differences between the reflection event being analyzed and the reflection event immediately above it are used to derive the interval velocity and thickness of the layer. In the approximate methods, only the total vertical delay times of the reflection being analyzed are used.

1D τ-p REFLECTION VELOCITY ANALYSIS

For 1D velocity analysis, we use equation 3. First, replace q_j by $(1 - p^2 v_j^2)^{1/2}/v_j$ and then $2\Delta z_j / v_j$ by Δt_j, the two-way normal time in each layer:

$$\tau_n(p) = \sum_{j=1}^{n} \Delta t_j (1 - p^2 v_j^2)^{1/2} \quad (4)$$

To solve for the interval velocity and two way normal time we can measure the delay times of any two reflection events, e.g., τ_n and τ_{n-1} at the same two or more ray parameters. The differences between the vertical delay times, $\Delta\tau_n(p) = \tau_n(p) - \tau_{n-1}(p)$ as a function of ray parameter are used to solve for the interval velocity. For example, if we observe $\Delta\tau_n(p)$ for two ray parameters, e.g., p_k and p_l we can solve for the interval velocity, v_n:

$$v_j = \left[\frac{\Delta\tau^2(p_k) - \Delta\tau^2(p_l)}{\Delta\tau^2(p_k)p_l^2 - \Delta\tau^2(p_l)p_k^2} \right]^{1/2} \quad (5)$$

Alternatively, if we have many $\Delta\tau_n(p)$ measurements, we can do a linear least squares estimate of $\Delta\tau_n^2$ versus p^2. The intercept will be Δt_n, and from the slope, $v_n^2 / \Delta t_n^2$ we can recover the interval velocity, v_n.

Either of the above methods require that we first interpret the τ-p data to identify the delay time of the reflections, i.e., we must 'pick' the event times. This is usually possible, but may be difficult in practice. An alternative is to 'NMO' correct the τ-p data, (Stoffa et al., 1981). That is, given the correct interval velocity function, the n^{th} reflection event can be corrected to its total two way normal time. The τ-p normal moveout correction is:

$$\Delta T_n(p) = T0_n - \tau_n(p) \tag{6}$$

where $T0_n$ is the total two-way normal time,

$$T0_n = \sum_{j=1}^{n} \Delta t_j \quad .$$

Thus,

$$\Delta T_n(p) = \sum_{j=1}^{n} \Delta t_j \left(1 - (1 - p^2 v_j^2)^{1/2}\right) \quad . \tag{7}$$

To actually do the NMO, we resample the discrete τ-p seismic waveform data for each plane wave seismogram, $f(\tau,p)$:

$$F(T0_n,p) = f(\tau,p)\,\delta\left(\tau - \sum_{j=1}^{n} \Delta t_j (1 - p^2 v_j^2)^{1/2}\right) \quad , \tag{8}$$

interpolating as required for discretely sampled data. This process is repeated for all $T0_n$'s and ray parameters of interest to construct the τ-p NMO corrected data, $F(T0_n,p)$.

For a 1D earth, this process is equivalent to imaging the plane wave data directly to depth. Using equation 3, we note that

$$\Delta z_j = \Delta\tau_j(p) / 2q_j \tag{9}$$

and

$$Z_n = \sum_{j=1}^{n} \Delta z_j = \sum_{j=1}^{n} \Delta\tau_j(p)/2q_j \tag{10}$$

The imaging to depth, or two-way normal time, can be done either in the τ-p domain by resampling the data as described by equation 8 or by phase shift followed by an integration over frequency in the ω, p domain. If the interval velocity function used for the imaging is correct, the reflections events for all the plane wave seismograms will be imaged to the same depth or two way normal time.

τ-p NMO is best implemented in a top down fashion. Once the velocity function is found for the section overlying the interval being analyzed, the τ-p data above this reflection can be NMO corrected and the next event's τ-p trajectory will be a single ellipse. At this point several methods can be used to define the residual τ-p delay times for the next event: In a workstation environment, τ-p travel time trajectories can be graphically superimposed on the data in an interactive fashion until visual agreement is reached (effectively 'picking' the next event delay times); the plane wave data can be iteratively τ-p NMO corrected until the event being analyzed is imaged to the same depth or two way normal time for all the plane wave seismograms; or, a two parameter semblance velocity analysis for interval two way normal time and interval velocity can be performed. For any of these approaches we have corrected for the overlying structure, i.e., we have 'layer stripped' or 'downward continued' to just above the layer being analyzed. Therefore, we have isolated the contribution of the current layer, and only a two parameter search is required

If the interval velocity of just one subsurface layer is of interest, it is not necessary to evaluate the entire overlying section. In this case, we can remove the effect of the overlying section by using an approximate velocity to correct the reflection event above the zone of interest. That is, we can do a single ellipse τ-p NMO (see below), followed by a residual statics correction, if necessary. Alternatively, the reflection above the zone of interest can be 'picked' and the data static shifted. Another possibility is to window the data to include only a reference reflection from just above the zone of interest and the reflection from the base of the zone of interest. Then, compute the autocorrelation function for each windowed plane wave seismogram. The lag times will now follow an elliptical trajectory and can now be analyzed to determine the interval velocity of the zone using any of the methods described above.

In some cases, insufficient source-receiver offset coverage and hence a small range of observed ray parameters, may make the above exact analysis unnecessary. For small ray parameters a single ellipse can be used to approximate the true τ-p delay time (Stoffa et al., 1981). In this case, we expand the term $(1-p^2 v_j^2)^{1/2}$ and then normalize using the total two-way normal time:

$$\tau_n(p) = \sum_{j=1}^{n} \Delta t_j (1 - \frac{1}{2} p^2 v_j^2 + \ldots = T0_n (1 - \frac{1}{2} p^2 \sum_{j=1}^{n} \frac{\Delta t_j v_j^2}{T0_n} + \ldots \tag{11}$$

We recognize in the second term the RMS velocity,

$$\overline{V}_n^2 = \frac{\sum_{j=1}^{n} \Delta t_j v_j^2}{T0_n} \tag{12}$$

and the above expansion to second order is equivalent to the expansion for:

$$\tau_n(p) = T0_n (1 - p^2\overline{V}_n^2)^{1/2} \tag{13}$$

By using the relations, $X = -\delta\tau/dp$, $\tau = T - pX$ and $p = dT/dX$, equation 13 can be shown to be exactly equivalent to the hyperbolic travel time equation:

$$T(X) = (T0_n^2 + X^2/\overline{V}_n^2)^{1/2} \quad . \tag{14}$$

Note that, in the above expansion only terms up to $p^2 v_j^2$ or sine squared were maintained. This indicates that when Snell's law ray bending increases to the point that contributions greater than the sine of the incidence angle squared become significant, the τ-p single ellipse delay time and the T(X) hyperbolic travel time approximation will fail.

