MYRES on
Heat, Helium, Hotspots, and Whole Mantle Convection

Dynamics of Thermal Boundary Layers and Convective Upwellings

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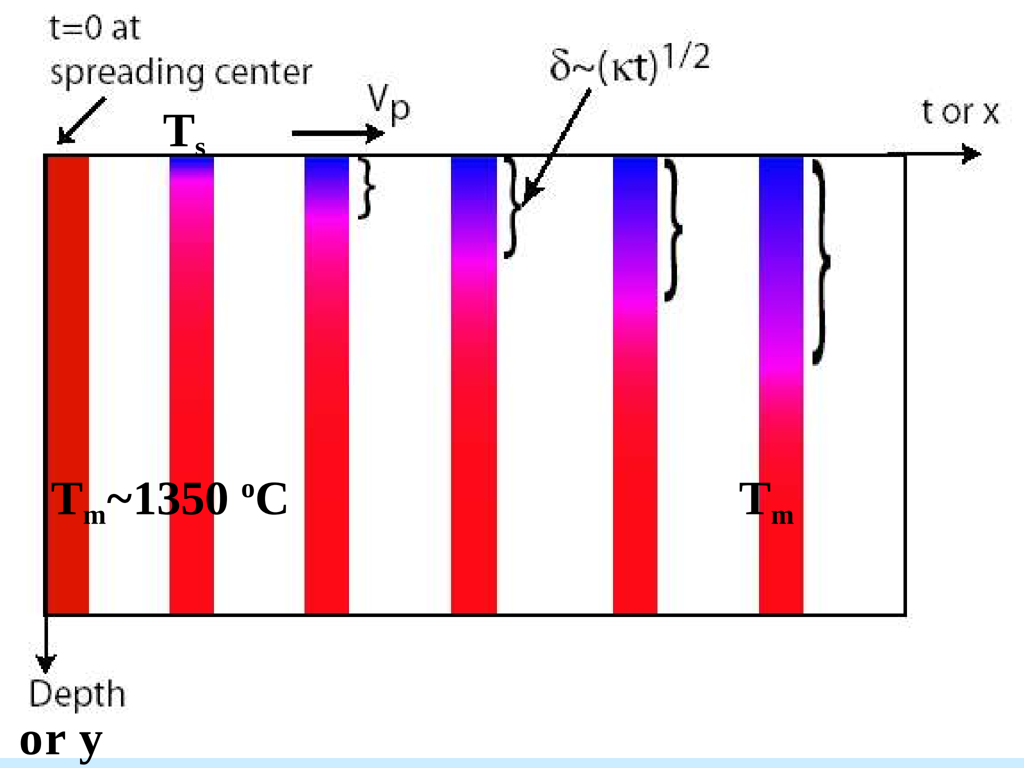
Outline

1. Introduction.
    a) Thermal boundary layers (TBL) and their dynamics.
    b) Layered versus whole mantle convection and heat budget.
    c) Plume heat flux.
5. Plume population and heat transfer.
6. Conclusions and remaining issues.

What is a thermal boundary layer (TBL)?

- ***A layer across which there is a significant temperature difference and the heat transfer is primarily via heat conduction, for example, the oceanic lithosphere.***



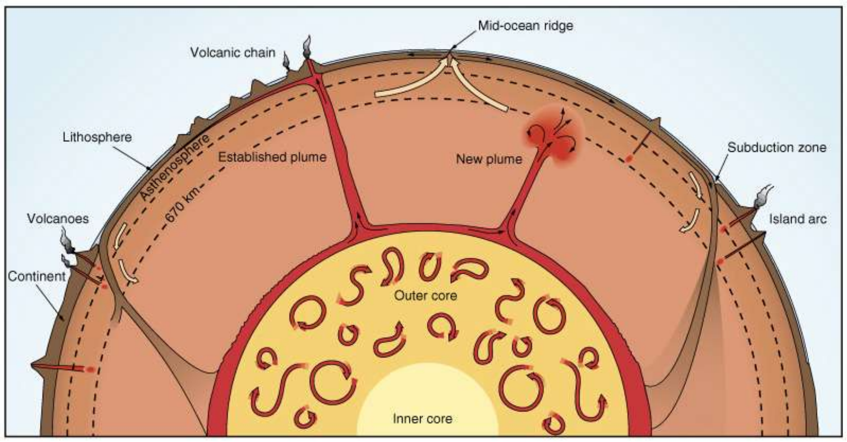
Temperature:

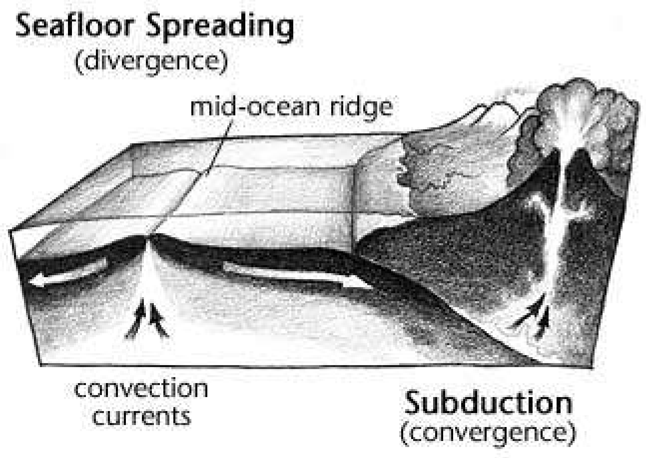
$$T=T_s+(T_m-T_s)\mathrm{erf}[y/(4\kappa t)^{1/2}]$$

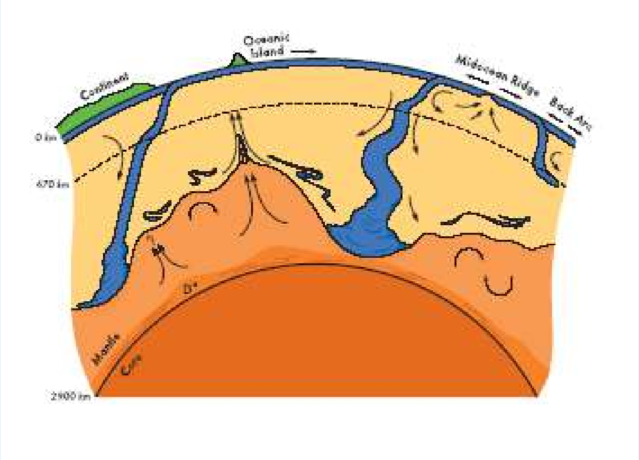
Surface heat flux:

$$Q \sim k(T_m-T_s)/\delta$$

How many TBLs are there in the mantle?

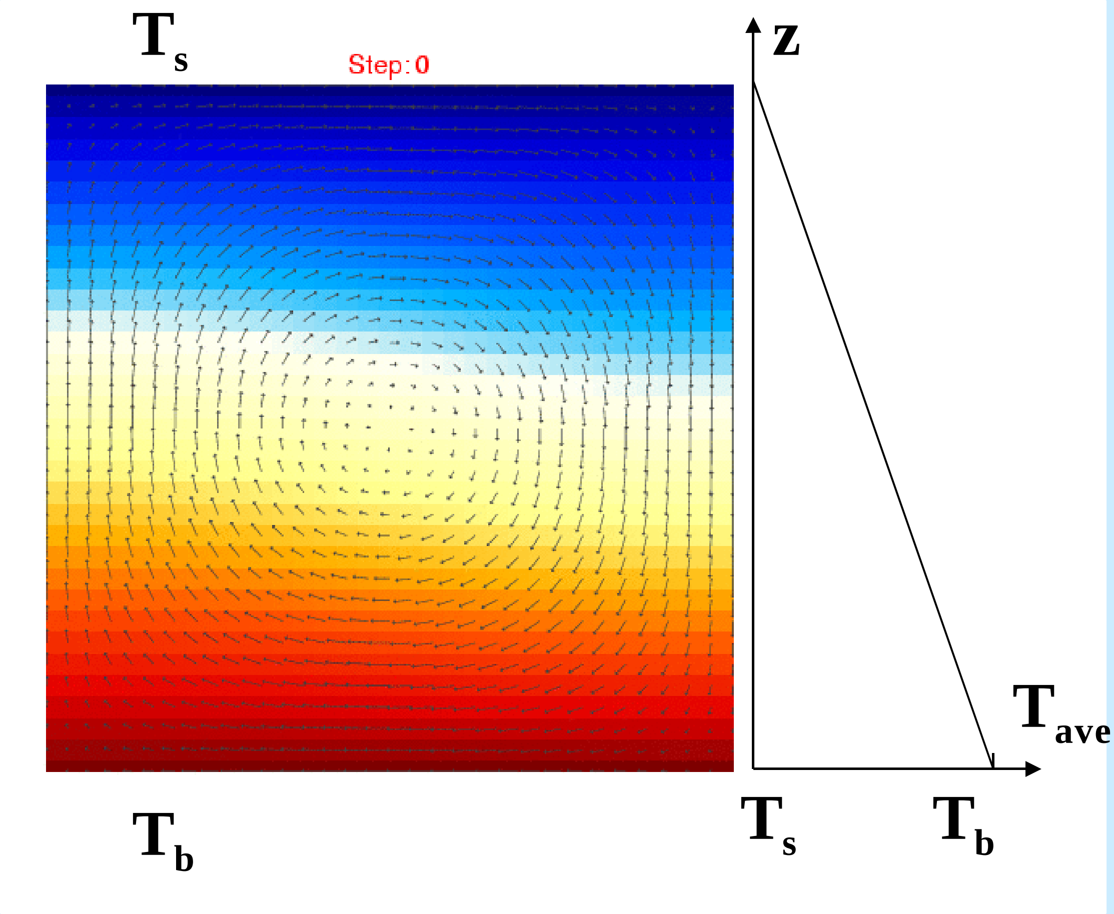






Why does a TBL form?

- *A TBL forms as a consequence of thermal convection.*
- *Why does thermal convection occur?*



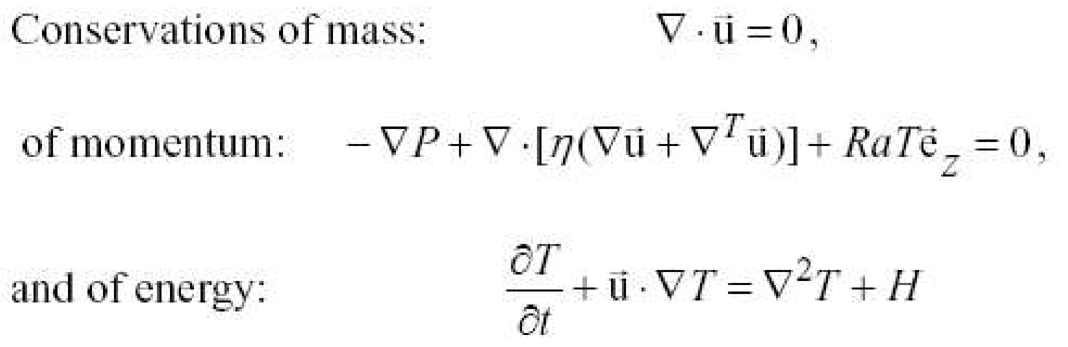
ρ, α, η_0, **and** κ.

***D*: box height; $\Delta T=T_b-T_s$**

At t=0, $T=T_s+(D-z)\Delta T/D$
$+\delta T$

$q_o \sim k\Delta T/D$

Governing equations for isochemical convection



Conservations of mass: $\nabla \cdot \vec{u} = 0,$

of momentum: $-\nabla P + \nabla \cdot [\eta(\nabla \vec{u} + \nabla^T \vec{u})] + RaT\vec{e}_Z = 0,$

and of energy: $\frac{\partial T}{\partial t} + \vec{u} \cdot \nabla T = \nabla^2 T + H$

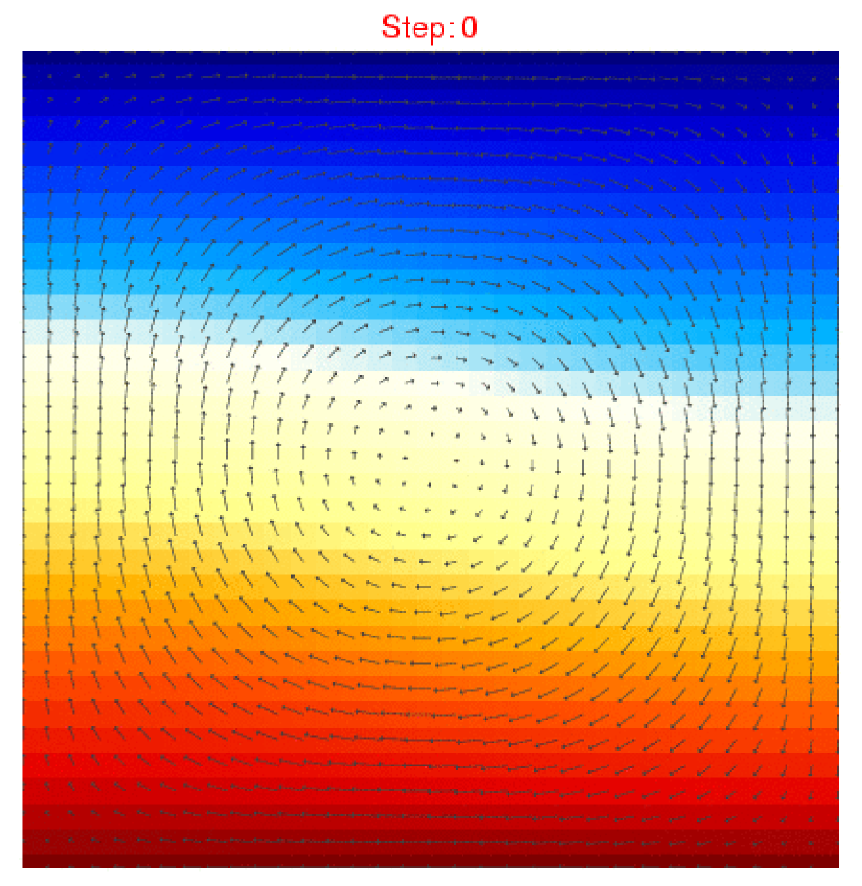
$Ra = \rho g \alpha \Delta T D^3/(\eta_0 \kappa)$ Rayleigh number.

