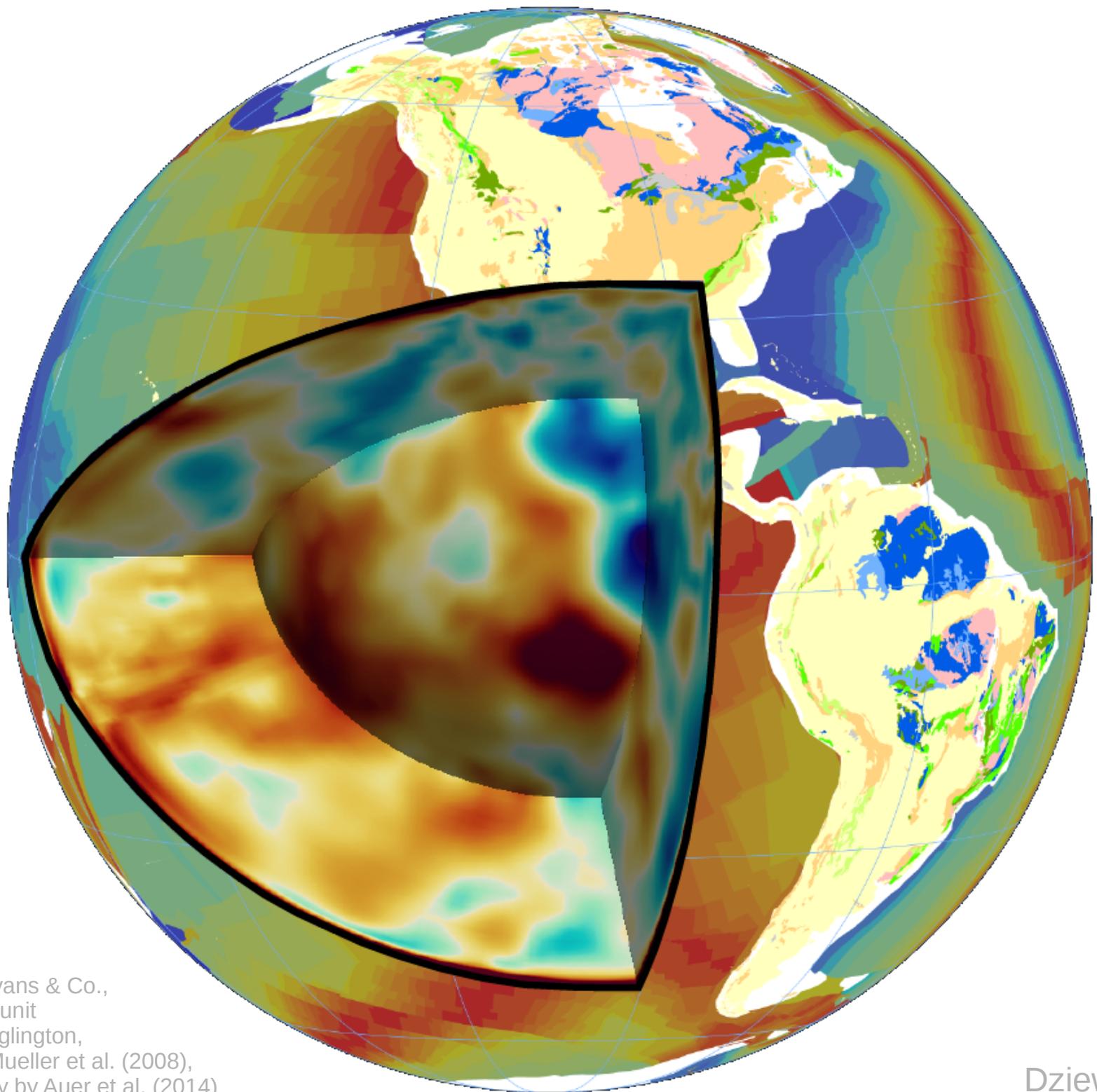


# Geodynamics III: Integrating geophysical observations into global mantle flow models

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CIDER Summer School *Flow In the Deep Earth*  
Santa Barbara CA  
July 2016



Composite by D. Evans & Co.,  
showing geological unit  
compilation by D. Eglinton,  
seafloor age from Mueller et al. (2008),  
SAVANI tomography by Auer et al. (2014)