2D τ-p REFLECTION VELOCITY ANALYSIS

1. Common Source and Common Receiver Profiles

For common source or receiver data we use equation 2 directly. In this case, the thicknesses, Δz_j, are referenced to either the shot or receiver position, whichever is fixed and the ray parameter measured is with respect to the element that changes position along the surface. For example, in land split spread profiles, the thicknesses are referenced to beneath the shot position and the receiver ray parameters are measured from the observed travel time data.

To solve for the interval velocity, thicknesses and dips directly using the vertical delay times is more complicated than in the 1D case. Here, we describe a method that is analogous to the

1D NMO correction, but takes into account dipping layers.To do a τ-p 2D NMO correction for fixed source or receiver profiles is also slightly more complicated. Beginning with a velocity thickness and dip model, we use equation 2 to predict the vertical delay time for each depth. We note two important points. First, we observe p_{b_1} , e.g., the receiver ray parameter in a split spread profile, but we do not observe p_{a_1} , the source ray parameter. Second, we must determine p_{b_2}, p_{b_3}, etc. from p_{b_1} by using Snell's law, the layer velocities and the dip angles. In practice, we split equation 2 into two terms. The first is the sum of the source vertical delay time contribution and the second is the sum of the receiver vertical delay time contributions:

$$\tau_n(p_b) = \tau_{s_n}(p_b) + \tau_{r_n}(p_b)$$

where $\tau_{s_n}(p_b)$ and $\tau_{r_n}(p_b)$ are the source and receiver vertical delay time contributions:

$$\tau_s(p_a) = \sum_{j=1}^{n} \Delta z_j q_{a_j}$$

$$\tau_r(p_b) = \sum_{j=1}^{n} \Delta z_j q_{b_j}$$

For a given velocity, thickness and dip model, we start with the observed surface receiver ray parameter, p_{b_1}, and downward continue it through each layer using Snell's law at each interface computing each p_{b_j} in sequence and accumulating the receiver vertical delay times. We then compute the reflection angle at the n^{th} interface followed by the source ray parameter, p_{a_n}. Now we can accumulate the source vertical delay times as we upward continue the source ray parameter to the surface using Snell's law. (As part of this process, we find p_{a_1}, but this is not of interest in this application.) In practice, we do this 'ray tracing' very quickly because we always downward continue the previous layer's receiver ray parameter and add the next receiver vertical delay time contribution to the sum of the previous contributions. The source contributions must, however, be explicitly computed because the reflection angle at the next interface will result in a different suite of source ray parameters. This procedure is the same as ray tracing the 2D structure, but no iterations are required to find the exact vertical delay times. This is in contrast to T(X) methods which are either approximate or require iteration to find the correct travel time at a specified source-receiver offset.

As in the 1D τ-p velocity analysis, each plane wave seismogram can be downward continued to the depth under the shot point, Z_n, or equivalently the vertical time:

$$T_{v_n} = 2\sum_{j=1}^{n} \Delta z_j / v_j \ .$$

Using the above procedure for a trial velocity, thickness and dip model, the 2D NMO correction is again performed by resampling the discrete τ-p seismic waveforms for each plane wave seismogram. Imaging to depth under the shot point position we have:

$$G(Z_n, p) = f(\tau, p)\, \delta(\tau - \sum_{j=1}^{n} \Delta z_j\, q_{a_j} - \sum_{j=1}^{n} \Delta z_j\, q_{b_j})$$

This analysis method can be used to find the correct velocity, thickness and dip model by doing the NMO correction in a top down fashion, layer by layer. The correct layer parameters are defined when the same reflection events as observed on all the plane wave seismograms are imaged to the same depth (or vertical time).

2. Common Mid-point Profiles

For common midpoint data, we can simplify the above development. However, we now must refer the layer thickness to the mid-point of the profile, Δz_{mp_j}, and the ray parameter we observe is $p_{mp} = (p_{a_1} + p_{b_1})/2$ Diebold and Stoffa, 1981. For 2D CMP velocity analysis, we again consider the interval delay time between layers. First, we rewrite equation 2 as:

$$\tau_n(p_{mp}) = \sum_{j=1}^{n} \Delta z_{mp_j} (q_{a_j} + q_{b_j}) \tag{15}$$

Substituting for q_{a_j} and q_{b_j} we have:

$$\tau_n(p_{mp}) = \sum_{j=1}^{n} 2\Delta z_{mp_j} \cos\omega_j \cos\gamma_j \,/v_j \tag{16}$$

where γ_j is the reflection angle at each interface. Defining $p_{\gamma_j} = \sin\gamma_j/v_j$ as the ray parameter referenced to the j[th] interface and noting that $\Delta t_{mp_j} = 2\Delta z_{mp_j} \cos\omega_j/v_j$ is the two way normal time in the j[th] layer referenced to the mid-point position we can write:

$$\tau_n(p_{mp}) = \sum_{j=1}^{n} \Delta t_{mp_j} (1 - p_{\gamma_j}^2 v_j^2)^{1/2} \tag{17}$$

In a common mid-point profile we measure p_{mp}, not p_{γ_j}. But, we can compute p_{γ_j} for a given velocity and dip model by using Snell's law and p_{mp}. Since $p_{mp} = (p_{a_1} + p_{b_1})/2$ and at the interface $\gamma_1 = b_1 + \omega_1 = a_1 - \omega$, we have:

$$p_{mp} = \sin(\gamma_1 + \omega_1)/2v_j + \sin(\gamma_1 - \omega_1)/2v_j$$

$$= \sin\gamma_1 \cos\omega_1 / v_1 \tag{18}$$

$$= p_{\gamma_1} . \cos\omega_1 \quad \text{or ,} \tag{19}$$

$$p_{\gamma_1} = p_{mp} / \cos\omega_1$$

It is also possible to develop an approximation for equation 2 that is similar to the 1D single ellipse, equation 13, for the case of common mid-point data.

Expanding as in equation 11 we find:

$$\tau_n(p_{mp}) \cong \sum_{j=1}^{n} \Delta t_{mp_j} \left(1 - \frac{1}{2} p_{\gamma_j}^2 v_j^2 + \ldots = T0_{mp_n} \left(1 - \frac{1}{2} \sum_{j=1}^{n} \frac{\Delta t_{mp_j} p_{\gamma_j}^2 v_j^2}{T0_{mp_n}} + \ldots \right.\right. \tag{20}$$

dropping all higher order terms and defining

$$T0_{mp_n} = \sum_{j=1}^{n} \Delta t_{mp_j} \tag{21}$$

and

$$\gamma_{mp_n}^2 = \frac{\sum_{j=1}^{n} \Delta t_{mp_j} p_{\gamma_j}^2 v_j^2}{T0_{mp_n}} \tag{22}$$

we have

$$\tau_n(p_{mp}) \cong T0_{mp_n} (1 - \gamma_{mp_n}^2)^{1/2} \tag{23}$$

For a 1D earth equation 23, reduces to equation 13 immediately. Consider now the one layer case:

$$\tau_1(p_{mp}) = T0_{mp_1}\,(1 - p_{mp}^2\, v_1^2/\cos^2\omega_1)^{1/2} \tag{24}$$

where we have substituted for $\gamma^2_{mp_1}$ in terms of the observed ray parameter p_{mp}. Using the relations $X = -\,\delta\tau(p_{mp})/\delta x$ and $\tau = T - p_{mp} \cdot X$ we have:

$$T(x) \cong (T0^2_{mp} + \frac{X^2\cos^2\omega_1}{v_1^2})^{1/2} \quad , \tag{25}$$

which is the expected single dipping layer travel time equation.