η=1 for isoviscous flow.

H=0 for basal heating or no internal heating.

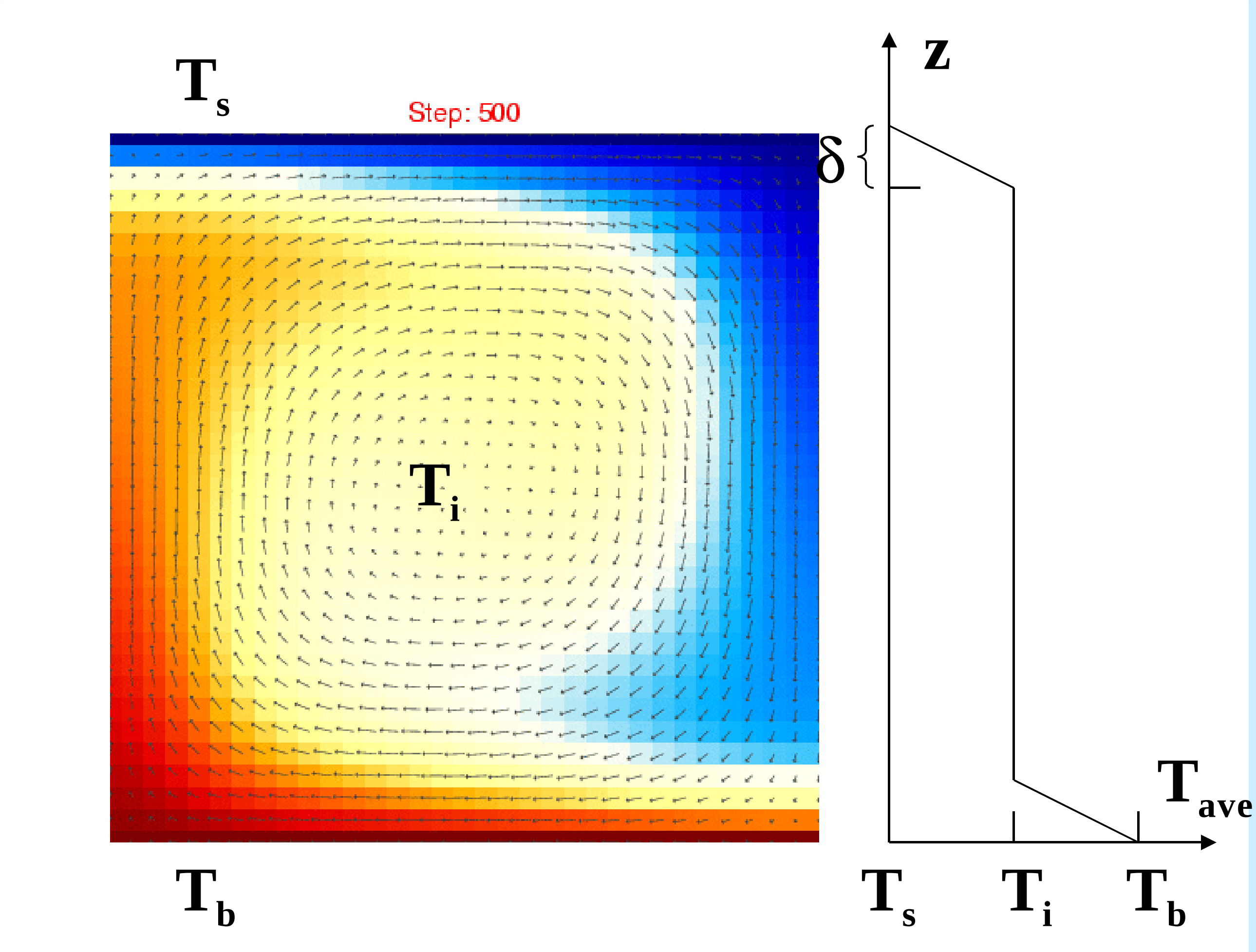
When $Ra > Ra_{cr} \sim 10^3$, convection.

Thermal convection with Ra=1e4 > Ra_{cr}



Basal heating and isoviscous

Convection transfers heat more efficiently



$q_s \sim k(T_i - T_s)/\delta$ or

$q_s \sim k(T_b - T_s)/(2\delta)$.

If no convection,

$q_o \sim k(T_b - T_s)/D$.

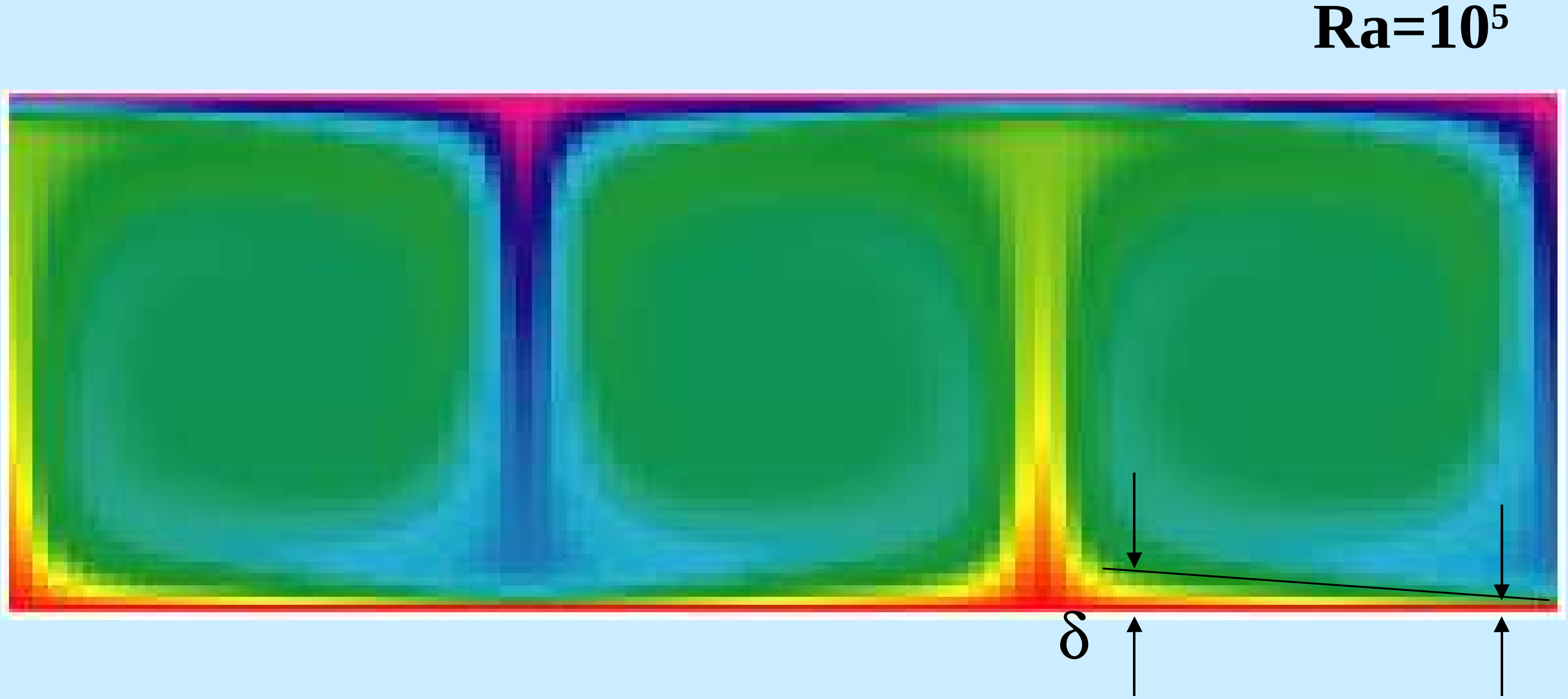
As $2\delta < D$, $q_s > q_o$.

$Nu = q_s/q_o > 1$.

Nu: Nusselt #

$q_b = q_s$ *for basal heating convection*

Control on the thickness of TBL, δ

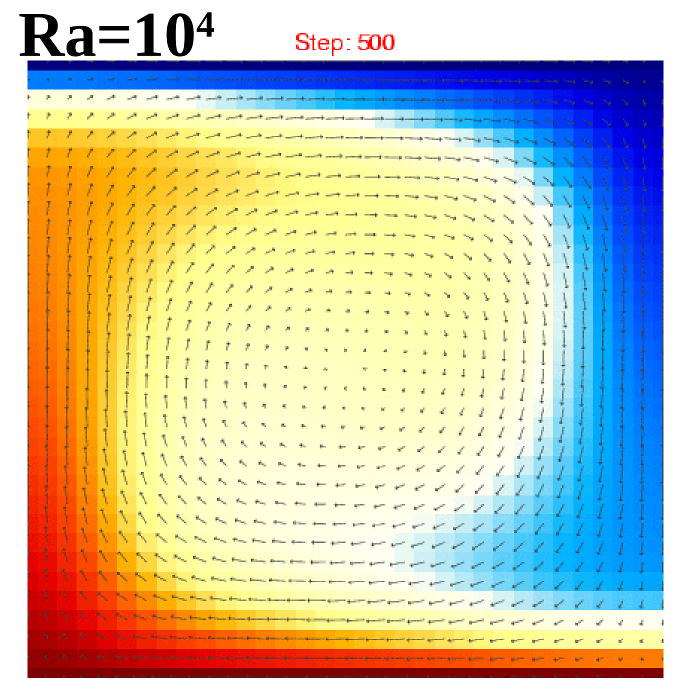


δ is limited by TBL instabilities such that
***Ra**$_\delta$* **= ρgα (T$_i$-T$_s$)δ^3/(ηκ) ~ Ra$_{cr}$ ~ 10^3. As a consequence, plumes form.**

$\delta \sim Ra^{-1/3}$ and $Nu \sim \delta^{-1} \sim Ra^{1/3}$

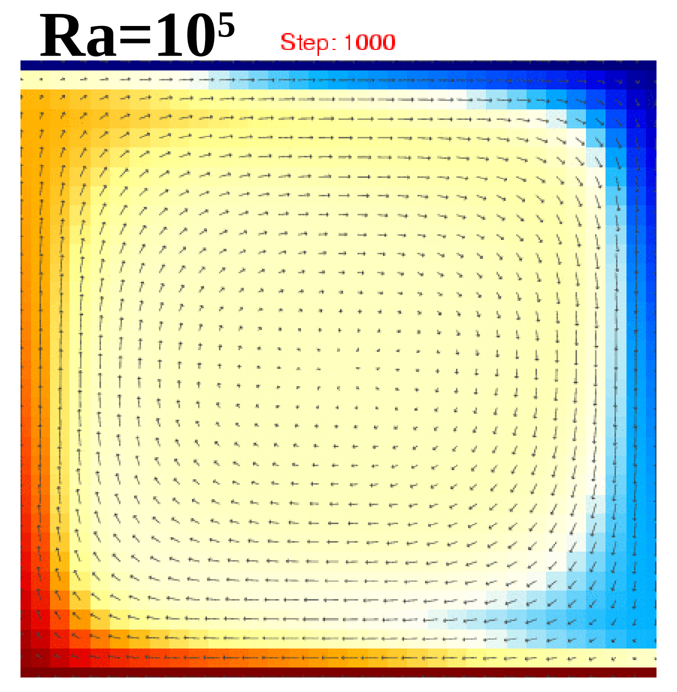
Control on the thickness of TBL, δ

$\delta \sim Ra^{-1/3}$ and $Nu \sim Ra^{1/3}$



$Ra=10^4$

Nu=4.88

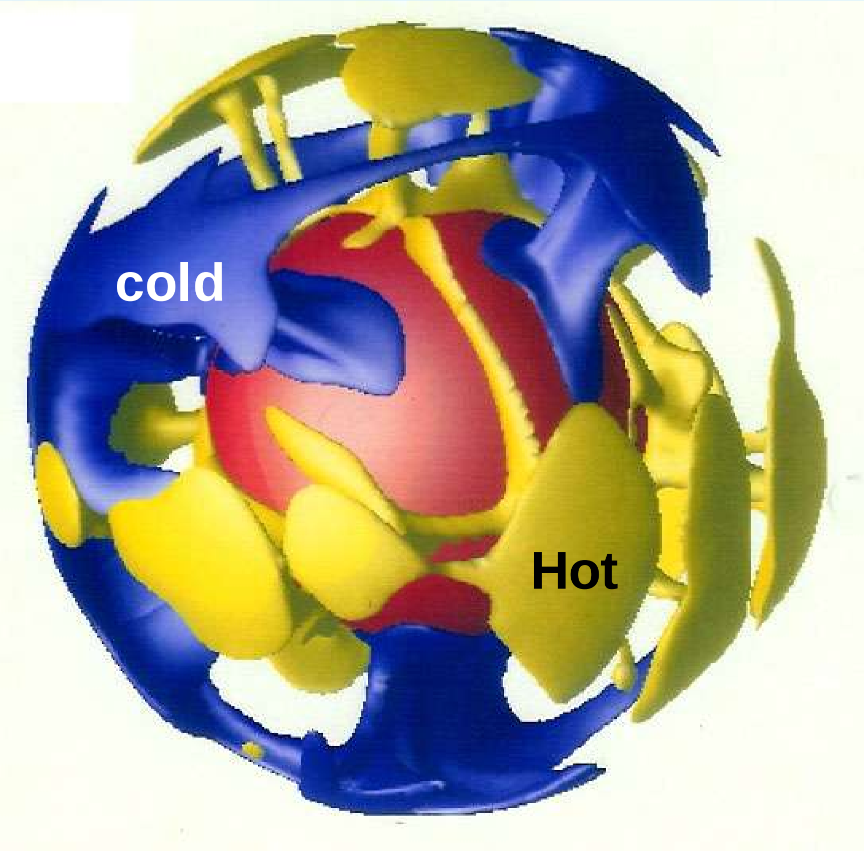


$Ra=10^5$

Nu=10.4

- **Davaille & Jaupart [1993]; Conrad & Molnar [1999]; Solomatov & Moresi [2000]; Korenaga & Jordan [2003]; Huang, Zhong & van Hunen [2003]; Zaranek & Parmentier [2004].**