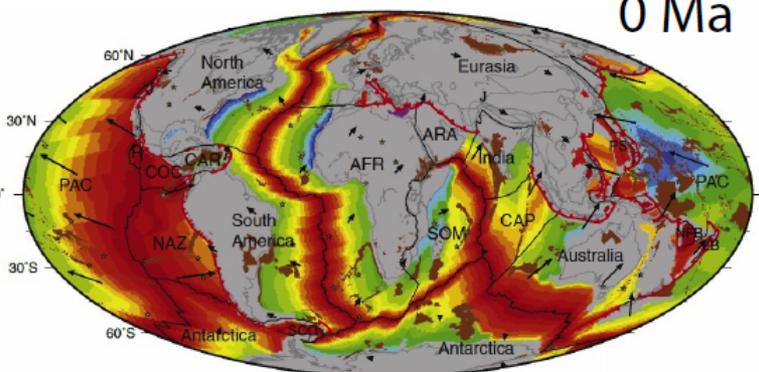
Dziewonski plot

*reconstructed seafloor age*

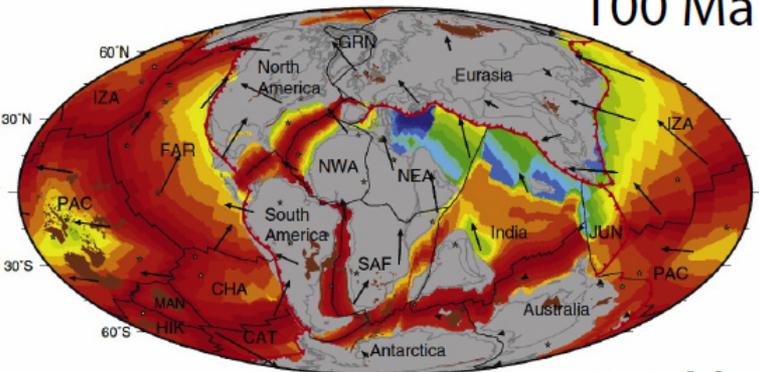
*hot temperature isosurface*

*dense chemical piles*

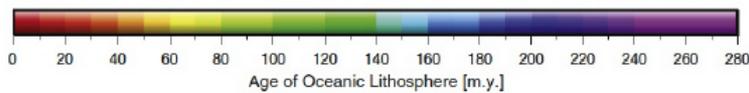
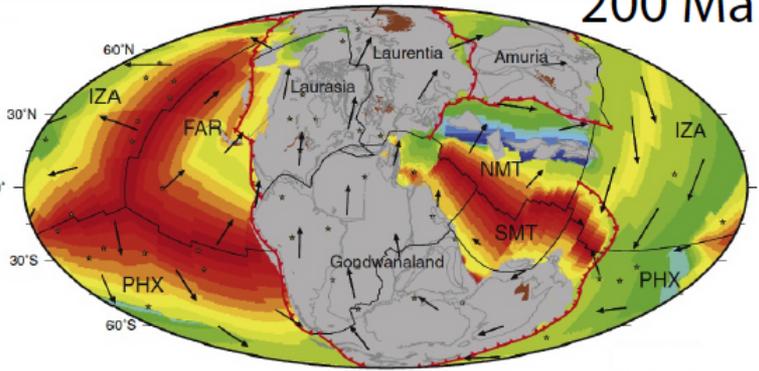
0 Ma



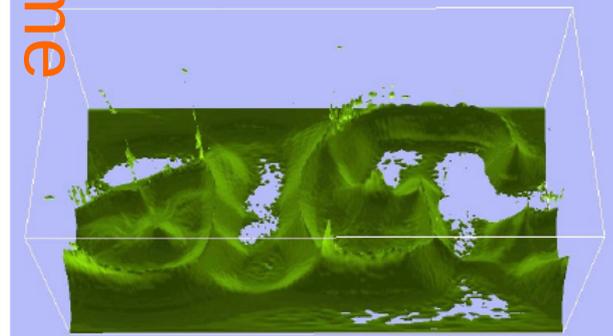
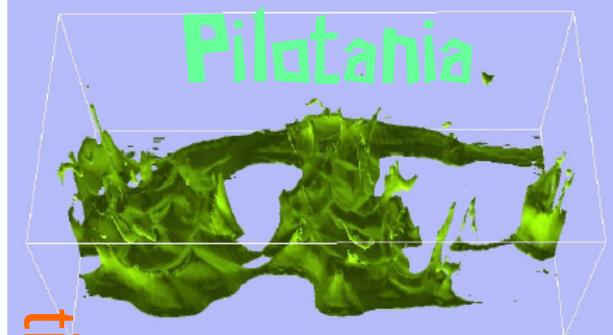
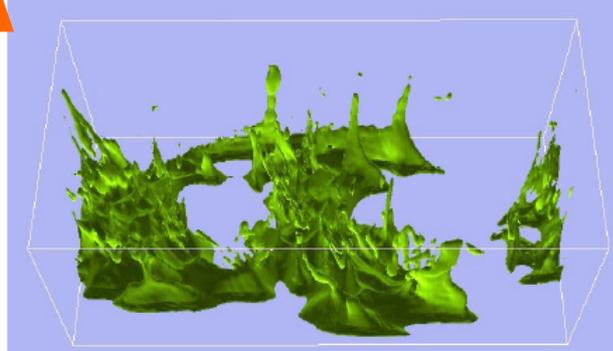
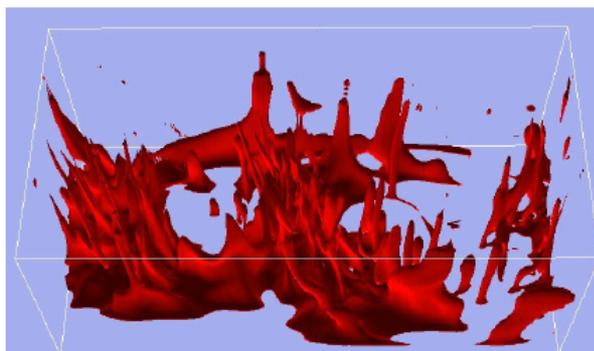
100 Ma