The implication of equations (17) and (23) for the correct normal moveout of common mid-point is obvious. For a known velocity, thickness and dip, the data can be either exactly, equation 17, or approximately, equation 23, corrected to either depth under the mid-point or the total two way normal time as defined by equation 21. The calculation of the p_{γ_j} terms required for either the exact or approximate 2D NMO equations is a straightforward and fast calculation that can be performed as part of the NMO process:

$$p_{\gamma_{j+1}} = \sin[\sin^{-1}[(v_{j+1}/v_j)] - \omega_j + \omega_{j+1}]/v_{j+1} \tag{26}$$

It is also clear that for a multi-layer dipping model that the p_{γ_j}'s must change and this makes it difficult to develop even an approximate algorithm that works in the T-X domain except for the case of a single dipping layer. In all other cases we must trace rays through the structure to determine the proper 2D T(X) NMO correction. This is implicitly done in either the exact or approximate τ-p 2D NMO, equations (17) and (23) by changing the ray parameter along each interface based on the dip angles. For either case, the NMO is accomplished by resampling the discrete τ-p seismic waveform data for each plane wave seismogram, $f_{mp}(\tau,p)$:

$$F_{mp}\,(T0_n,p_{mp}) \;=\; f_{mp}(\tau,p_{mp})\;\;\delta(\tau - \tau_n(p_{mp}))$$

where $\tau_n(p_{mp})$ is defined by either equations 17 or 23 and the velocity, thickness, and dip model.

As described earlier, this 2D NMO procedure can be performed iteratively until the reflection events for all the ray parameter seismograms are downward continued to the same depth or two-way normal time. Thus, the velocity analysis procedures are similar for 1D and 2D shot point and common mid-point data. The differences are in the underlying τ-p equation used for the analysis and whether dip is required as a model parameter.

VELOCITY ANALYSIS EXAMPLE

An interactive τ-p velocity analysis program was written for a Sun computer and Landmark workstation. Plane wave data generated on the Cray are transferred either by tape or over the network to the Sun or Landmark for analysis. The interpreter first defines the τ-p reflection event time and then interactively changes the velocity and dip for the event. As the model changes the τ-p reflection trajectories are superimposed on the data display. Once reasonable agreement is achieved, the data can be NMO corrected using the algorithm appropriate for the data being analyzed as described above. The geophysical analysis can now proceed again in the NMO domain. The procedure is performed in a top down fashion, deriving the model parameters for the shallowest layers and then the deeper layers in succession. The data can be windowed in vertical delay time and ray parameter to suit the interpreter's needs. Once the interpreter completes the analysis, the model parameters are saved for future use.

A program to do the plane wave decomposition, i.e., the τ-p transform, using a Cray computer was also written and optimized. Both synthetic and real data have been transformed using this program. For illustration, the data of Figure 1a were generated using a finite difference algorithm and the earth model in Figure 1b. The data were then transformed to the τ-p domain, Figure 1c. Because of the dipping layers, the τ-p trajectories deviate from ellipses centered about p = 0. These data were 2D NMO corrected to vertical time under the shot point position, Figure 1d. (Vertical time is the cumulative time it would take for a wave to travel vertically through each layer and back. This is the sum of the source and receiver vertical delay times discussed earlier.) In this example, the correct velocity, thickness and dip model was used so the plane wave seismogram reflection arrivals are imaged to the same vertical time (or equivalently depth). A similar suite of diagrams illustrate the process applied to 240 channel real data, collected of the east coast of the United States by the GECO *My* in 1988. Figure 2a is the original shot gather, Figure 2b is the τ-p transform, and Figure 2c is the data moved out with interval velocities obtained after interpretation of an x-t stacking velocity analysis.

IMAGING

Although knowledge of the velocity-depth model which best fits the data is an important goal of seismic data processing, it is also desirable to make an image of the subsurface directly from the seismic waveforms. If the local earth structure can be approximated by planar dipping layers, imaging can be done quite easily once the data are in the τ-p domain. The imaging procedure is a natural extension of the NMO velocity analysis. During the iterative velocity analysis the data is moved out to either vertical time or depth. Since the velocity structure is known when the τ-p move out analysis is complete, the data can easily be displayed in either vertical time or depth. Simply stacking the moved out data will result in a trace where large amplitudes will exist at the depths corresponding to the depths of the layers beneath the shot. If this trace is placed beneath the shot location in an image space, and the same is done with other nearby shots, the result will be an image of the subsurface. This is the equivalent of a pre-stack reflection migration.

The major computational work is done in the original τ-p transform and the 2D NMO is (by comparison) not computationally intensive.

To illustrate this imaging procedure, note the ray diagram in Figure 3a. Every 10th ray path for shot point 50 in a model of 100 shot points is displayed. When a moved out τ-p reflection is stacked, the energy reflected from point A through point C is summed and the result is placed at point B. Point B is at the true depth (or vertical) time of the reflector directly beneath the shot point. Waveforms are never moved away from the reflector by improper sorting or stacking. (In x-t the data that we use are common-mid-point profiles and the summed energy for a dipping layer structure are placed at the normal incidence point which lies up dip and at an early time. This is why migration is required to correct the CDP stacked image.) In Figure 3b, fixed source (or receiver) not common-mid-point data, are 2D τ-p moved out and then stacked. The positions of the reflectors are imaged correctly.