Linear and Plume structures in 3D thermal convection with η(T) and 40% internal heating



A simulation from CitcomS [Zhong et al., 2000]

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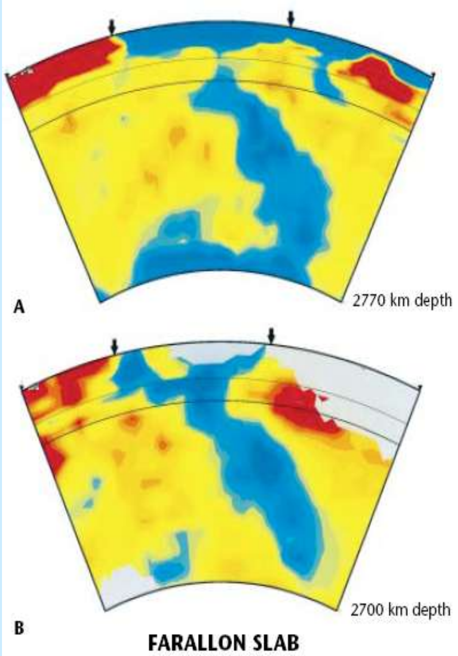
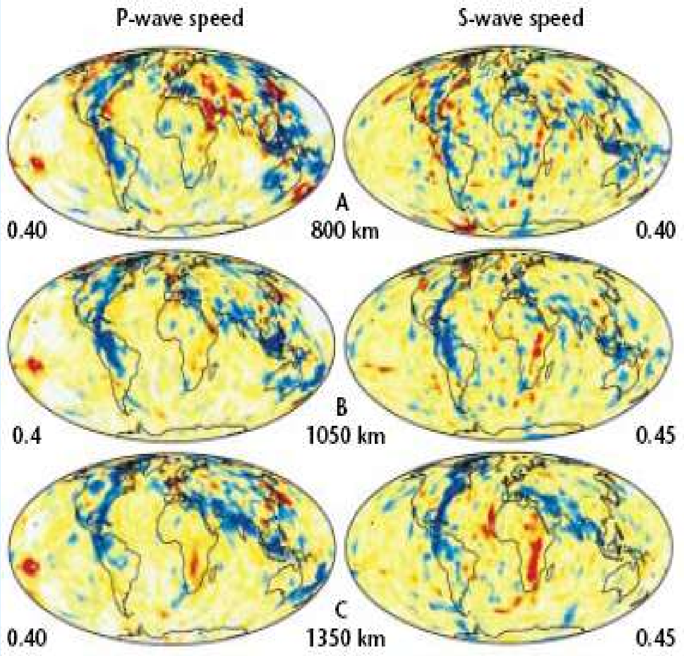
Whole mantle convection

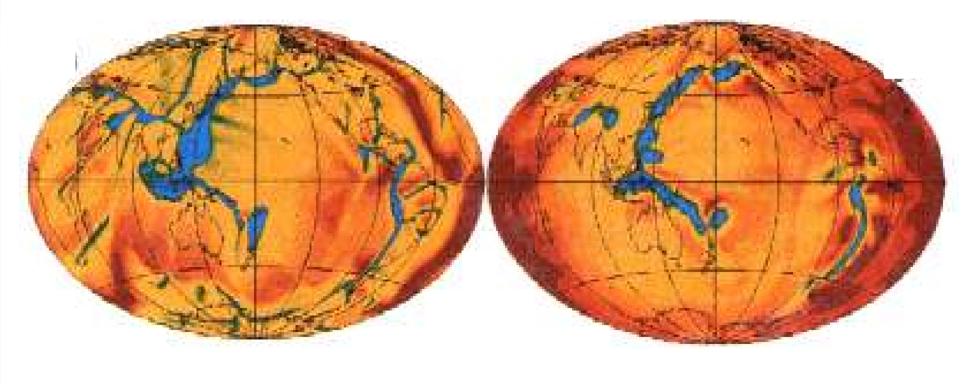
Seismic structure





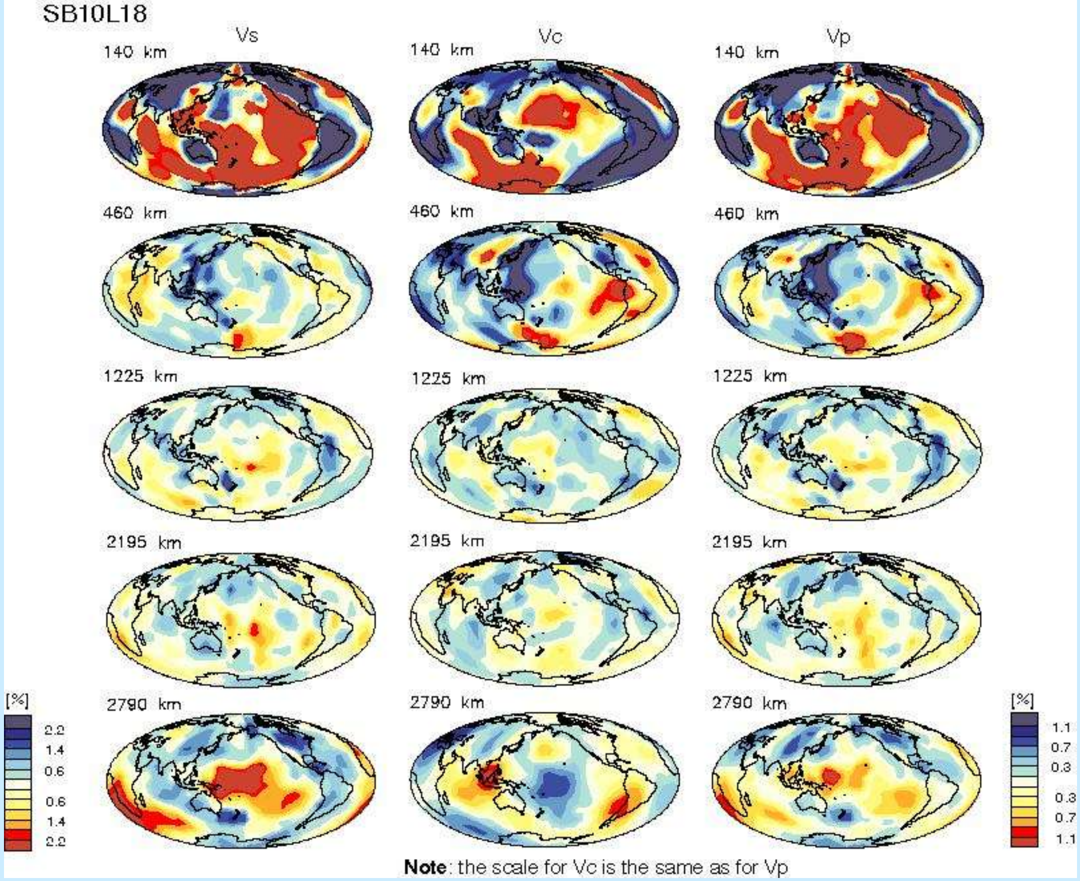
Grand,
van der Hilst, &
Widiyantoro [1997]

- **Long-wavelength geoid [Hager, 1984].**
- **Coupling plate motion to the mantle [Hager & O’Connell, 1981].**



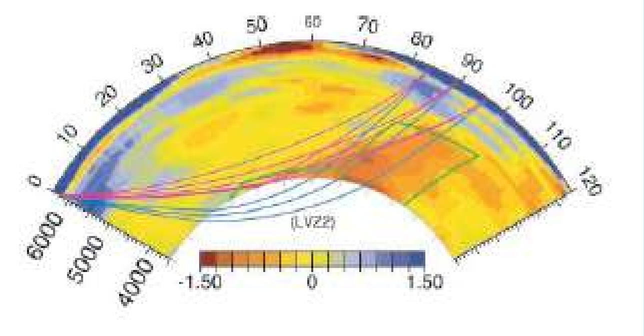
Bunge &
Richards [1996]

Seismic evidence for compositional anomalies at the base of the mantle



Note: the scale for Vc is the same as for Vp

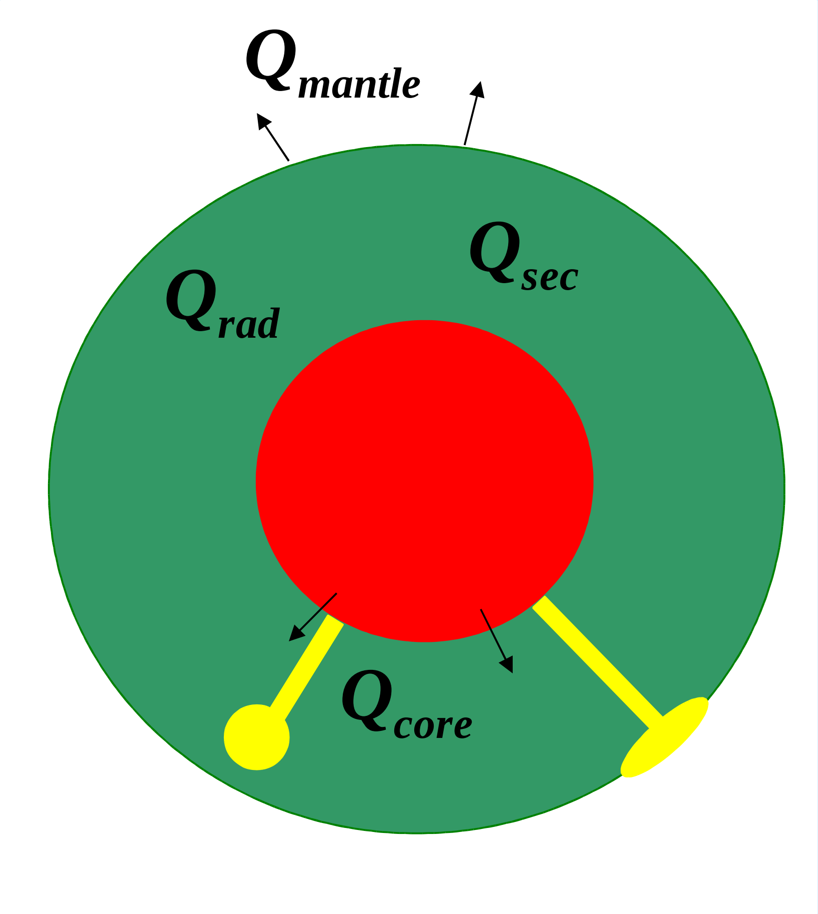
Masters et al. [2000]



Ni et al. [2002]

Heat budget of the Earth

(A modified version for the whole mantle convection [Davies, 1999])