200 Ma



Seton et al. (2012)



time

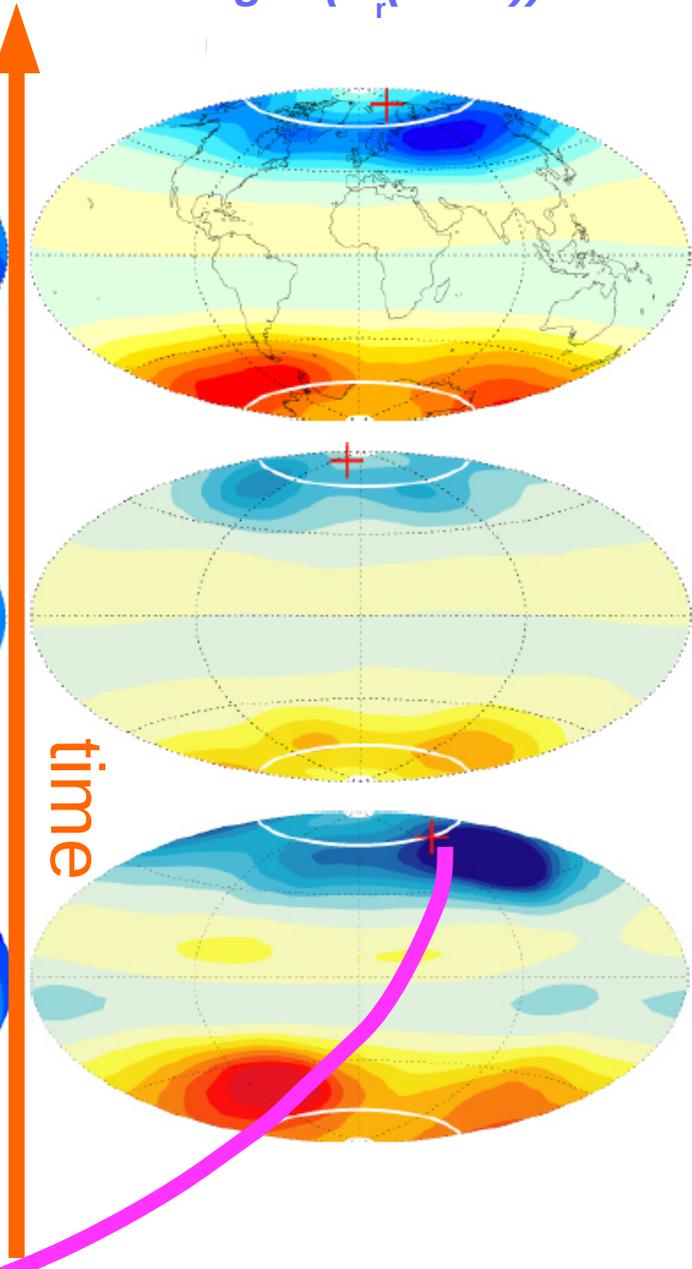