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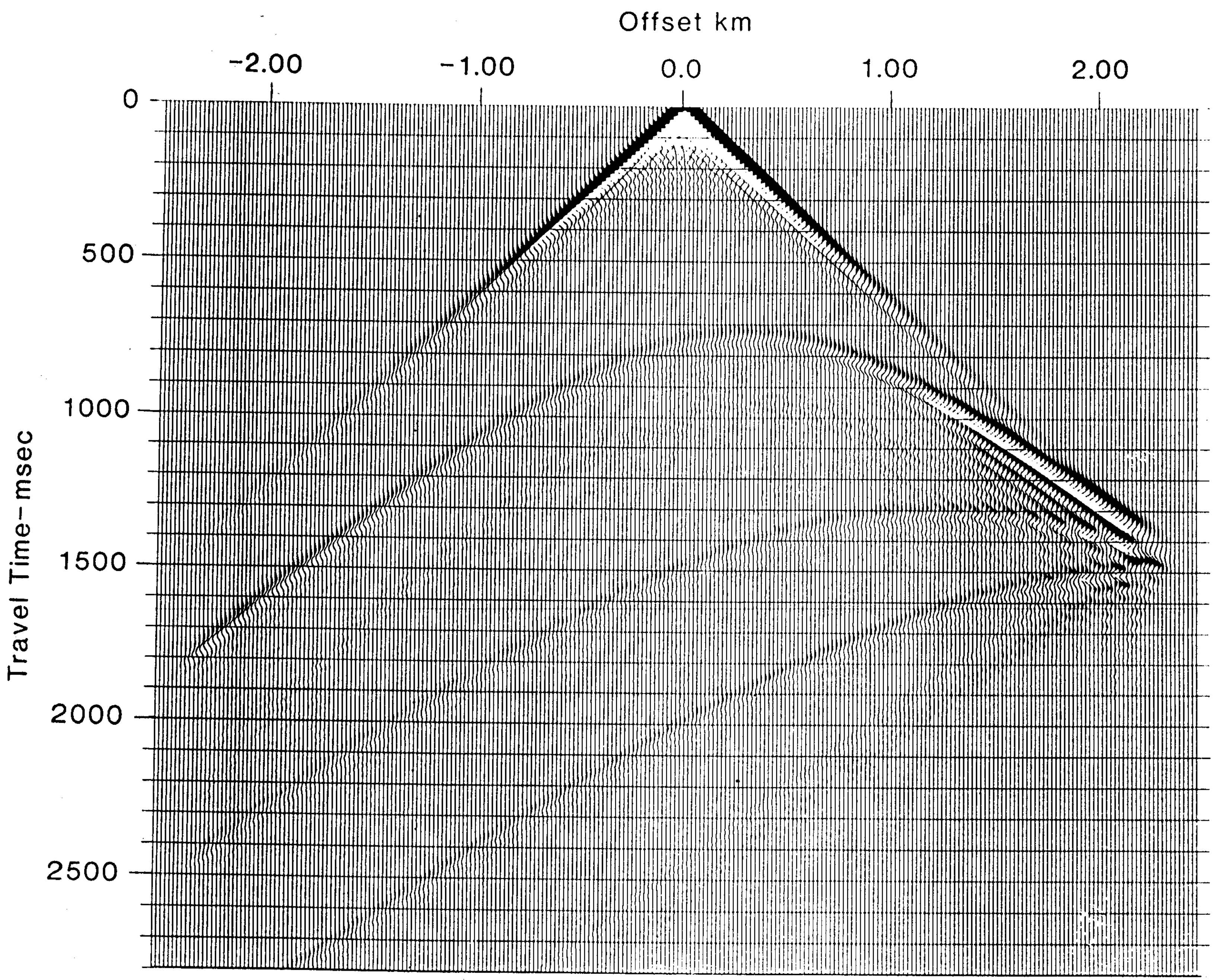

Figure 1a. Synthetic seismic data generated using a finite difference algorithm and the 3 layer earth model of Figure 1b.

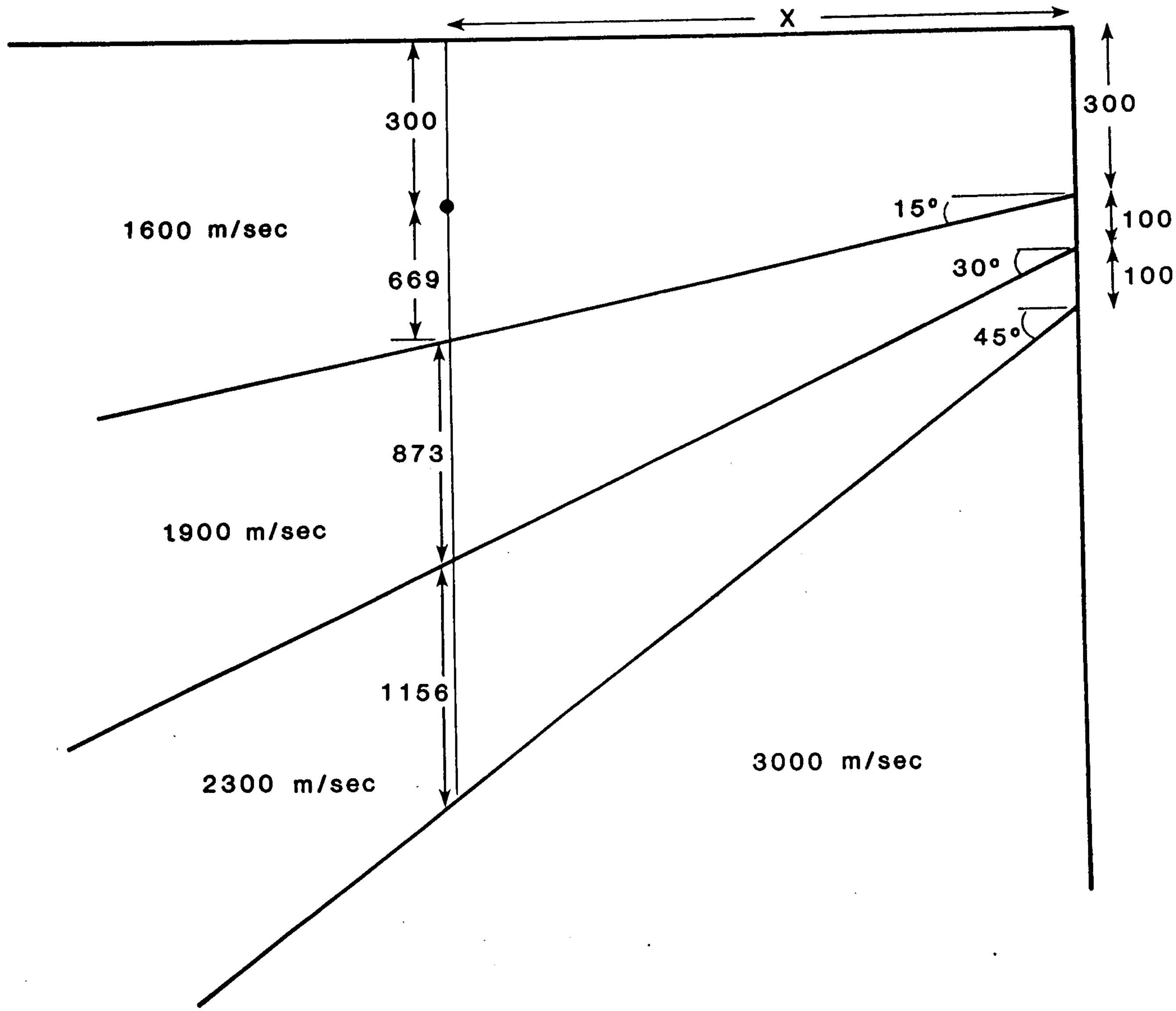

Figure 1b. Earth model consisting of 3 isovelocity layers with dips of 15°, 30°, and 45 °, over a half space.

