USC GEOL557: Numerical Modeling of Earth Systems Spring 2016

Instructor

Prof. Thorsten W. Becker (ZHS269; (213)740-8365; twb@usc.edu), with help from Dr. Rob Porritt

Class Logistics

Class/lab Times: Tue and Thu 12:30-2 pm

Locations: ZHS264.

Software/harware: Students need to install `Matlab` on their laptops *before class starts* to be able to do classwork. (The new computer lab, ZHS100, only has a few PCs with Matlab installed, sorry).

Notify the instructor if you do not have your own laptop, we can arrange loaners.

Outline

This class discusses the numerical solution of problems arising in the quantitative modeling of Earth systems. The focus is on continuum mechanics problems as applied to geological processes in the solid Earth, but the numerical methods have broad applications including in geochemistry or climate modeling. We briefly review math and continuum mechanics fundamentals, then discuss ordinary differential equations (ODEs), and spend the majority of the class covers finite difference (FD) and finite element (FE) solutions to partial differential equations (PDEs).

The class is for advanced undergraduates and graduate students from the Earth sciences, and consists of lectures and joint and individual computer lab exercises. Grading is based on home work/programming project assignments, a final project, and class participation.

- Students have to pick a final project in week ten, after consultation with the instructor, and are encouraged to start work early, in week 12 at the latest.
- Problem sets are due one week after they were issued, Thursday after class. *No late problem sets will be permitted.*
- The final project (in terms of a 15 page, PDF writeup along with Matlab routines to run the computations, to be submitted electronically) is due on May 3. *No late projects will be accepted, nor will extensions be granted.*

- Prerequisites: Familiarity with `Matlab`. Recommended: GEOL425, GEOL440, GEOL534, GEOL540, Calc-I, II, III, Linear Algebra.
- Grading
  - 50% home work assignments
  - 40% final project
  - 10% class participation

Syllabus

In the following syllabus, all reading and assignments refer to the lecture notes by Becker and Kaus (2016), available from `geodynamics.usc.edu/~becker/Geodynamics557.pdf`, and updated throughout the course. Individual assignments can also be downloaded from the course website, `http://geodynamics.usc.edu/~becker/teaching-557.html`.

Week 1: Jan 12

Introduction: Numerical modeling in the Earth Sciences. Overview of numerical methods. Example problems. Ordinary differential equations. Finite difference, finite element, spectral element solution for PDEs.

Overview of scientific computing: Computer hardware. Computer Languages. Principles of Programming.

Reading: Sections 2.1 to 2.3. Review content of chapters 6 to 8.

Problem set: Scaling analysis (section 2.4)

Week 2: Jan 19, 21

Ordinary differential equations I: Definition of ODEs. Initial value problems. Euler method. Taylor expansions. Accuracy of numerical methods. Midpoint method.

Ordinary differential equations II: 4th order Runge Kutta.

Reading: Chapter 3

Problem set: Program and solve Lorentz equations (section 3.3)

Week 3: Jan 26, 28

Finite differences I: Approximations to derivatives. Accuracy. 1-D heat equation. Explicit solution of diffusion problems. Stability.

Finite differences II: Implicit methods. Crank-Nicolson method. Order of spatial and temporal accuracy. Stability conditions. Neumann and Dirichlet boundary conditions. Sparse matrices, triangularity. Linear systems of equations. Heat equation in 1-D.

Reading: Sections 4.1 to 4.5

Problem set: Explicit FD, Implicit FD methods (sections 4.2 and 4.4)

Week 4: Feb 2, 4

Finite differences III: Non-linear equations. Darcy flow equation for pressure-dependent diffusivity.

Finite differences IV: Two-dimensional heat equation, solution with fully explicit and fully implicit methods. Comparison with analytical solutions.

Reading: Sections 4.6 and 4.7

Notes/problem set: 2-D FD heat equation (section 4.7.4)

Week 5: Feb 11

Finite differences V: Advection equation for heat transport. FTCS method and stability. Lax method, Courant criterion. Upwind schemes. Staggered leapfrog.

Notes/problem set: Section 4.8

Problem set: Section 4.8.3, exercises 1-4

Week 6: Feb 16, 18 (Rob Porritt)

Finite differences VI: Finite differences in seismology. Staggered grid.

Reading: Section 4.11

Problem set: Wave propagation (section 4.11)

Week 7: Feb 25

Finite differences VII: Semi-Lagrangian methods. Advection-diffusion combos in 2-D, operator splitting.

Notes: Section 4.8

Problem set: Semi-Lagrangian in 2D, section 4.8.3, exercises 5-8

Week 8: Mar 1, 3

Finite elements I: Introduction to the finite element method. Strong and weak forms of PDEs. Discretization of domains into finite elements. Shape functions. Bilinear forms. Variational approaches, virtual work. Galerkin method.

Finite element II: Solution of large linear systems of equations, direct and iterative methods. LU decomposition, Cholesky. Jacobi, Gauss-Seidel, Conjugate gradient, and multigrid methods. Sparseness and banded ness. Node ordering and bookkeeping.

Reading: Sections 5.1 and 5.2

Problem set: 1-D heat with FE (section 5.3)

Week 9: Mar 8, 10

Finite elements III: Local and global coordinate systems. Change of variables during integration. Matrix assembly.