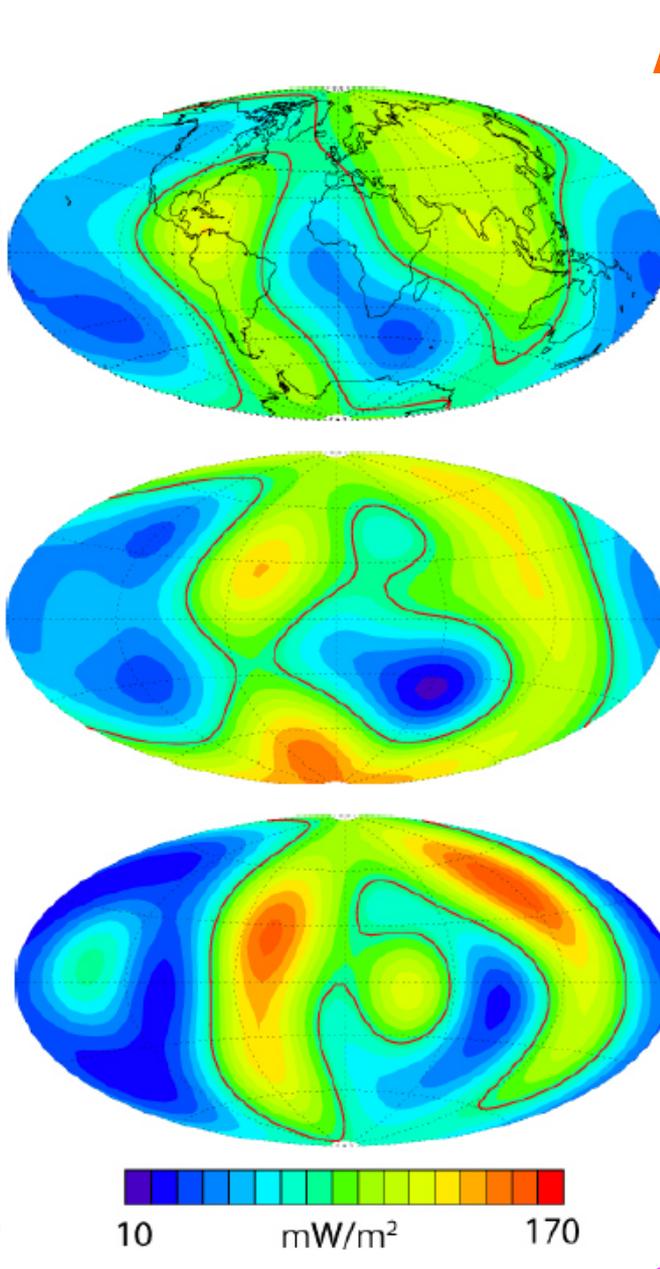
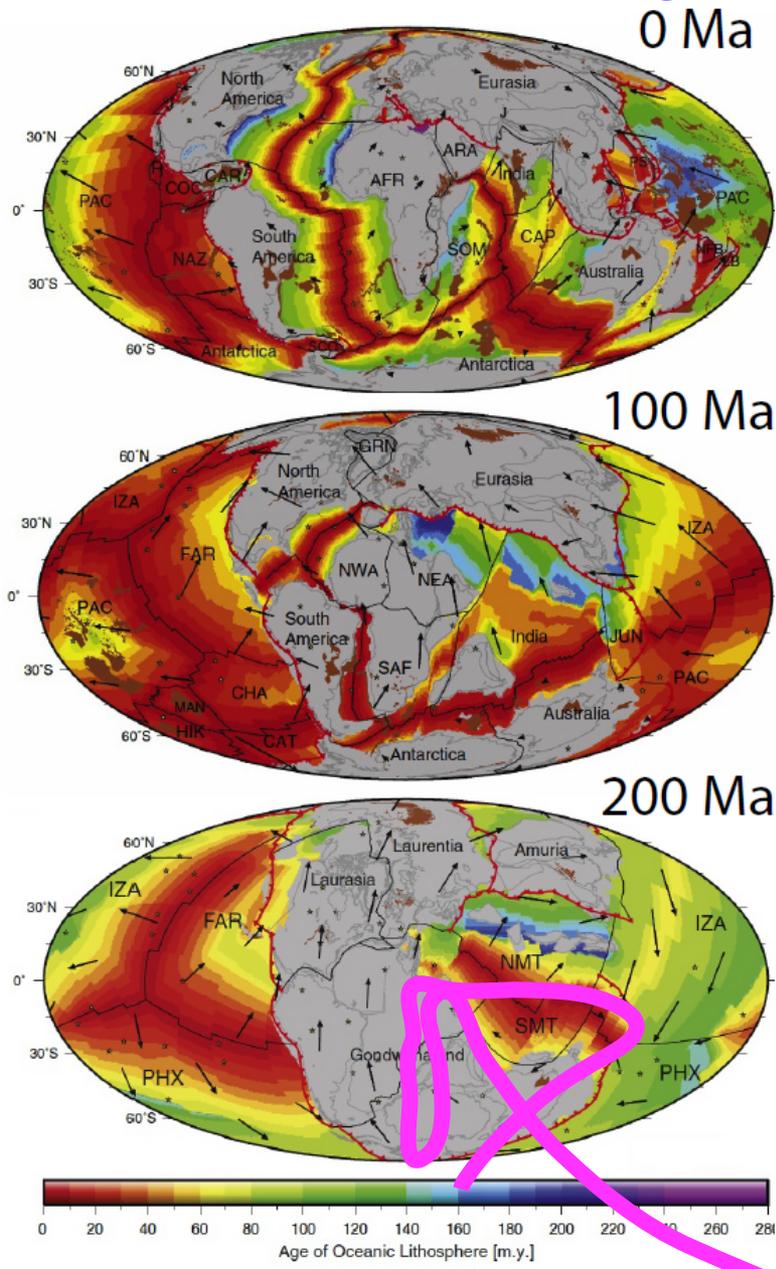
Olson et al. (2016)