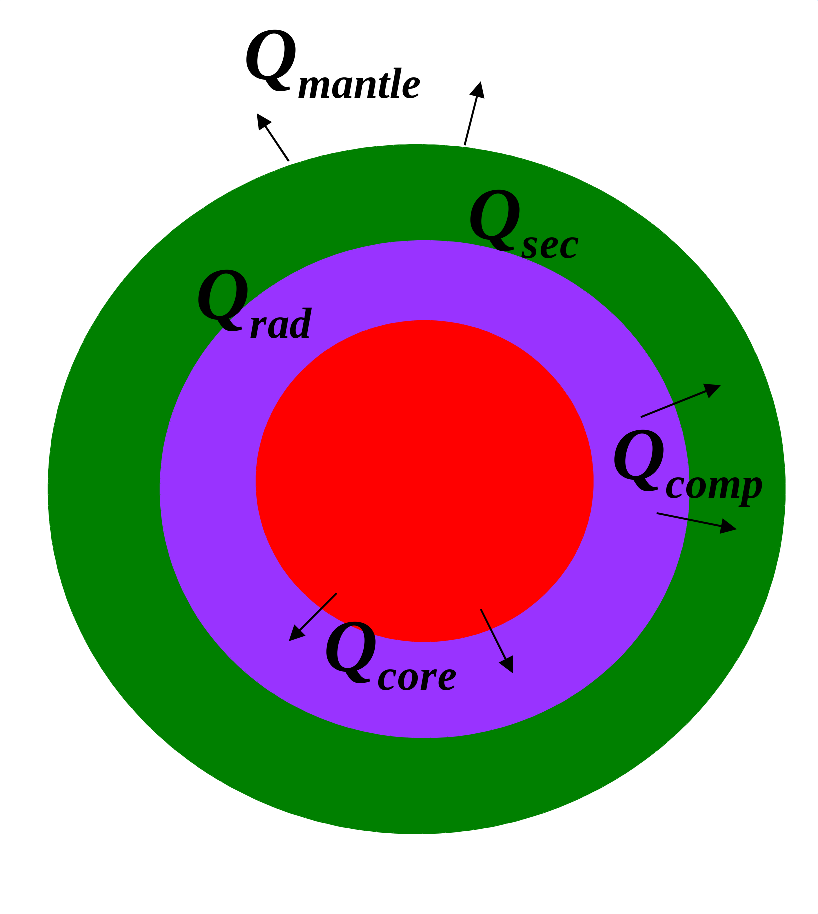


- $Q_{total} \sim 41$ *TW.*
- $Q_{mantle} \sim 36$ *TW.*
- $Q_{sec} \sim 9.3$ *TW (70 K/Ga).*
- *For a mantle with the MORB source material,* $Q_{rad} \sim 3\text{-}7$ *TW (???).*
- $Q_{core} \sim 3.5$ *TW (plume flux ???).*
- ***Unaccounted for:***

$$Q_{mantle}-Q_{rad}-Q_{sec}-Q_{core}=18\ TW$$

Two TBLs: the surface and CMB

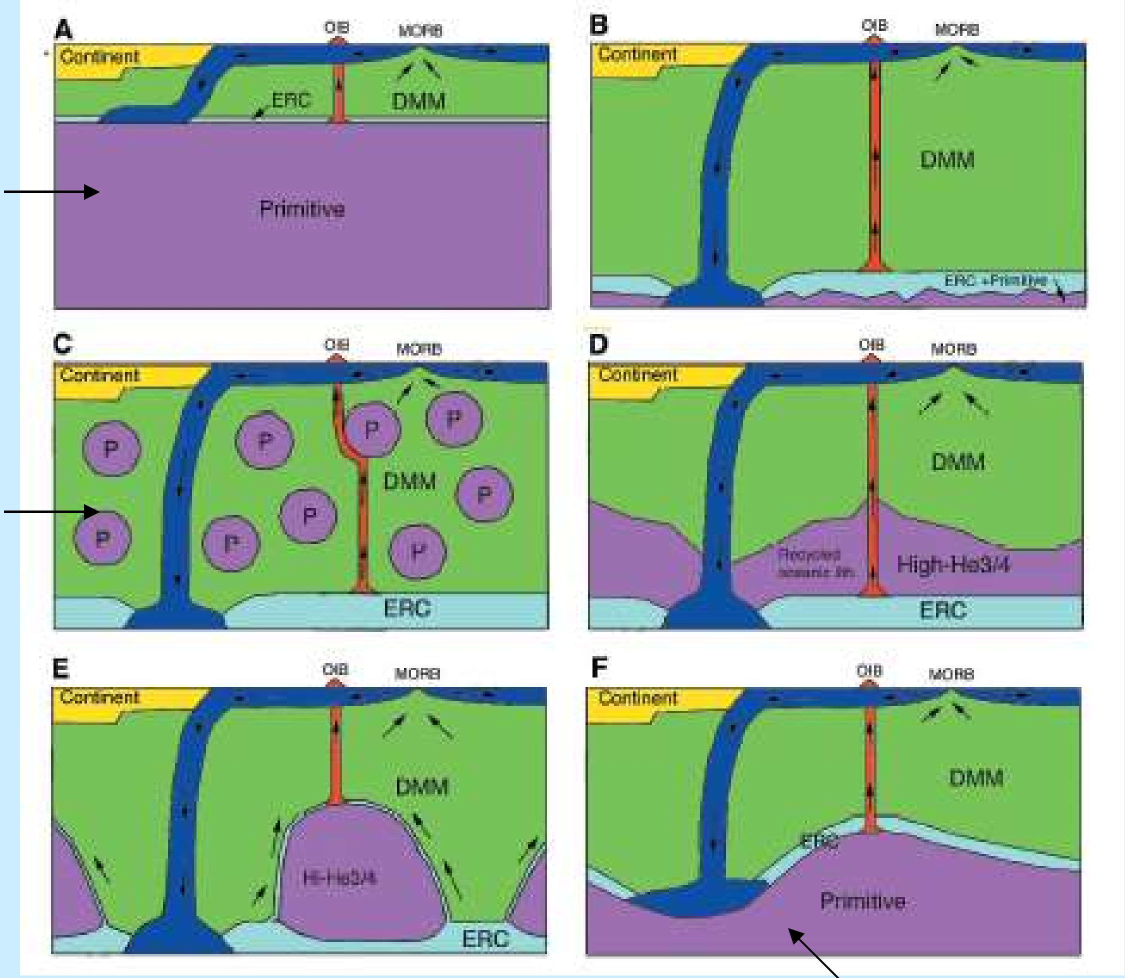
A layered mantle with an enriched bottom layer



- ***To increase Q_{rad} in the bottom layer, Q_{rad_btm} .***
- ***$Q_{comp} = Q_{core} + Q_{rad\_btm}$.***

Three TBLs: the surface, CMB, and the interface.

A variety of layered mantle models (Tackley, 2002)



Hofmann [1997] →

Becker et al. [1999] →

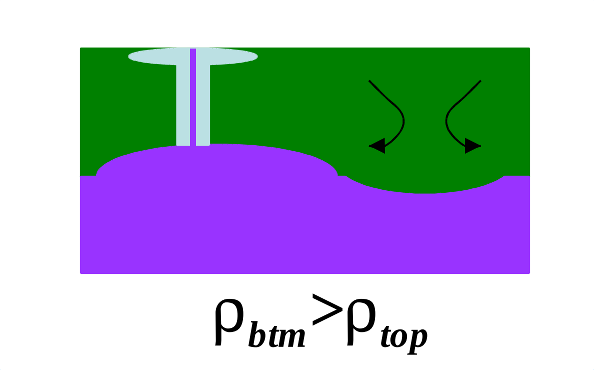
L. Kellogg et al. [1999]

Review of thermochemical convection studies, I

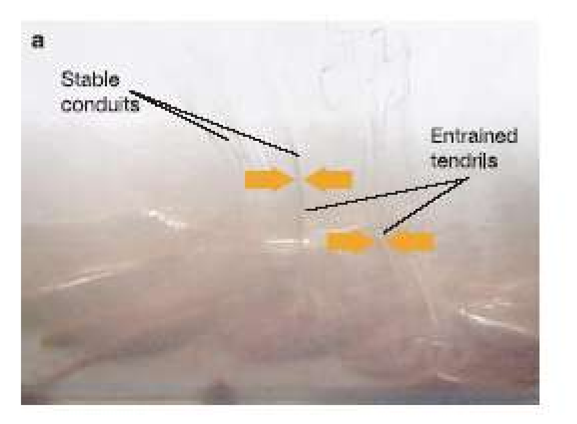
- ***Stability***

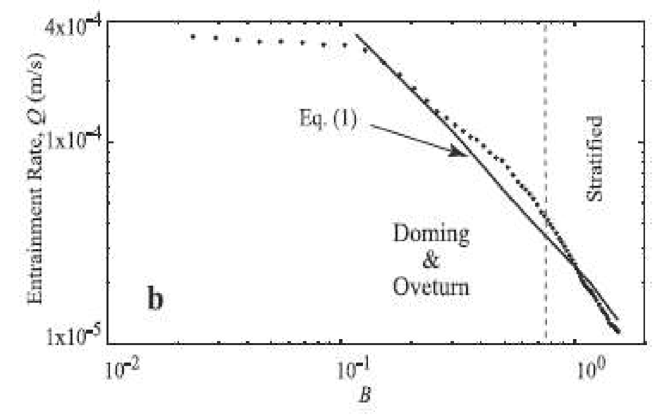
  i) against overturn.

  ii) against entrainment.
- ***Structure***



$\rho_{btm} > \rho_{top}$





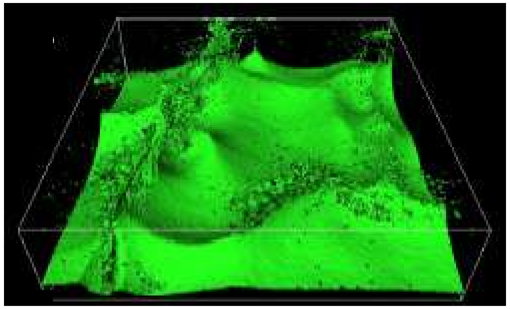
Jellinek & Manga [2002]

Gonnermann et al. [2002]

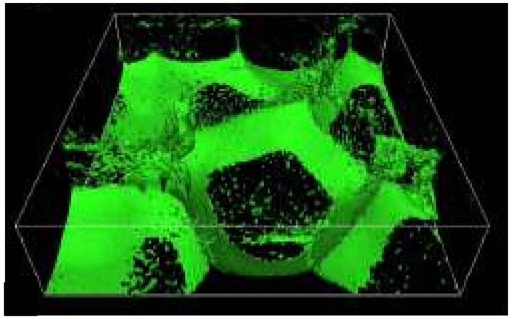
Other studies: Sleep [1988]; Davaille [1999]; Zhong & Hager [2003]

Review of thermochemical convection studies, II

Isolated Piles



Thick bottom layer

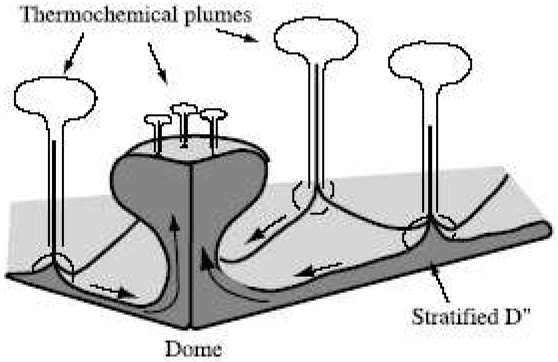


Thin bottom layer

Tackley, 2002

Favor a thin bottom layer.

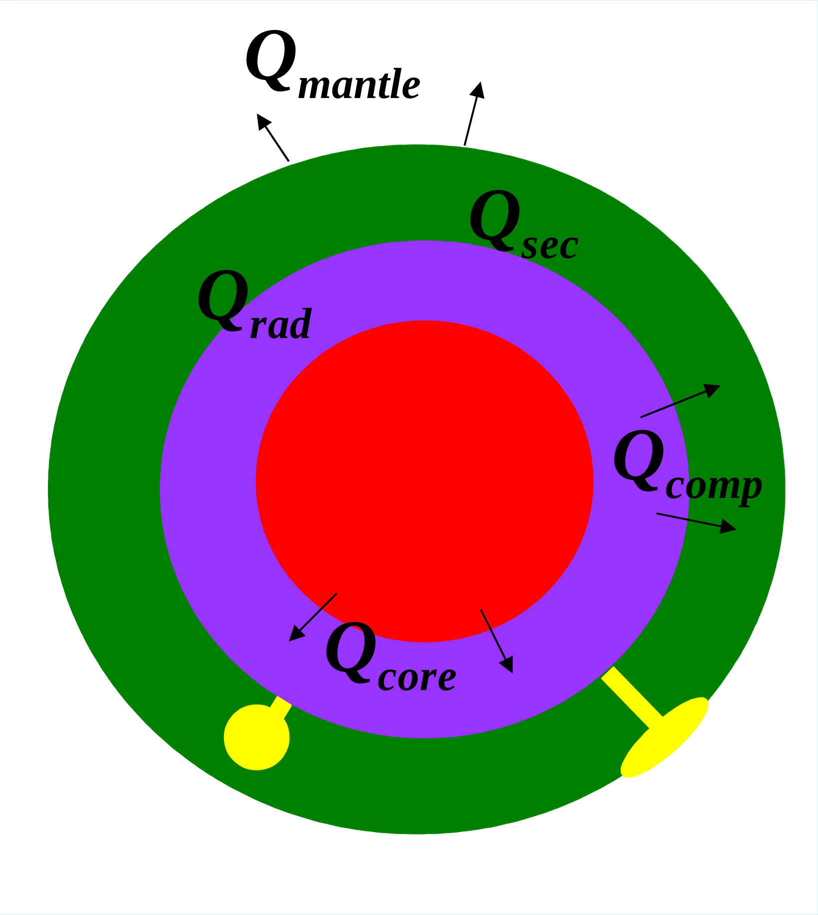
Domes



Davaille et al., 2002

Require the bottom layer more viscous. But how?

Q_{core} ~ plume heat flux Q_{plume}, for a layered mantle?