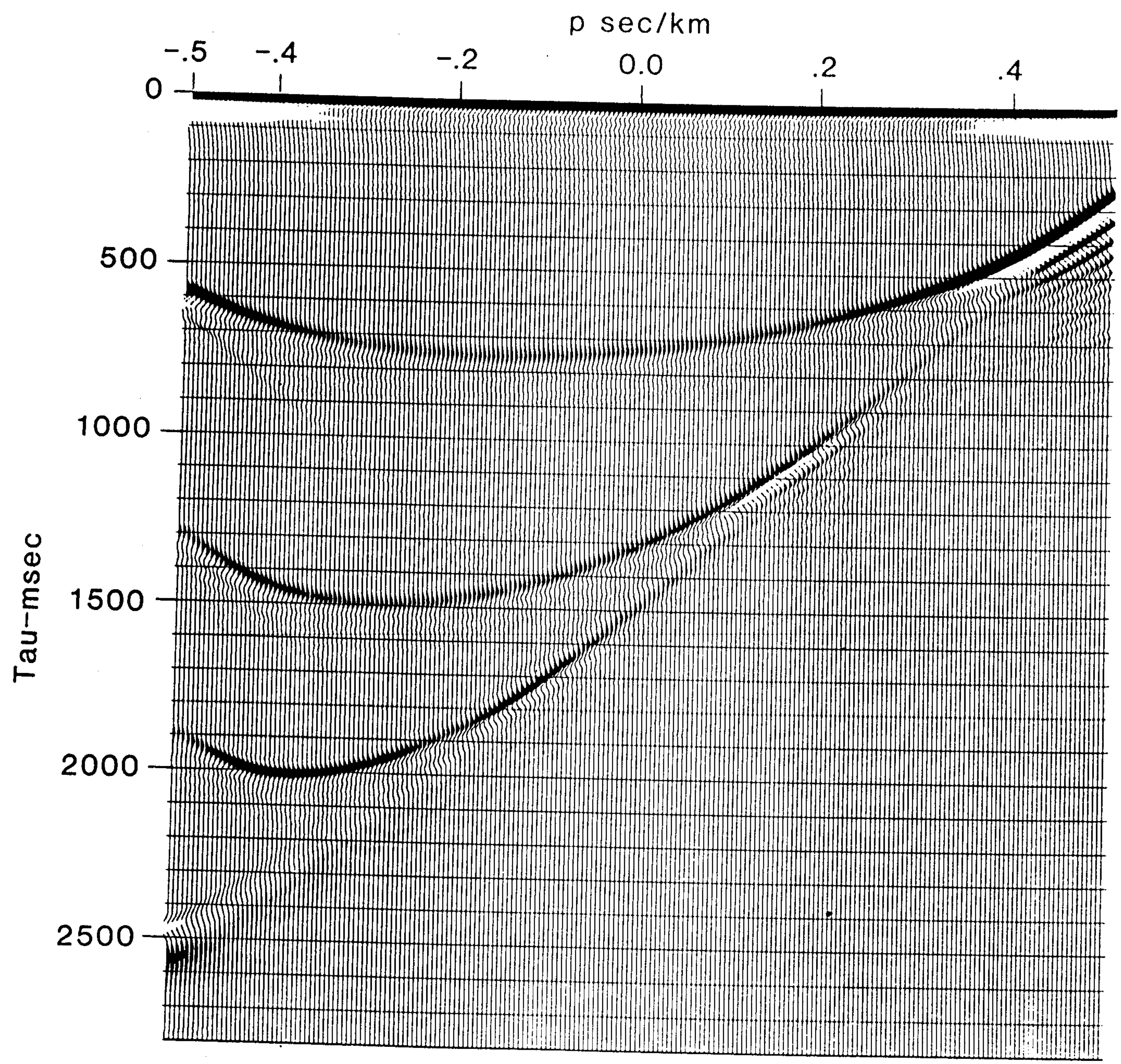

Figure 1c. τ-p transform of the finite difference data of Figure 1a.

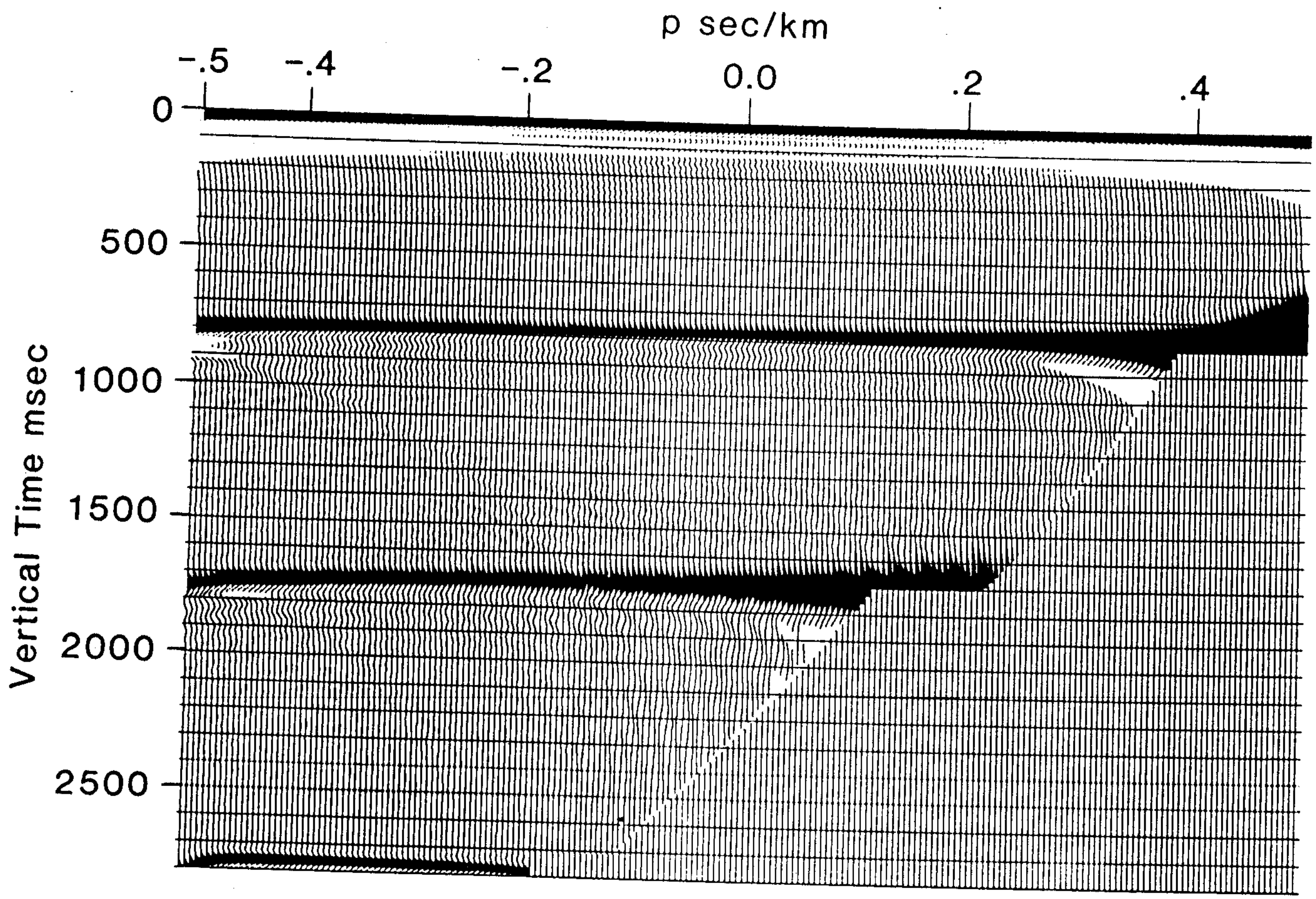

Figure 1d. τ-p data of Figure 1c after application of the 2D τ-p normal moveout. In this case the data were corrected to vertical time. This is the equivalent of a pre-stack migration of these data.