cf. McNamara and Zhong (2005) and many others

reconstructed seafloor age

CMB heat flow

magnetic field strength ( $B_r(\text{CMB})$ )



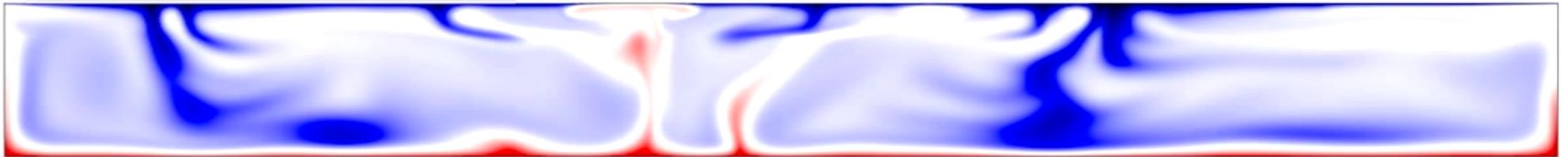
Seton et al. (2012)

Olson (2016)

**We know how  
plate tectonics  
works**

(~true, and often assumed)

# Plate tectonics is the top boundary layer of thermo-chemical convection

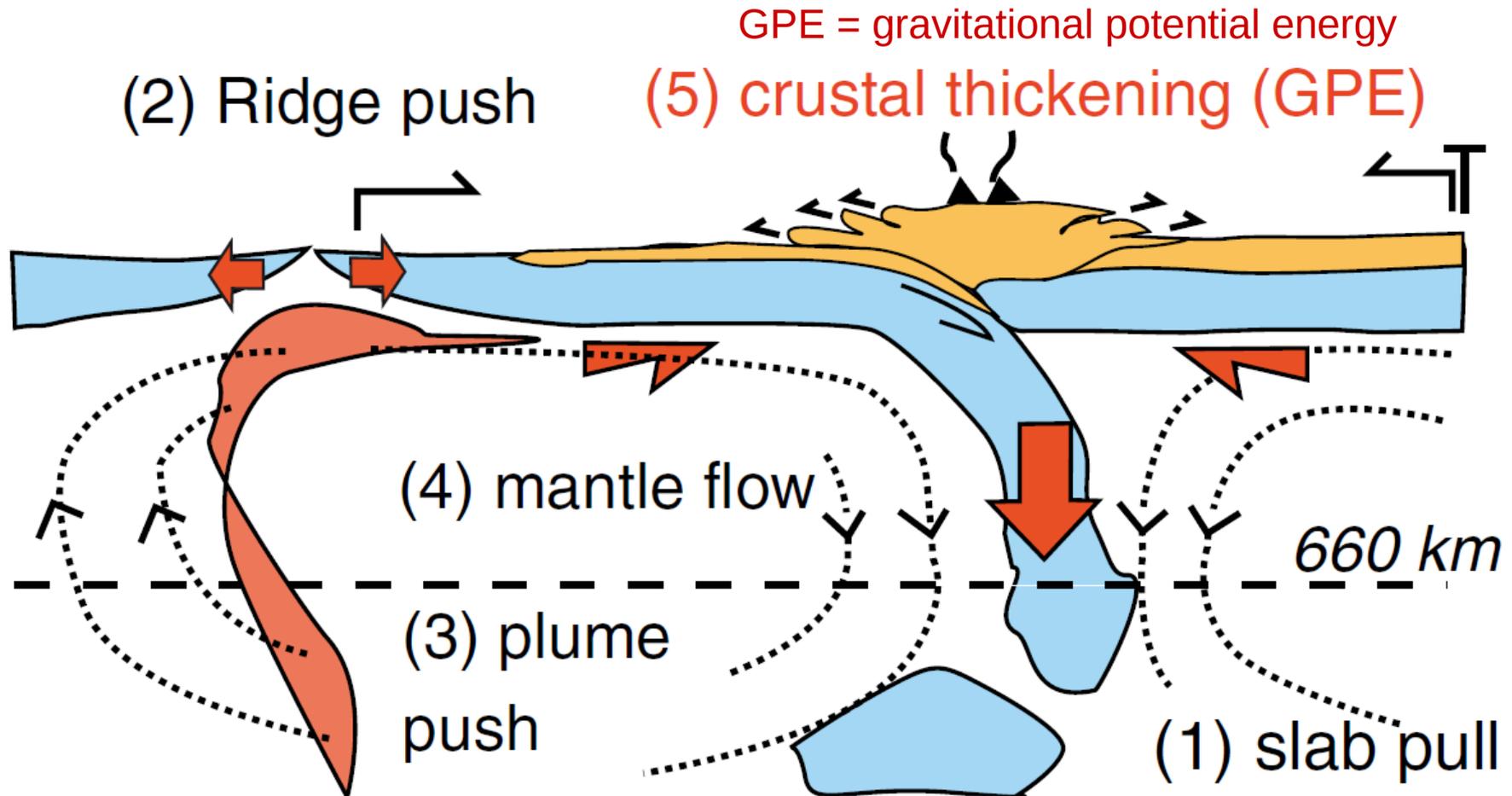


Symmetry of upwellings and downwellings of Rayleigh-Taylor convection is broken by:

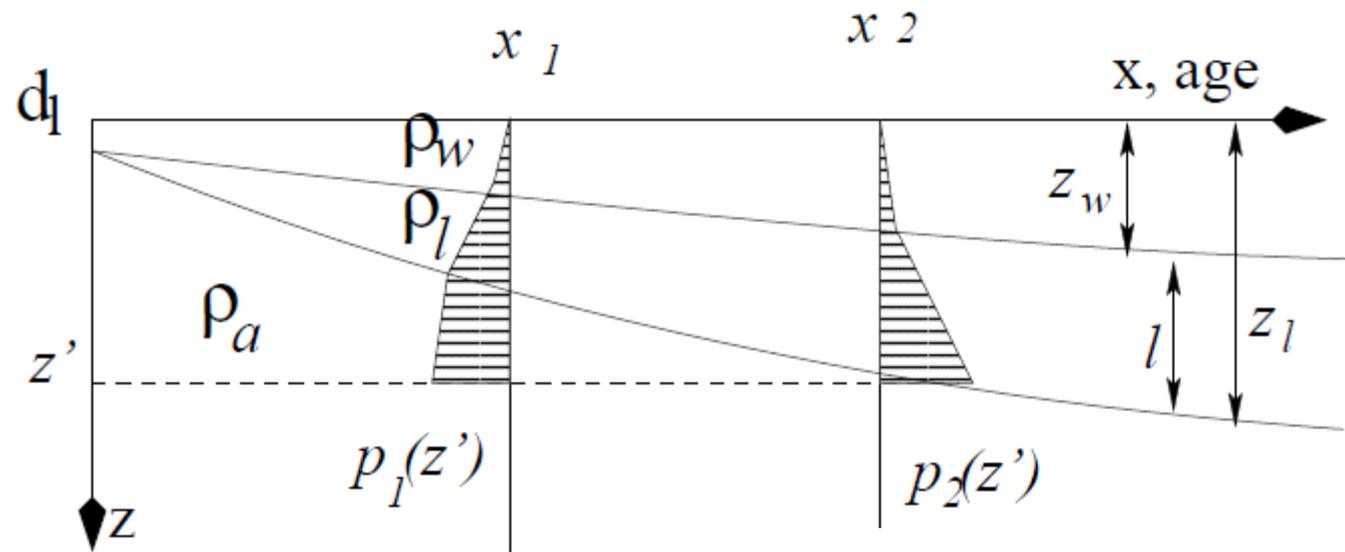
- **temperature dependence of viscosity**
- **depth-dependent viscosity**
- internal vs. bottom heating
- fractionation (e.g. continents and thermo-chemical piles)
- ...