- ***Q_{core} ~ 3.5 TW becomes really questionable, as it was estimated from Q_{plume}, assuming a whole mantle convection and <u>other things</u>* [Davies, 1988; Sleep, 1990].**

- ***At best, Q_{plume} of 3.5 TW should now be ~ Q_{comp} = Q_{core} + Q_{rad_btm}.***

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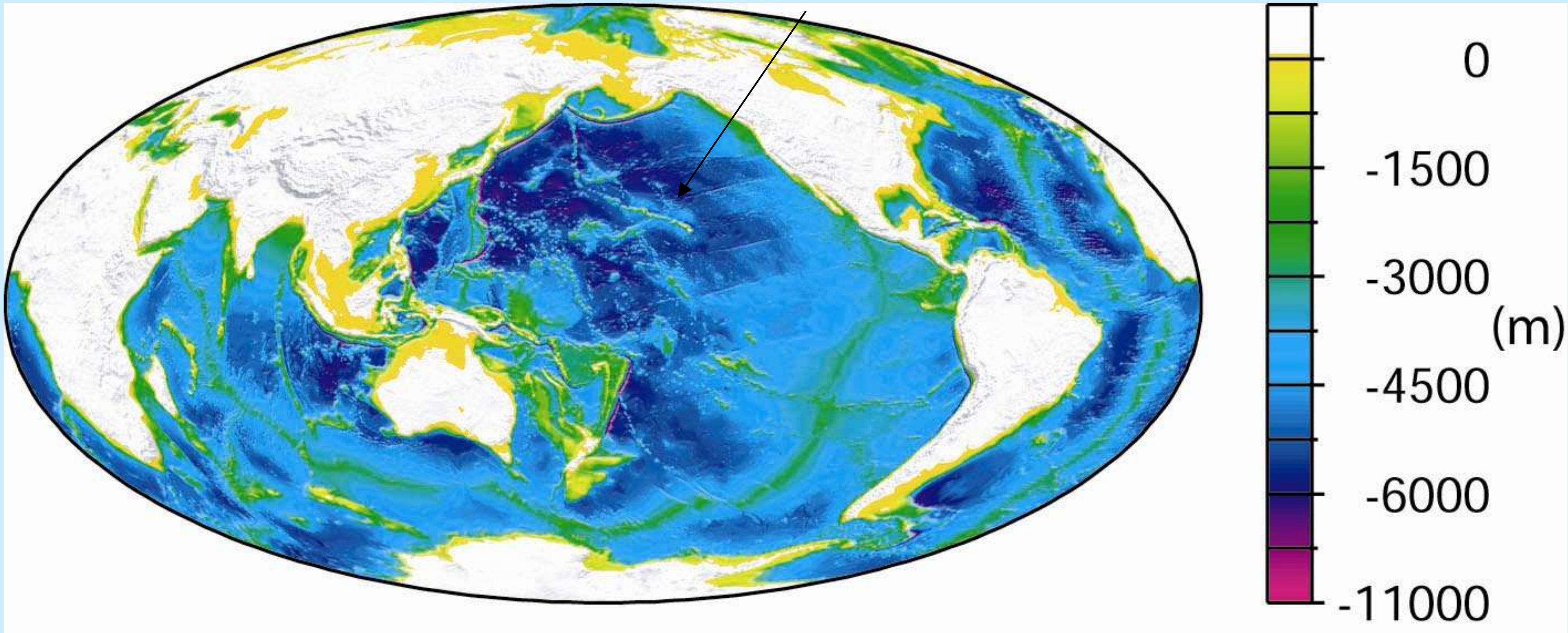
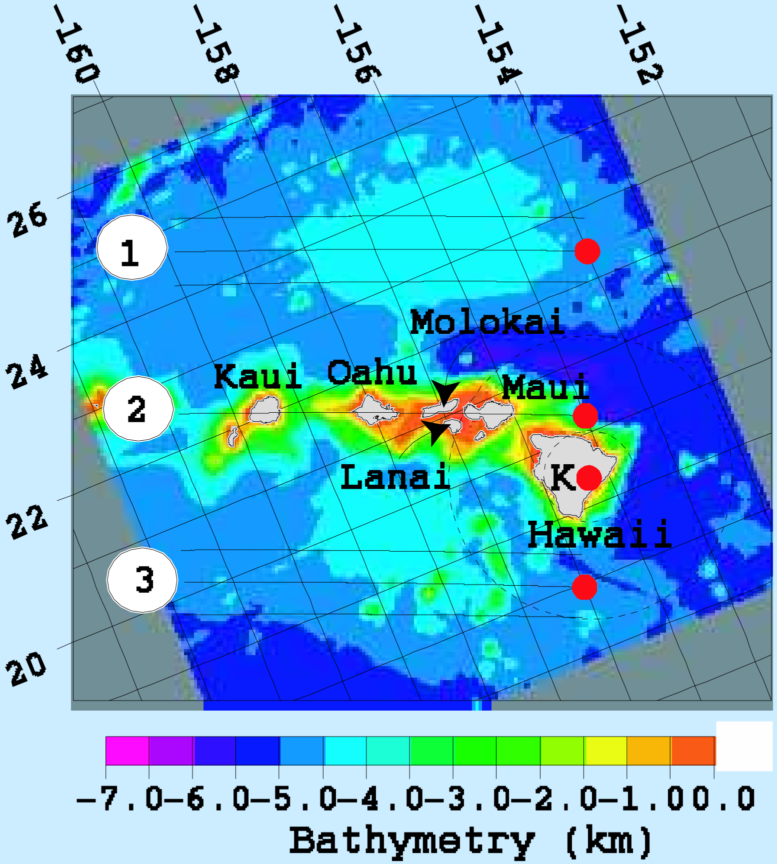
Swell topography and hotspots



Volcanic chain and swell

Hawaiian Swell and Islands

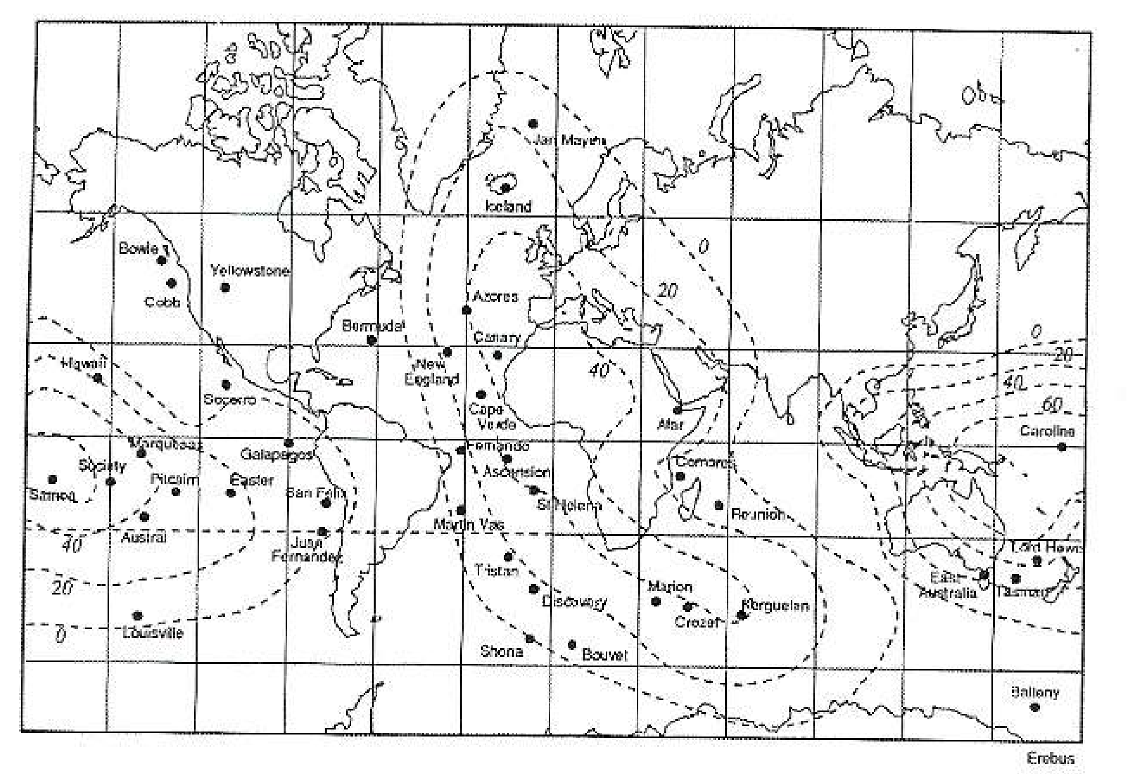


Swell width~1200 km;

Swell height~1.35-1.5 km.

Best quantified by Wessel [1993] and Phipps Morgan et al. [1995].

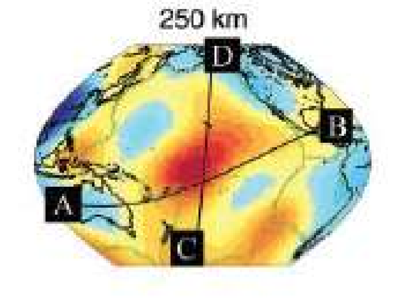
Origins of the hotspots and swell topography

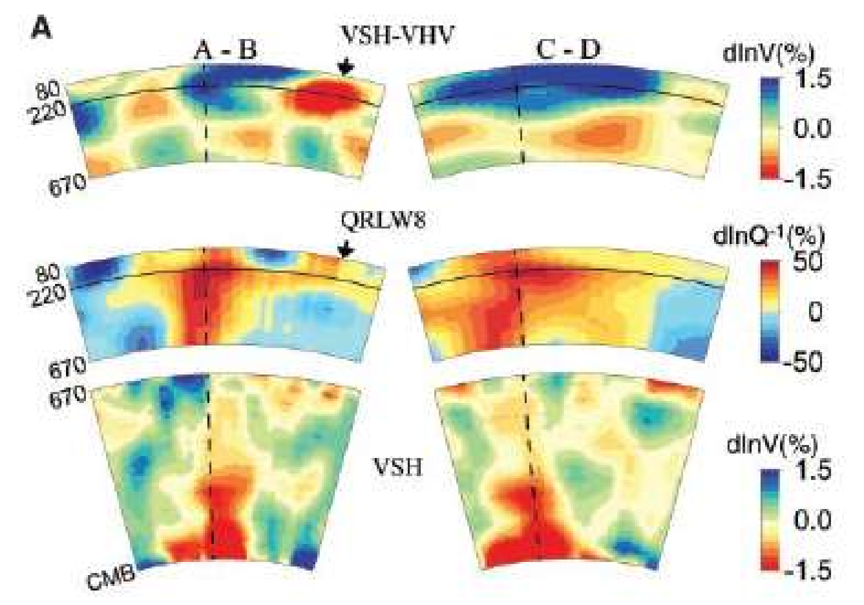


- ***Shallow origins (fractures [Turcotte and Oxburgh, 1972]).***
- ***Deep origins (plumes [Morgan, 1971]).***

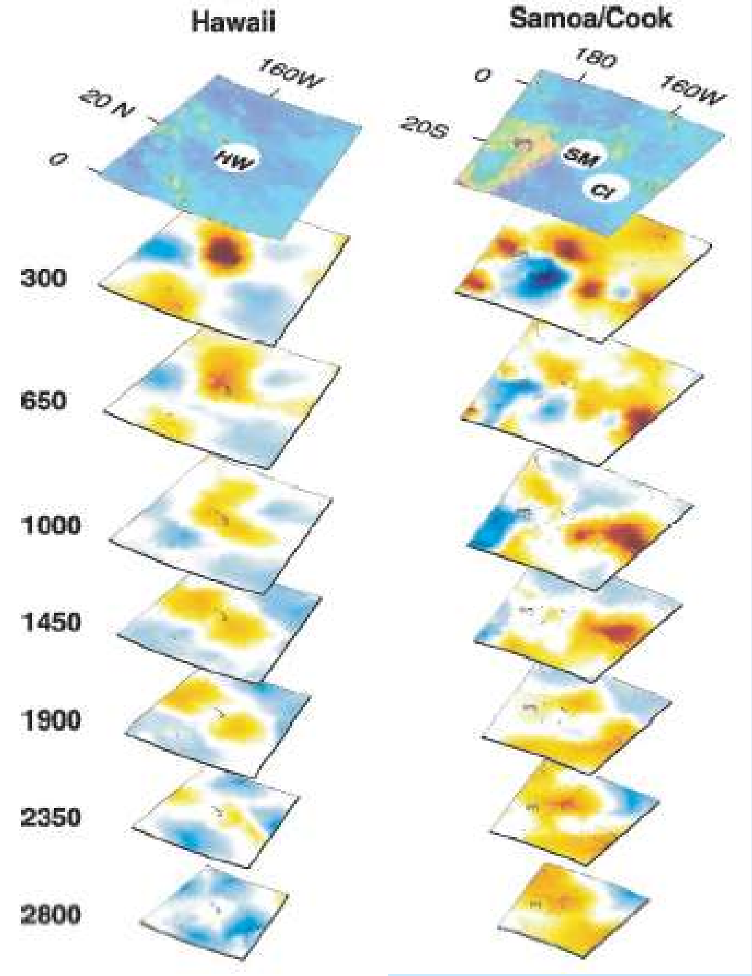
10s [Crough, 1983] to 5000 plumes [Malamud & Turcotte, 1999].