Finite elements IV: 2D boundary value problems. Isoparametric elements. Jacobian; global and element-local coordinates. Numerical integration using Gauss quadrature. Triangular and quadrilateral shape functions. Meshing using triangles. Solution of 2-D heat equation.

Reading: Section 5.5

Problem set: 2-D FE heat equation (section 5.6)

Week 10: *Spring break*

Week 11: no class

Week 12: Mar 31

Finite elements V: Compressible elastic problems. Elastic moduli, plane stress, plane strain. Gradient operator, elasticity matrix, engineering strain convection. Visualization of stress states, eigensystems.

Problem set: 2-D FE elastic (section 5.7)

Week 13: Apr 5, 7

Finite elements VI: Compressible and incompressible elasticity and Stokes flow. Mixed formulation with discontinuous pressure. Powell-Hestenen iterations.

Finite elements VII: Time-dependent problems

Reading: Sections 5.8 and 5.10

Problem set: 2-D FE incompressible Stokes (section 5.9)

Week 14: Apr 14

Solving large systems of equations.

Reading: Section 5.4

Problem set: Multigrid, section 5.4, exercises 1-3

Week 15: Apr 19, 21

Collaborative work on projects.

Week 16: April 26, 28 Collaborative work on projects.

Final projects are due May 3

Textbooks and notes

The main reference for this class are our lecture notes.

- Becker, T. W. and Kaus, B. J. P.: *Numerical Modeling of Earth Systems. An introduction to computational methods with focus on solid Earth applications of continuum mechanics.* Lecture notes (221 pages). University of Southern California, Los Angeles, 2016.

  Available online at `http://geodynamics.usc.edu/~becker/Geodynamics557.pdf`, with accompanying Matlab exercises at `http://geodynamics.usc.edu/~becker/teaching-557.html`, continuously updated throughout the class.

No text book is required, but the following openly available sources contain required reading:

- Zhong et al. (2007)
- Spencer, R. L. and Ware, M.: Introduction to Matlab, Brigham Young University, 2006. Available online at `http://www.physics.byu.edu/faculty/spencer/physics430/matlab.pdf`, accessed 09/2011.
- Spiegelman, M. Myths and Methods in Modeling. Columbia University, 2004. Available online at `http://www.ldeo.columbia.edu/~mspieg/mmm/course.pdf`, accessed 09/2011.

Recommended textbooks include:

- *Numerical Recipes* by Press et al. (1993), 2^{nd} or 3^{rd} edition
- *The finite element method* by Hughes (2000), for details on finite elements (FE).

  Other good texts on the finite element method include Kwon and Bang (1996), which provides a a clear step-by-step introduction to the FEM method with many Matlab program examples. The classic by Bathe (2007) also makes for a good reference.
- Ismail-Zadeh and Tackley (2010) for a broader, less-detailed overview of general methods in computational geodynamics.
- Gerya (2009) for more details on finite difference approaches to geodynamics.

Statement for Students with Disabilities

Any student requesting academic accommodations based on a disability is required to register with Disability Services and Programs (DSP) each semester. A letter of verification for approved accommodations can be obtained from DSP. Please be sure the letter is delivered to me (or to TA) as early in the semester as possible. DSP is located in STU 301 and is open 8:30 a.m.5:00 p.m., Monday through Friday. The phone number for DSP is (213) 740-0776.

Statement on Academic Integrity

USC seeks to maintain an optimal learning environment. General principles of academic honesty include the concept of respect for the intellectual property of others, the expectation that individual work will be submitted unless otherwise allowed by an instructor, and the obligations both to protect one's own academic work from misuse by others as well as to avoid using another's work as one's own. All students are expected to understand and abide by these principles. Scampus, the Student Guidebook, contains the Student Conduct Code in Section 11.00, while the recommended sanctions are located in Appendix A: `http://www.usc.edu/dept/publications/SCAMPUS/gov/`. Students will be referred to the Office of Student Judicial Affairs and Community Standards for further review, should there be any suspicion of academic dishonesty. The Review process can be found at: `http://www.usc.edu/student-affairs/SJACS/`.

References

Bathe, K.-J. (2007). *Finite Element Procedures*. Prentice-Hall, London.

Becker, T. W. and Kaus, B. J. P. (2016). *Numerical Modeling of Earth Systems. An introduction to computational methods with focus on solid Earth applications of continuum mechanics*. University of Southern California, Los Angeles. Lecture notes (224 pages), available online at `http://www-udc.ig.utexas.edu/external/becker/Geodynamics557.pdf`, accessed 09/2017.

Gerya, T. (2009). *Introduction to Numerical Geodynamic Modelling*. Cambridge University Press, Cambridge UK.

Hughes, T. J. R. (2000). *The finite element method*. Dover Publications.

Ismail-Zadeh, A. and Tackley, P. (2010). *Computational Methods for Geodynamics*. Cambridge University Press.

Kwon, Y. W. and Bang, H. (1996). *The Finite Element Method Using Matlab*. CRC Press.

Press, W. H., Teukolsky, S. A., Vetterling, W. T., and Flannery, B. P. (1993). *Numerical Recipes in C: The Art of Scientific Computing*. Cambridge University Press, Cambridge, 2 edition.

Zhong, S. J., Yuen, D. A., and Moresi, L. N. (2007). Numerical methods in mantle convection. In Schubert, G. and Bercovici, D., editors, *Treatise in Geophysics*, volume 7, pages 227–252. Elsevier.