# Plate driving forces:

Integrals over individual components of thermo-chemical convection



# Lithospheric thickening (AKA ridge push, oceanic GPE)

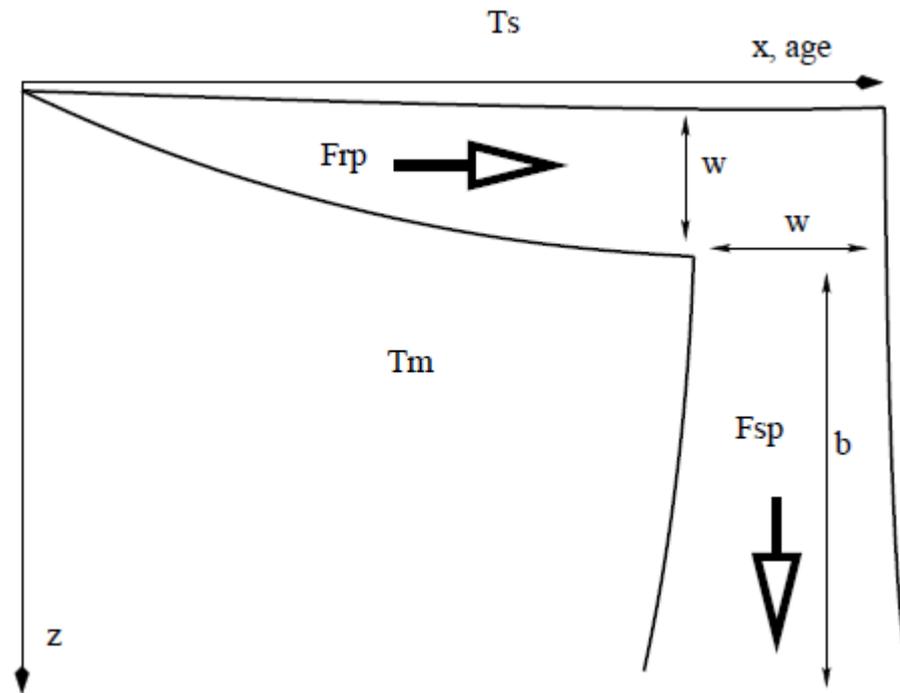


$$l = a(z_w - d_l) = a\Delta z_w \quad a = \frac{\rho_w - \rho_a}{\rho_a - \rho_l} \quad z_w(x) = c\sqrt{\frac{x}{u_0}}$$

$$F = \frac{ac^2}{2u_0} g \Delta \rho \Delta x$$

# Slab pull

(subducting thermal boundary layer)

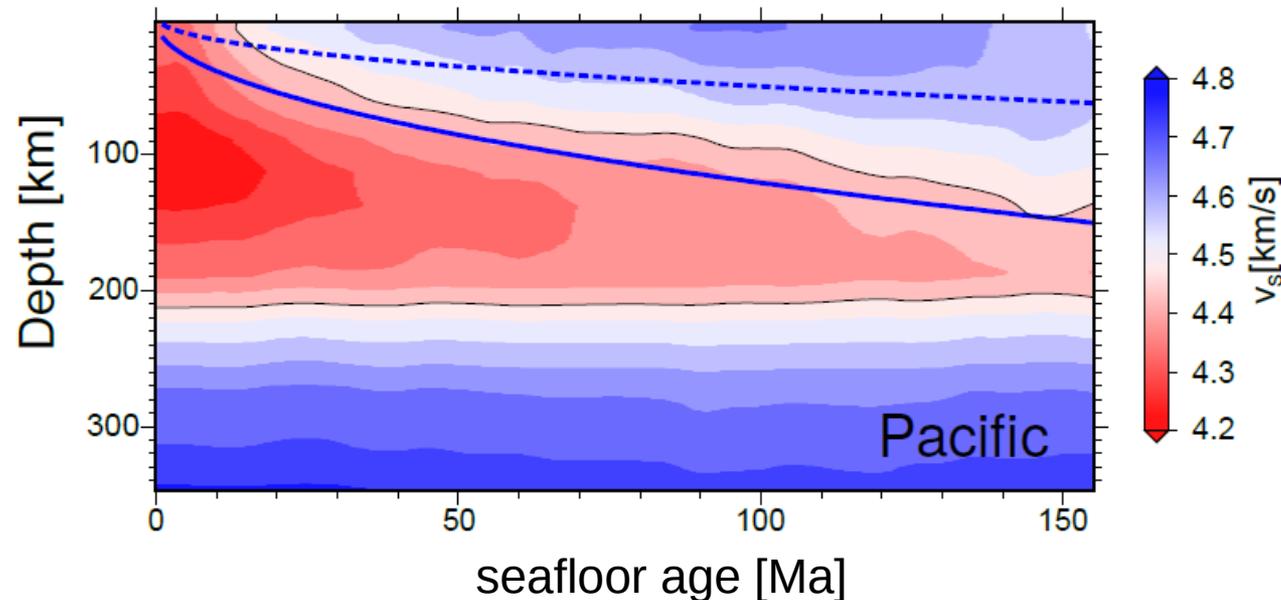


$$F_b = -2\rho_0 g \alpha b (T_m - T_s) \sqrt{\frac{\kappa t_s}{\pi}}$$

# Force estimates from half-space cooling

(hugely important reference model and  
achievement of geodynamics)