Hotspot and thermal plumes



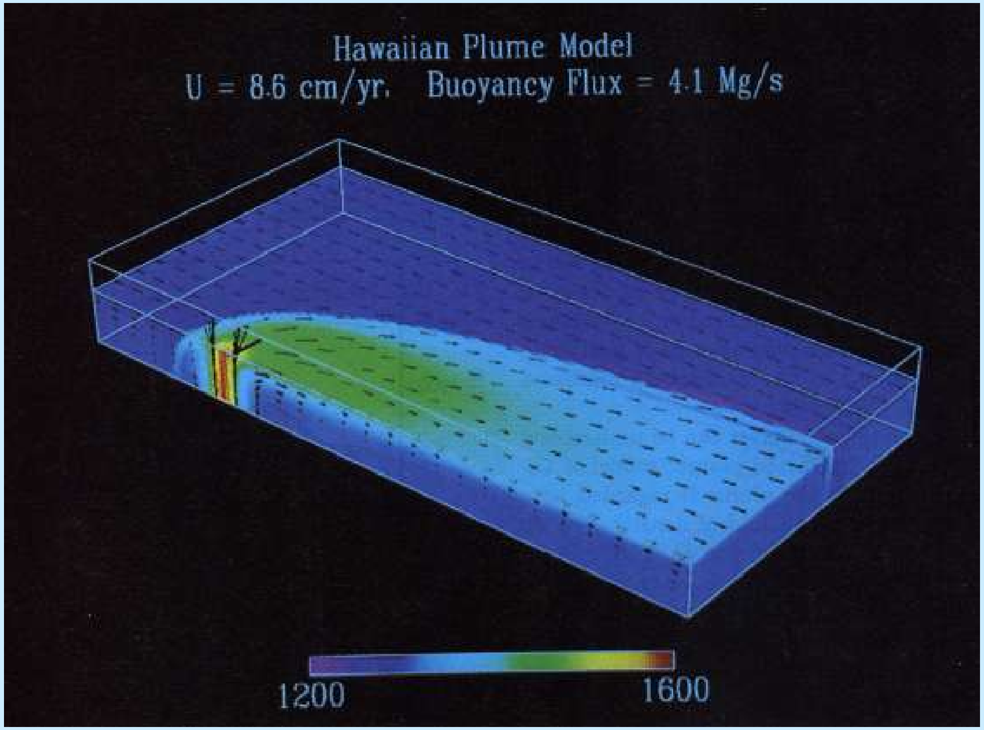


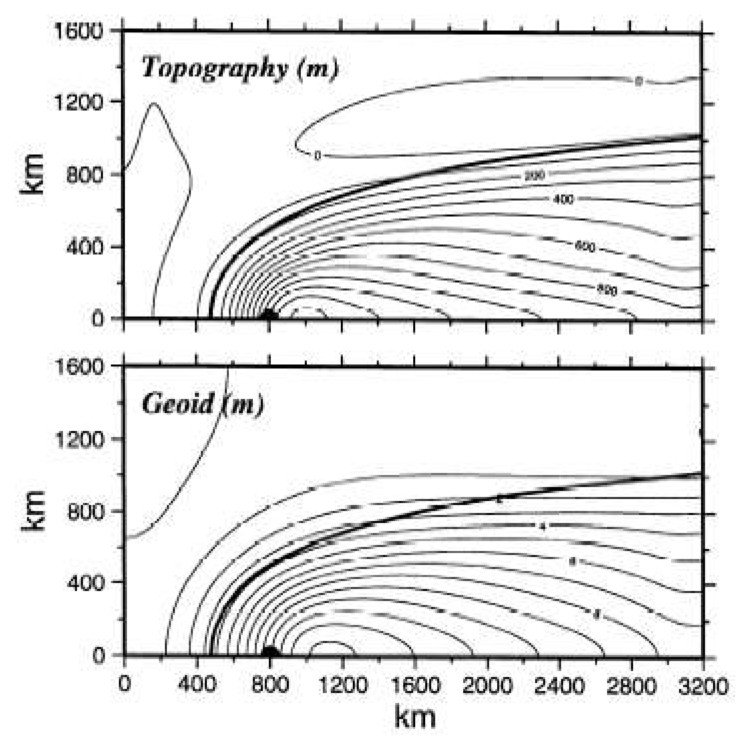
Romanowicz and Gung [2002]



Montelli et al. [2004]

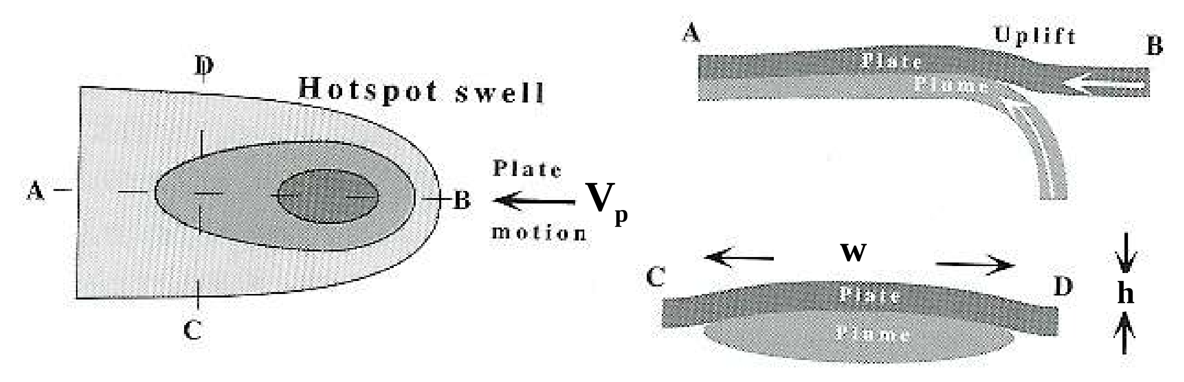
A plume model for Hawaiian swell





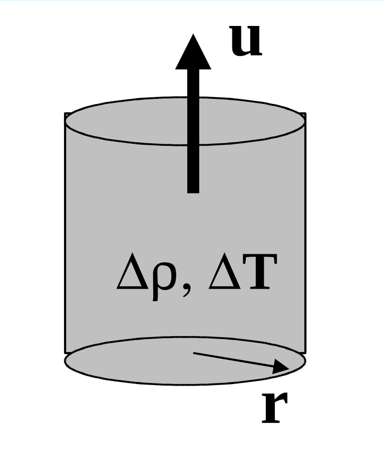
Ribe and Christensen [1994]

Estimate plume heat flux [Davies, 1988; Sleep, 1990]



The rate at which new surface mass anomalies are created due to the uplift:

$$M = hwV_p(\rho_m - \rho_w)$$



Plume flux of mass anomalies:

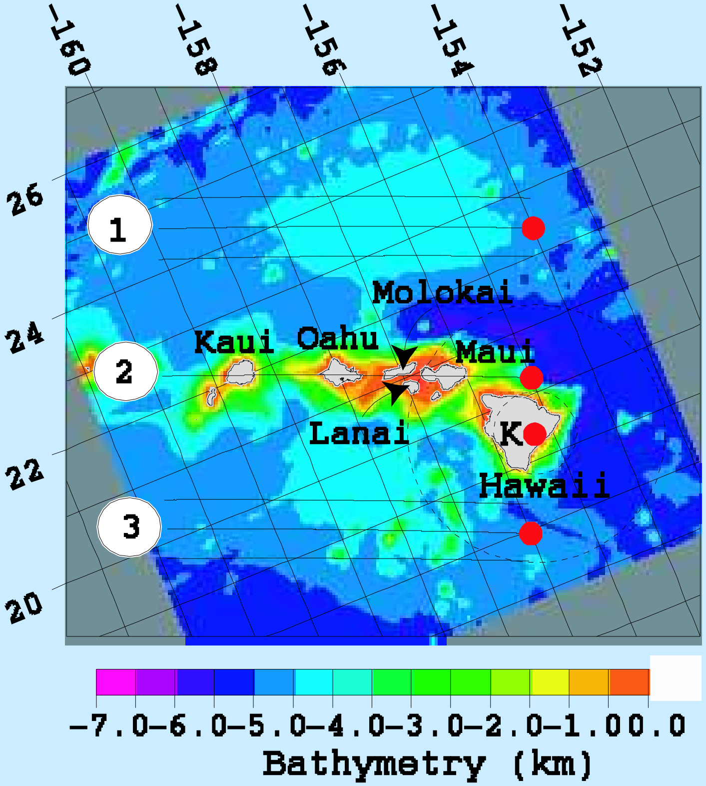
$$B = \pi r^2 u \Delta\rho = \pi r^2 u \rho \Delta T \alpha$$

$$M = B$$

Plume heat flux: $Q = \pi r^2 u \rho \Delta T C_p = BC_p/\alpha$

$$Q = MC_p/\alpha = hwV_p(\rho_m - \rho_w)C_p/\alpha$$

Hawaiian swell as an example



$w \sim 1000$ km; $h \sim 1$ km; $V_p \sim 10$ cm/yr;

$\rho_m - \rho_w = 2300$ kg/m3; $\alpha = 3\times10^{-5}$ K^{-1};

$C_p = 1000$ J $kg^{-1}K^{-1}$

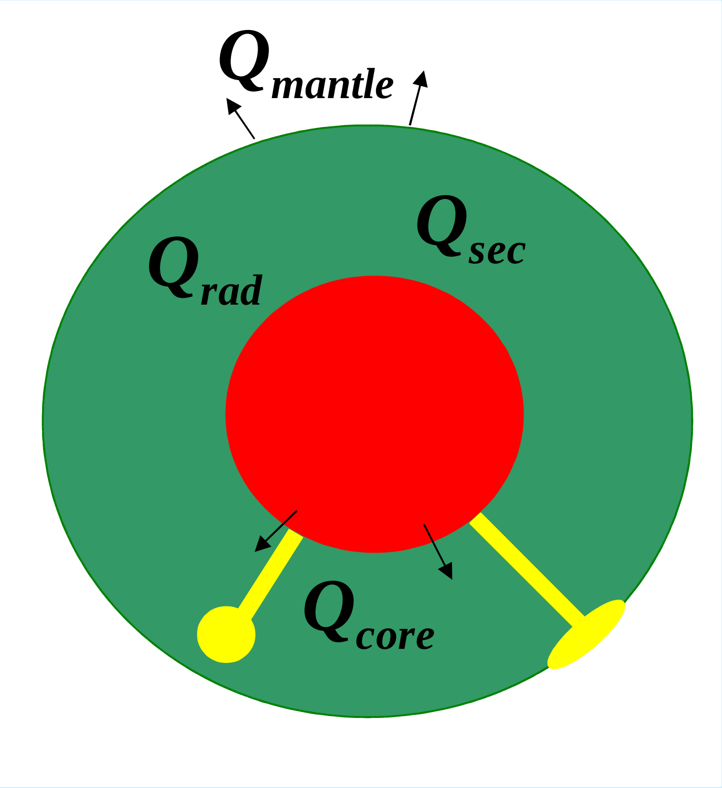
$$Q = hwV_p(\rho_m - \rho_w)C_p/\alpha$$

$$Q \sim 0.24 \text{ TW} \sim 0.7\% \text{ of } Q_{mantle}$$

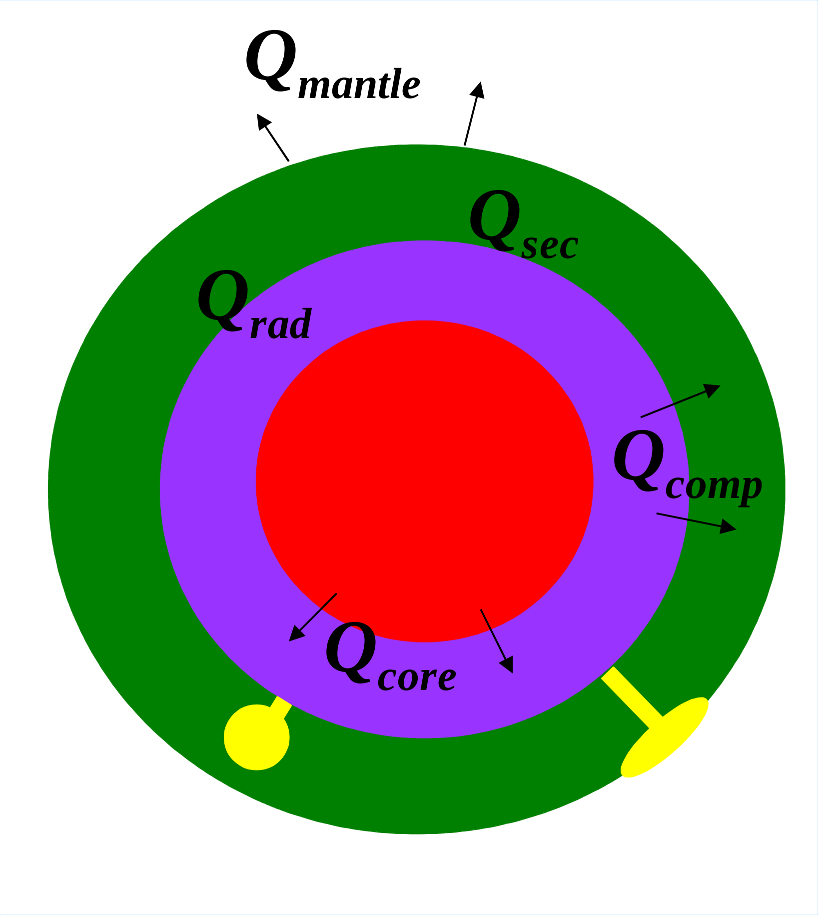
Total plume heat flux

[Davies, 1988; Sleep, 1990]

- ***Q_{plume} ~ 3.5 TW from ~30 hotspots.***
- ***Considered as Q_{core}, in a whole mantle convection, as plumes result from instabilities of TBL at CMB (???).***
- ***Further considered as evidence for largely internally heating mantle convection, as Q_{core}/Q_{mantle}~90%*** **[Davies, 1999] (???).**



$Q_{core} \neq Q_{plume}$ *for a layered mantle!*



- $Q_{plume} \sim Q_{comp} = Q_{core} + Q_{rad\_btm}$ ***because plumes result from TBL instabilities at the compositional boundary, if the proposal by Davies and Sleep is correct.***
- ***If so,*** Q_{plume} ***poses a limit on how much*** Q_{rad_btm} ***into the bottom layer!***