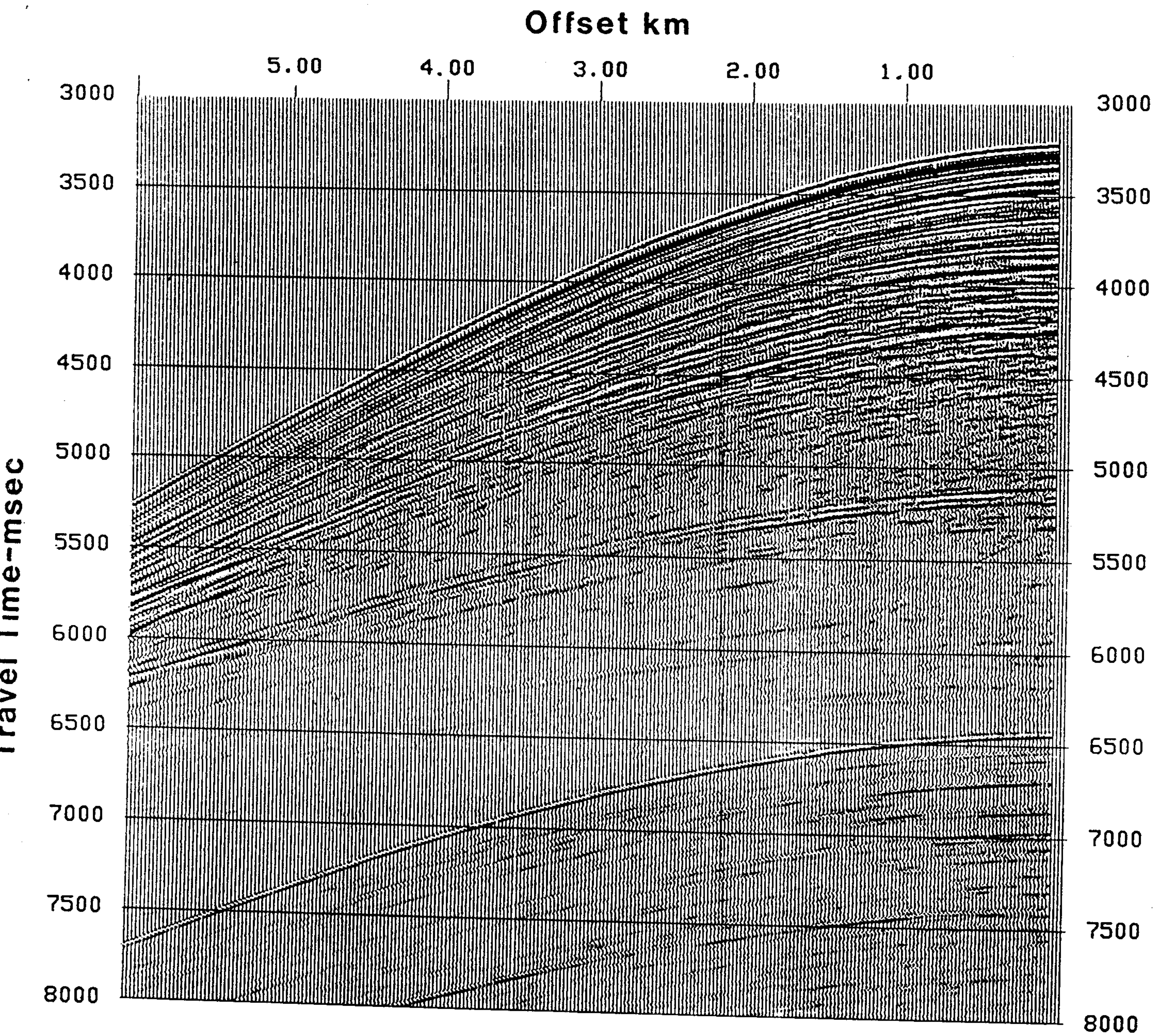

Figure 2a. A shot gather collected with a 240 channel, 6 km receiving array off the east coast of the United States.