- Ridge push (lithospheric thickening)  $\sim 10^{12}$  N/m
- Slab pull  $\sim 10^{13}$  N/m



# Slab pull

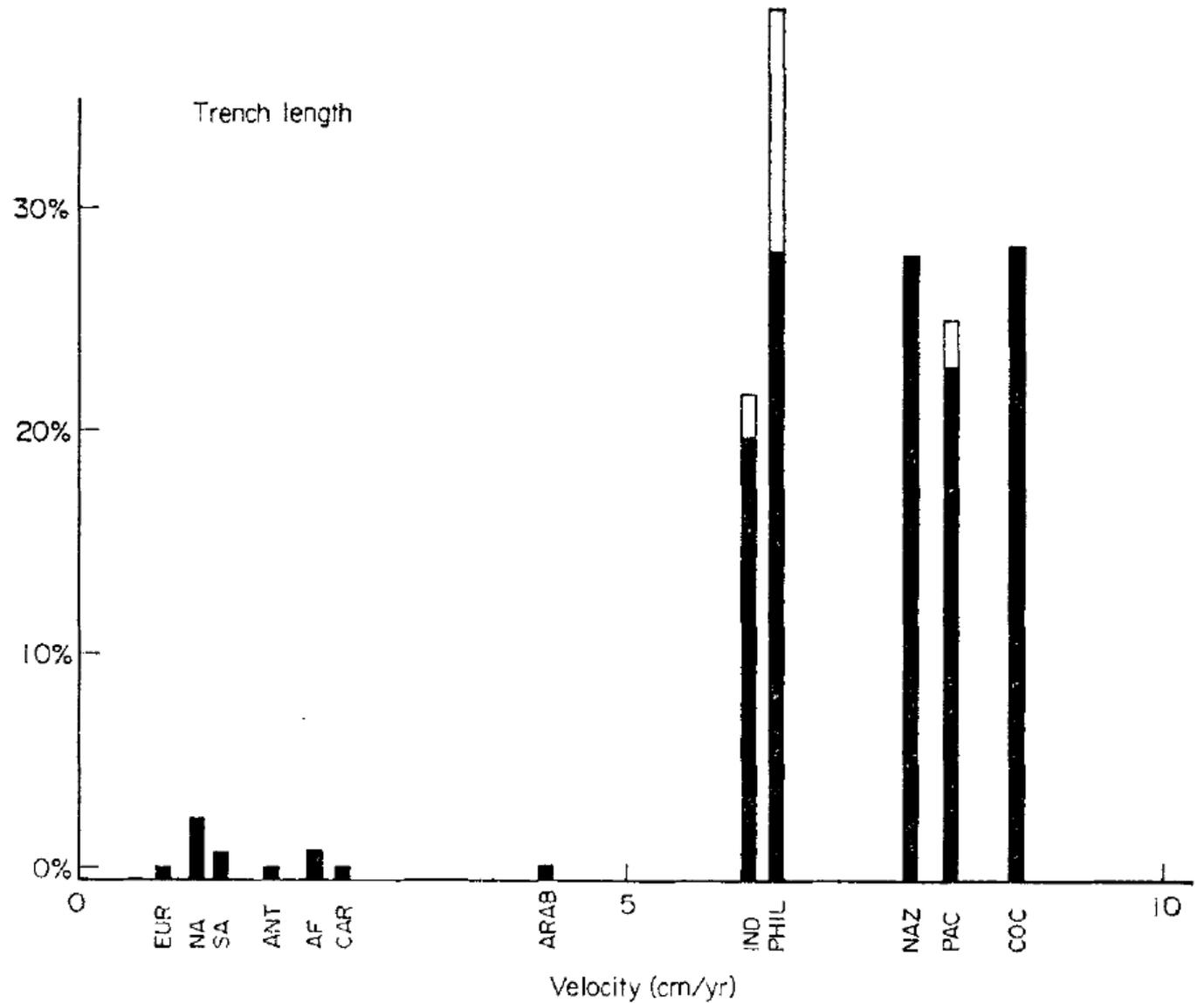
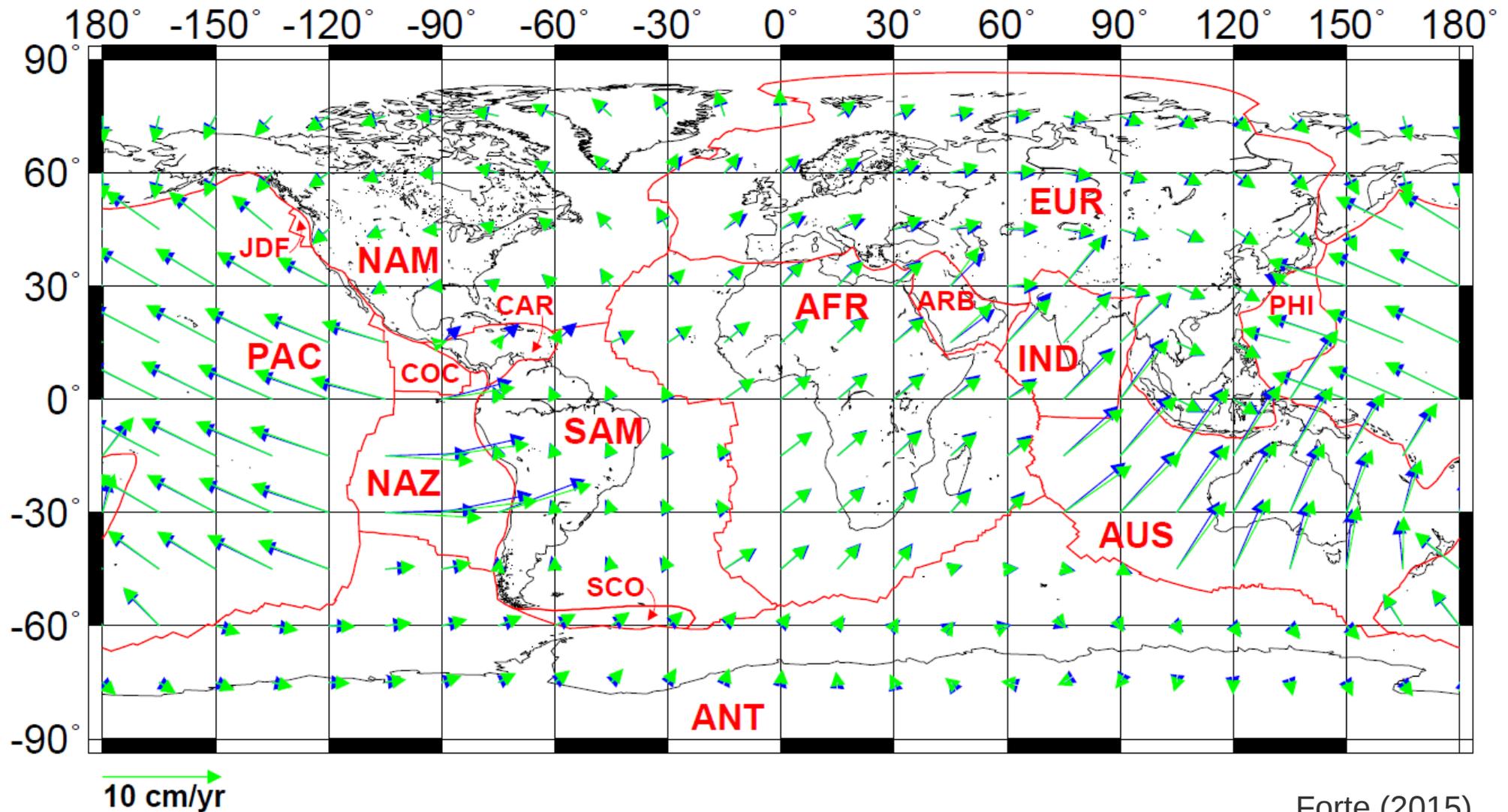