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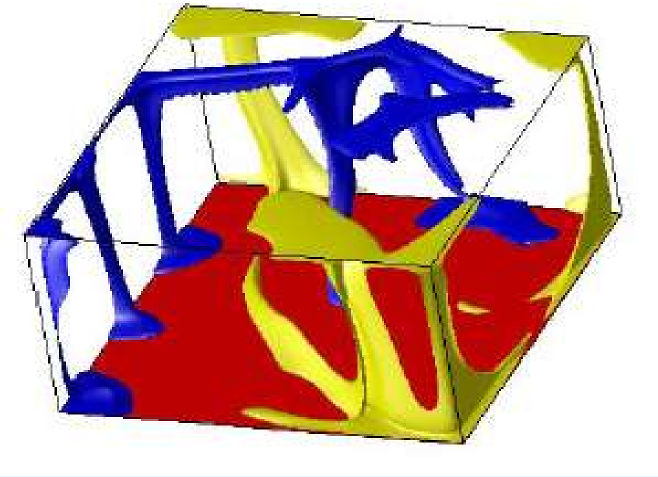
Questions

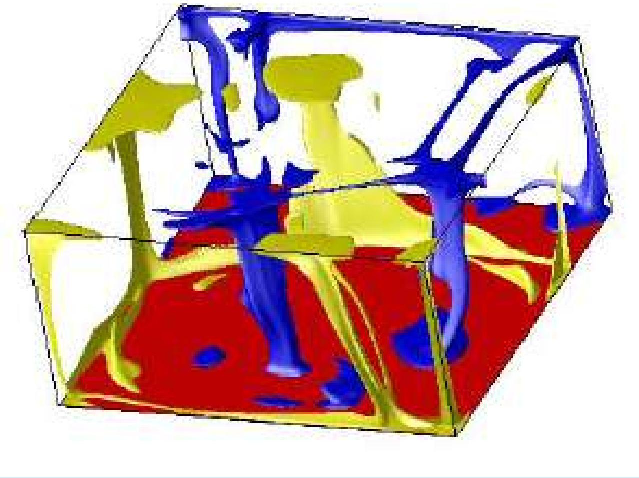
1. ***Should we expect thousands of small plumes that transfer significant amount of heat but produce no surface expression in terms of topography and volcanism (i.e., invisible)? as suggested by Malamud & Turcotte [1999].***
2. ***To what extent does Q_{plume} represent Q_{btm} of the convective system including surface plates?***
3. ***Should we care at all about Q_{plume} ?***

Dependence of plume population on Ra

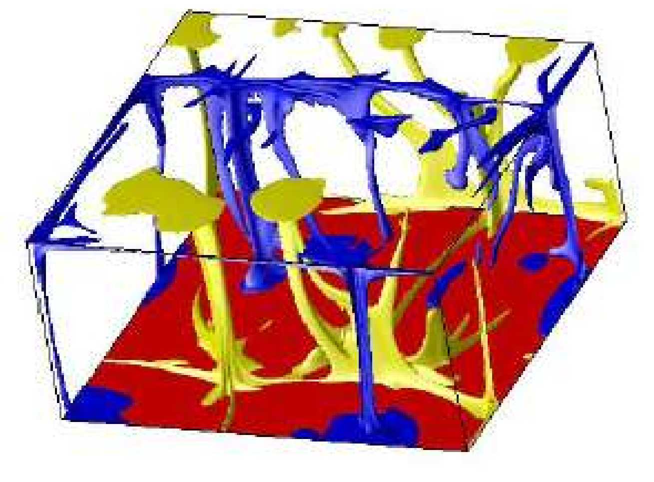
Ra=3x10^{6}



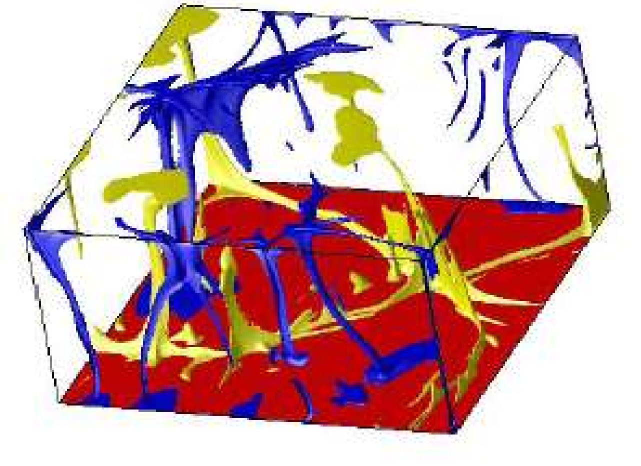
Ra=10^{7}



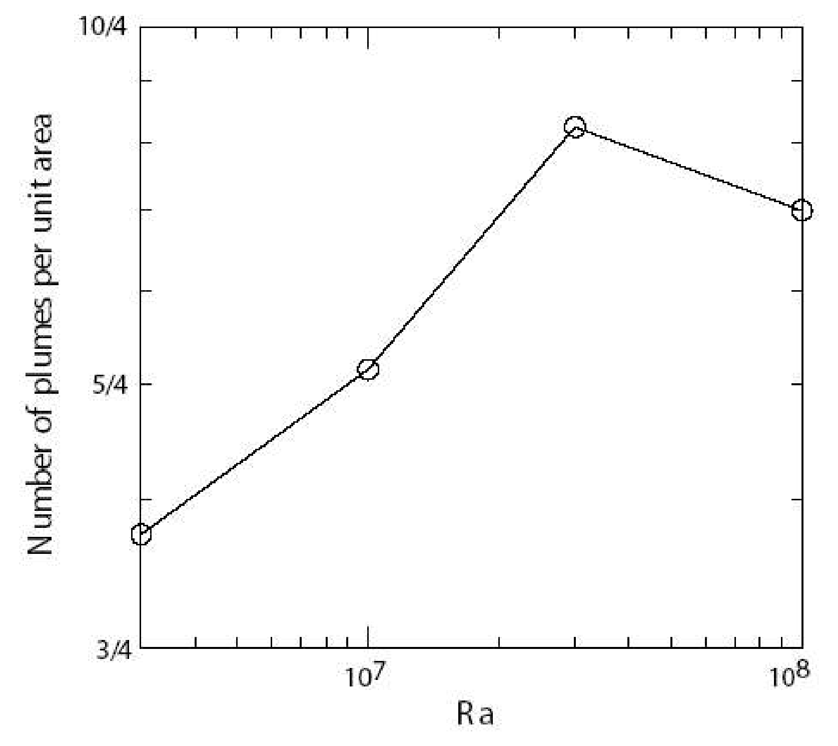
Ra=3x10^{7}



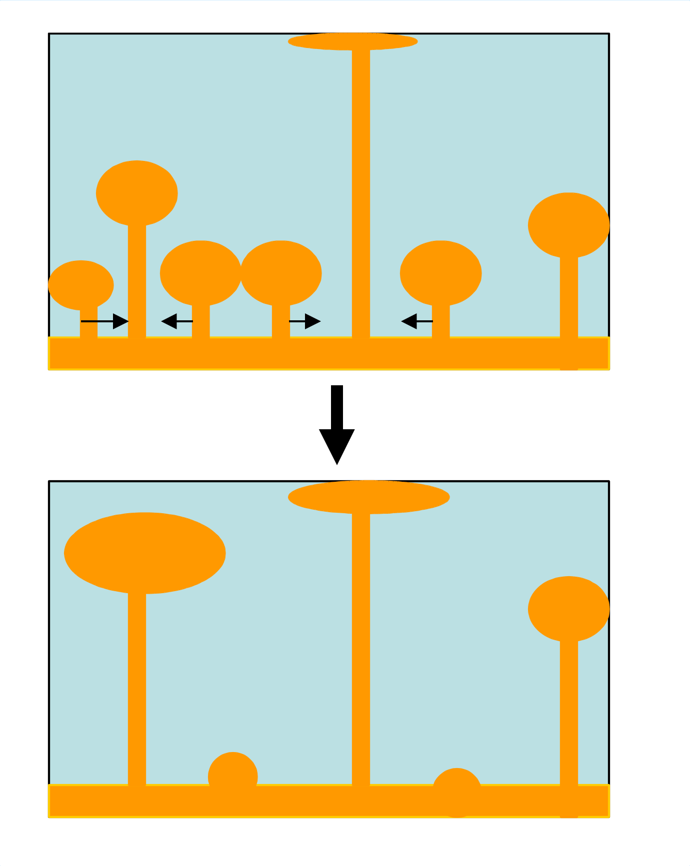
Ra=10^{8}



There is a limit on number of plumes

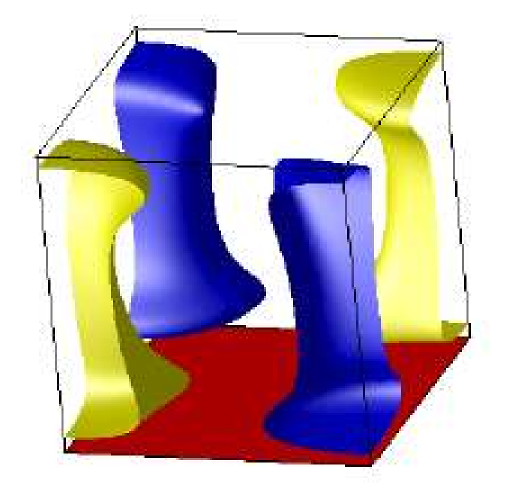


Plumes merge



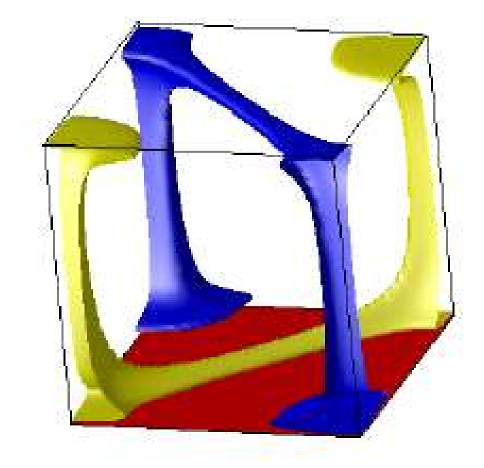
The limit is ~75 plumes, if scaled to the Earth’ s mantle.

Heat transfer by thermal plumes



$Ra=10^4$

$Nu=4.72$



$Ra=3x10^5$

$Nu=16.10$

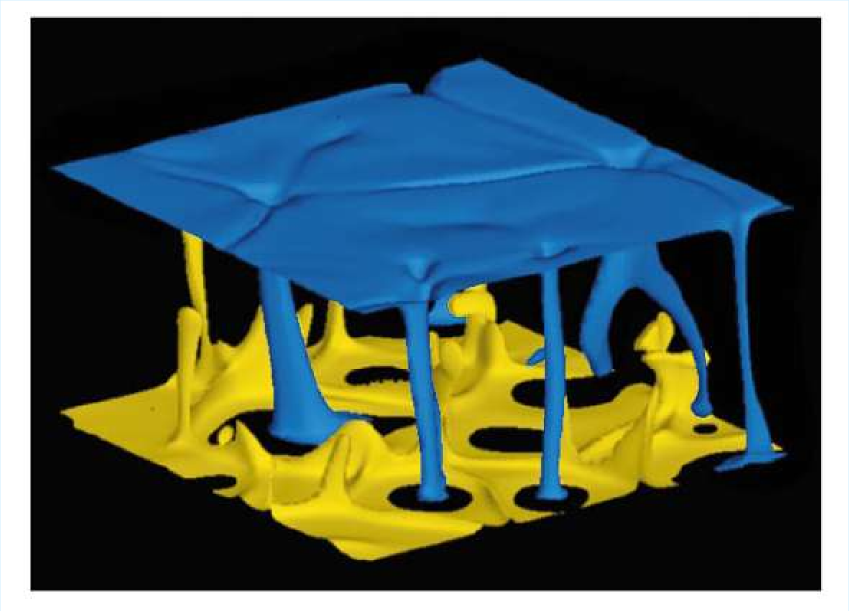
Convective heat flux: **$q \sim \rho c u_z(T-T_{ave})$, important outside of TBLs.**

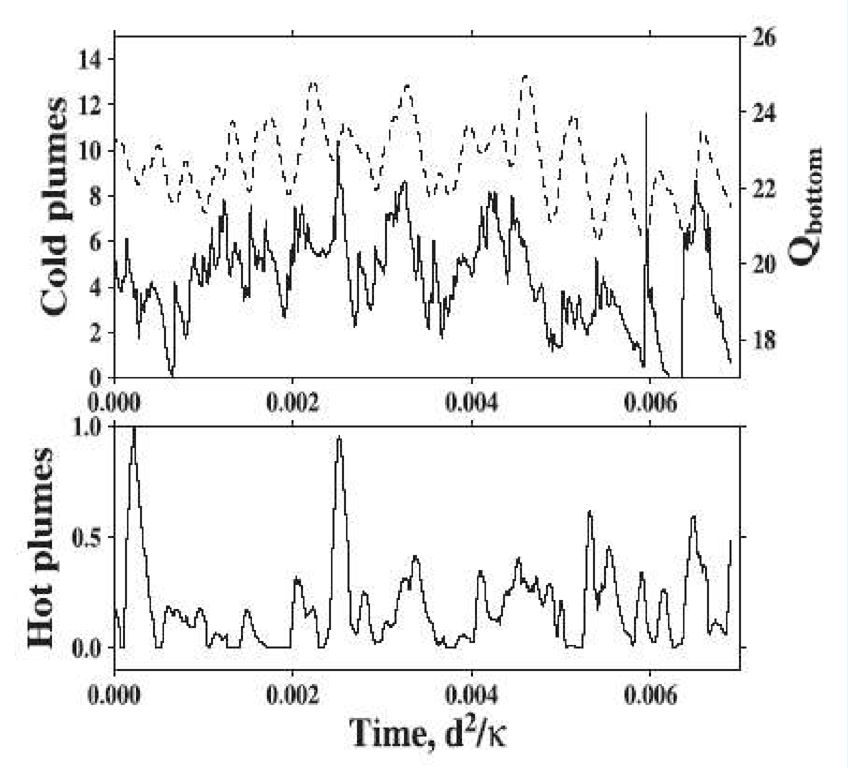
For hot upwellings, $T-T_{ave} > 0$ and $u_z > 0$, so $q_{uw} > 0$.