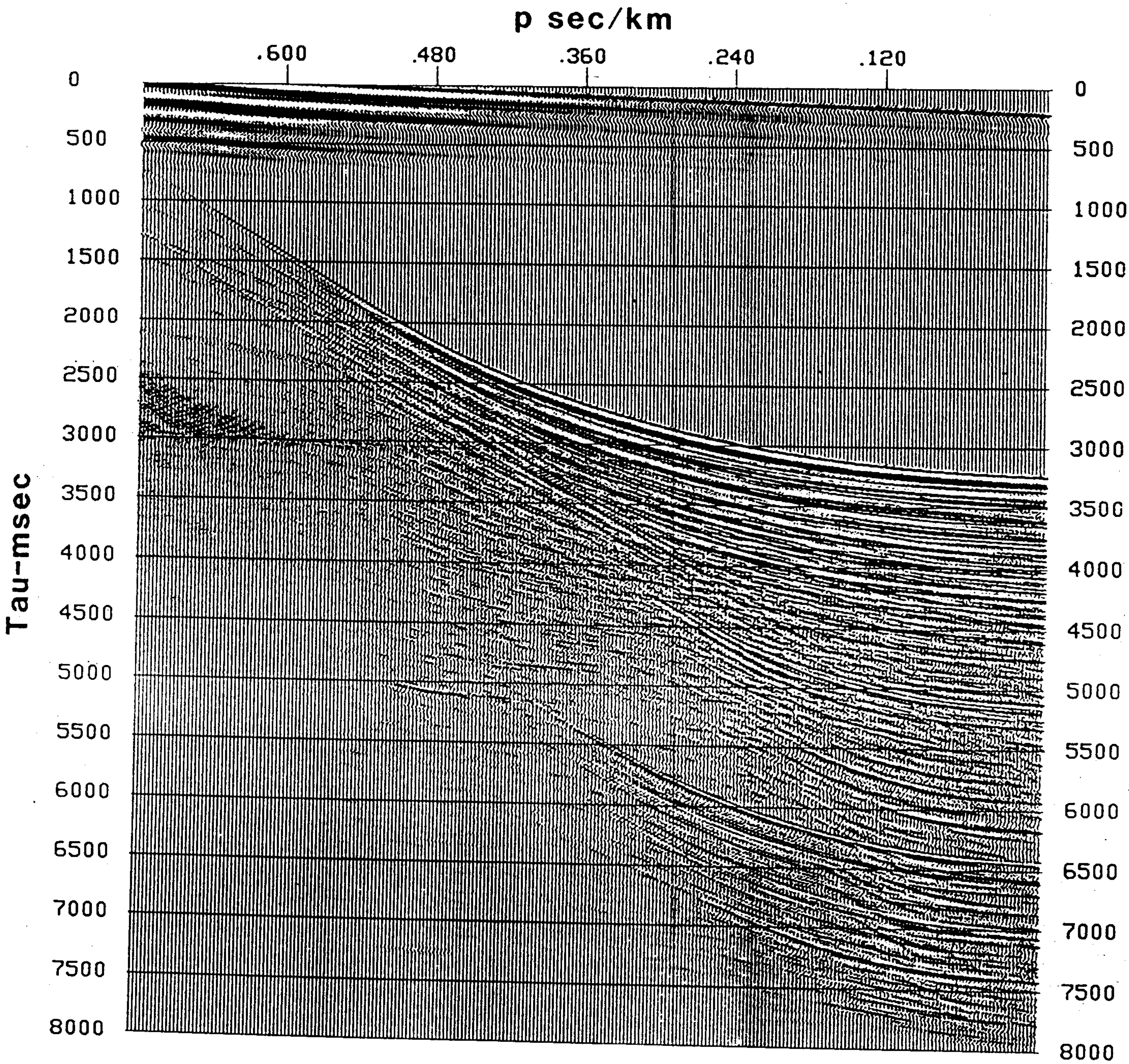

Figure 2b. Data of Figure 2a after a line source τ-p transform. Note the water bottom multiple at 6450 ms.

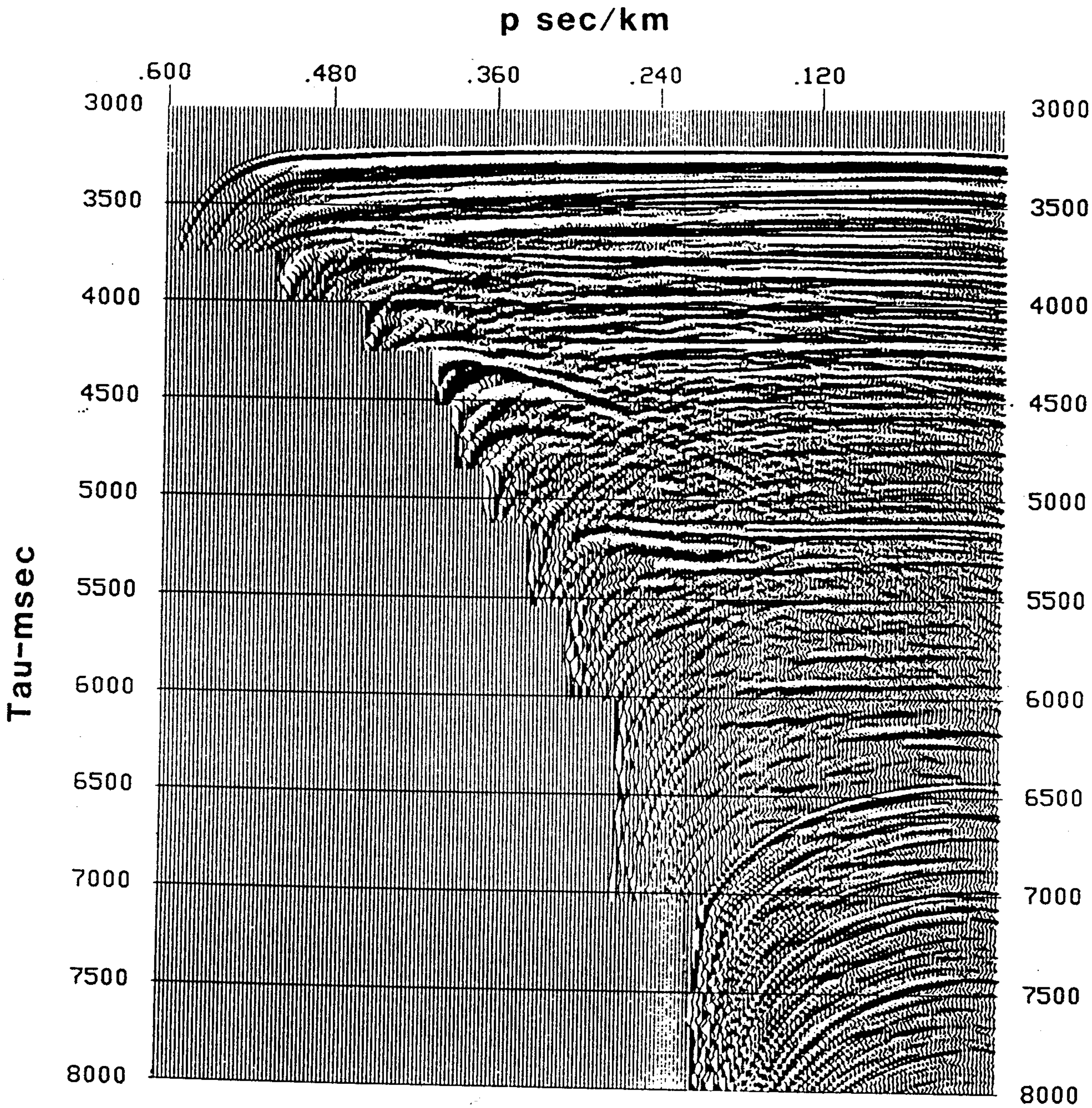

Figure 2c. Data of Figure 2b after the 2D τ-p NMO. Velocities were obtained from a nearby velocity analysis in x-t and were converted to interval velocities for the τ-p NMO. Note the under corrected water bottom multiple.

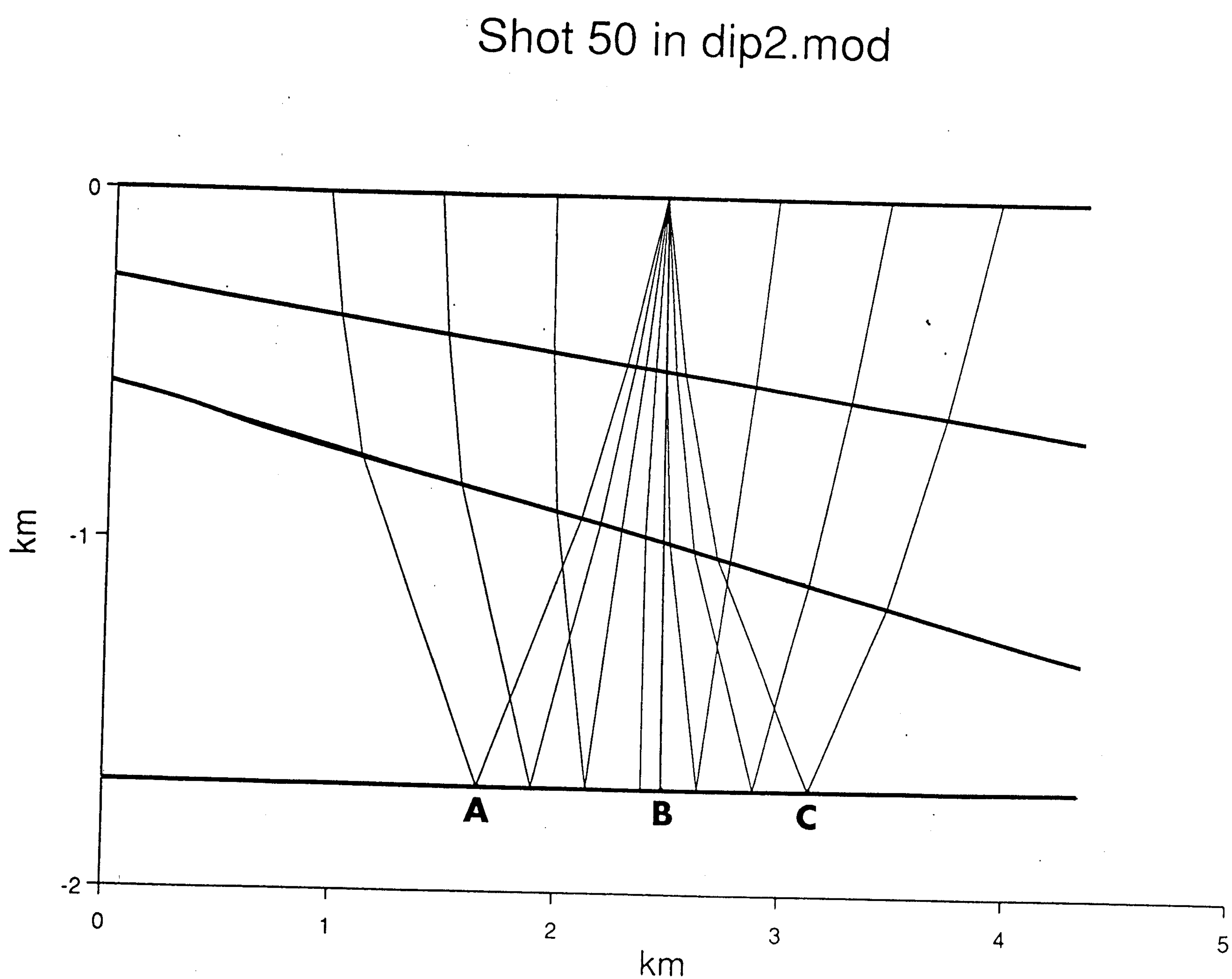

Figure 3a. Ray path diagram showing every 10th ray path of shot point 50 in a dipping layer model of 100 shots. To produce a pre-stack reflection image, the seismic energy reflected from points A through C are summed and placed at B, the true depth (or vertical time) beneath the shot point.

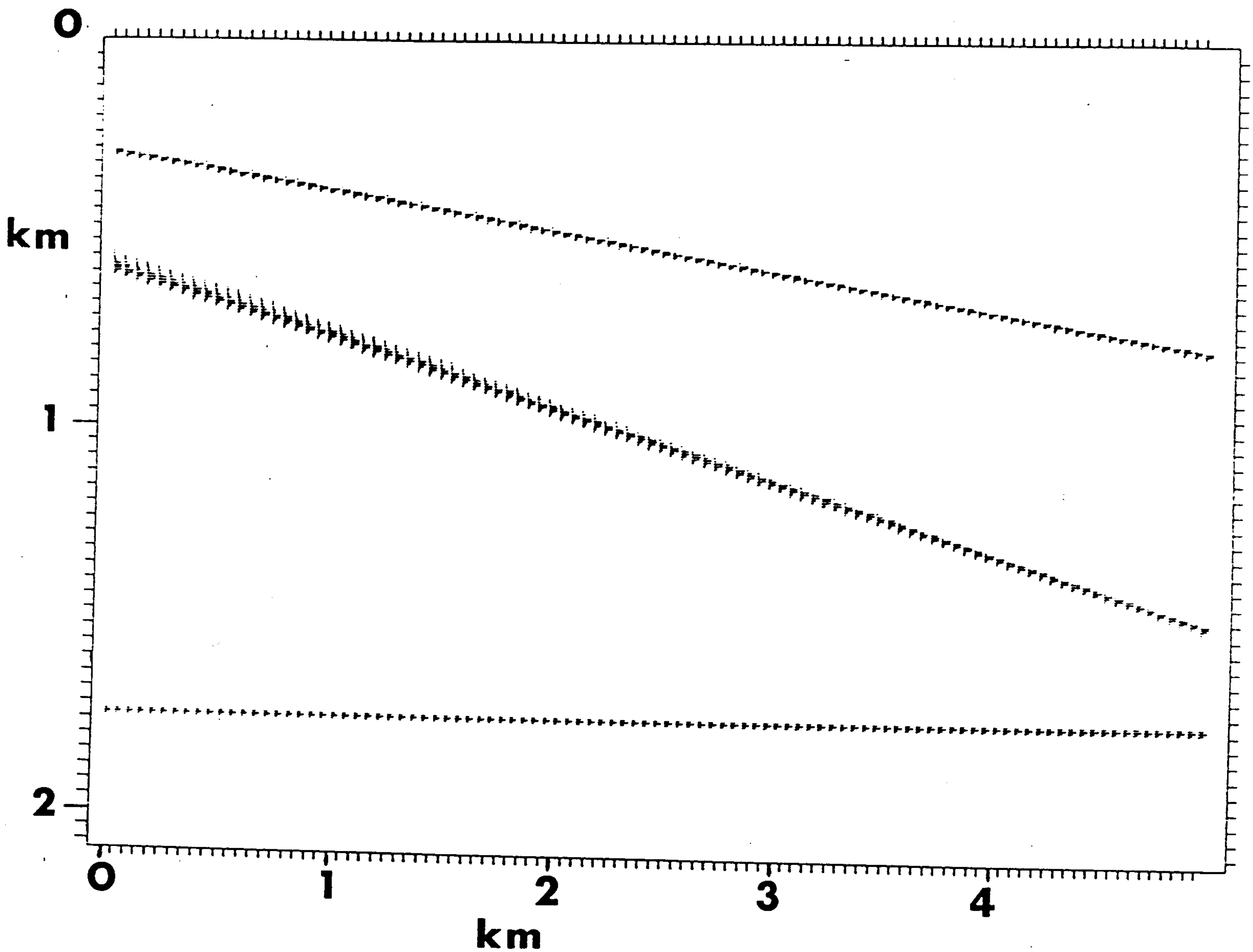

Figure 3b. Since the energy is placed at the correct depth or vertical time, the layers are immediately imaged to the correct position and do not need to be migrated further.