For cold downwellings, $T-T_{ave} < 0$ and $u_z < 0$, so $q_{dw} > 0$ as well.

For these basal heating cases, $q_{uw} \sim q_{dw} \sim 1/2q_s = 1/2q_b$, *i.e.*,
upwelling plumes only transfer ½ of heat flux from the bottom!

The cooling effect of downwellings on Q_{btm}



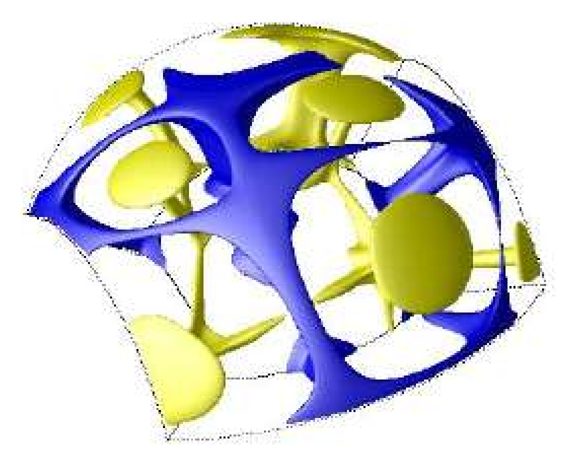


Labrosse, 2002

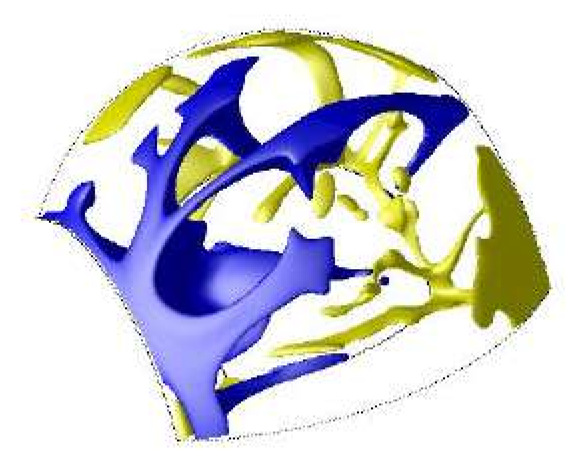
Quantifying Q_{uw}

[internal heating + η(T)+spherical geometry]

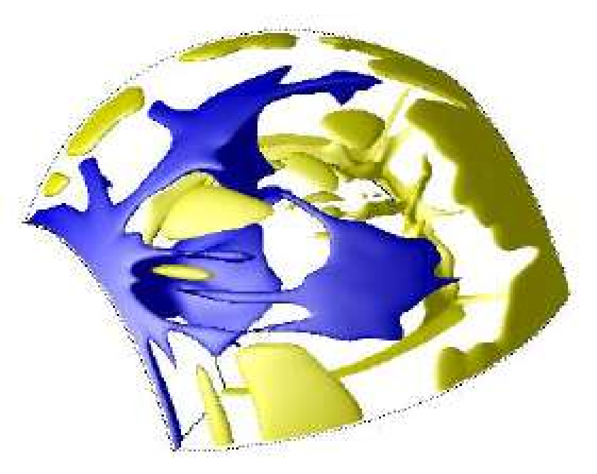
$Q_i/Q_s=0$



$Q_i/Q_s=26\%$



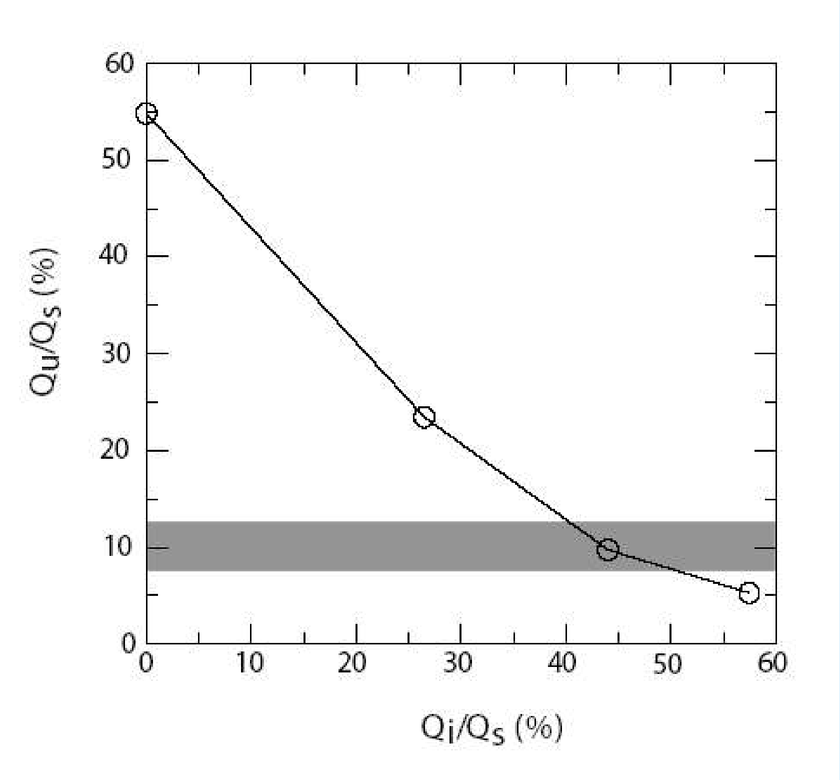
$Q_i/Q_s=57\%$



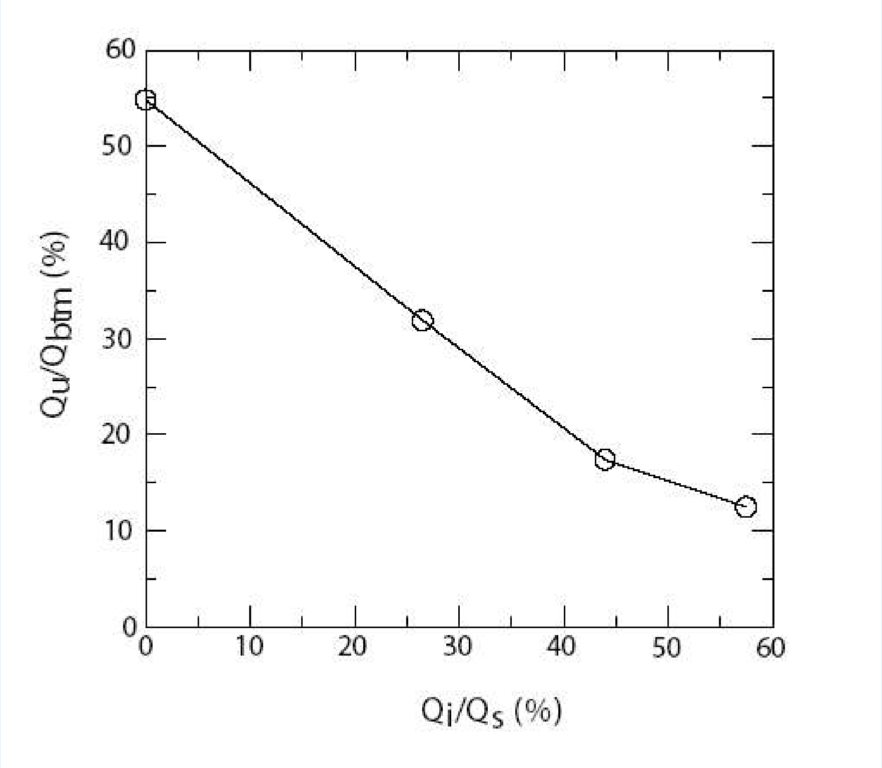
How does Q_{uw}/Q_s (or Q_{plume}/Q_s) depend on internal heating rate Q_i/Q_s?

How does Q_{uw}/Q_{btm} depend on internal heating rate Q_i/Q_s?

Now the answers …



Remember 90% internal heating rate suggested based on Q_u/Q_s~10%?

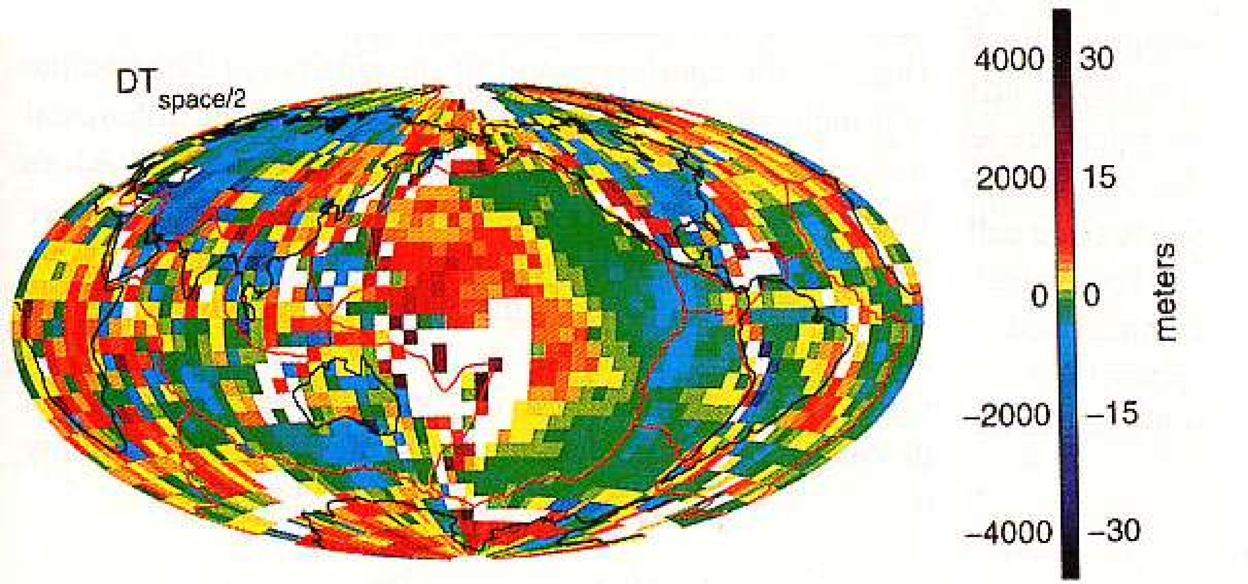


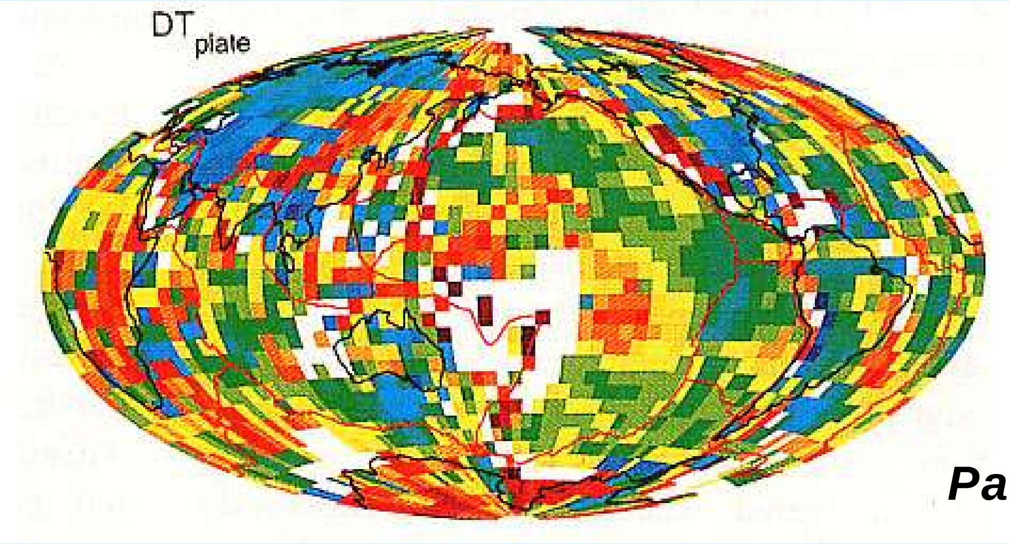
If Q_i/Q_s~40%, then Q_u/Q_{btm}~20%. As Q_u~3.5 TW, Q_{btm}~17 TW.

Summary

- ***Plume heat flux remains a constraint on the heat from the bottom layer (core or the bottom layer of the mantle).***
- ***Q_i/Q_s~40% and Q_{plume}/Q_{btm}~20%, or Q_{btm}~17TW (??).***
- ***A thin layer (100’ s km) at the base of the mantle, D” ?***
- ***Expect some (10’ s) plumes that produce observable surface features.***

“ Dynamic (residual)” Topography





Panasyuk and Hager, 2000

Remaining issues

- ***Heat budget:***

  i) Plume heat flux: super-plumes (What are they?) and the role of weak asthenosphere.

  ii) Secular cooling.

  iii) Wish list (easy to say but hard to do, perhaps). Try to estimate uncertainties for both seismic and geochemical models.

We have a long way to go …