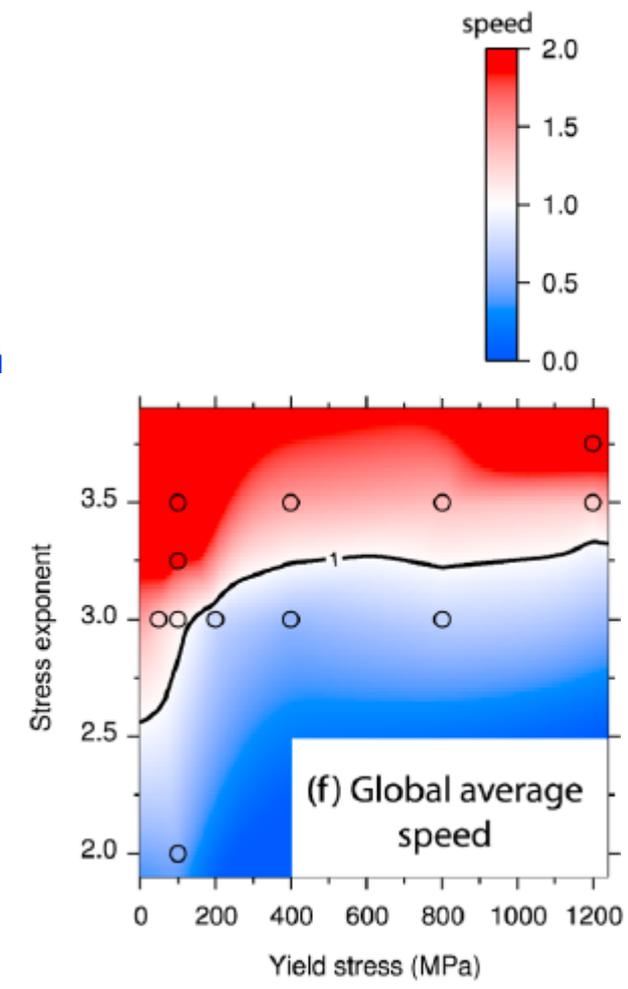
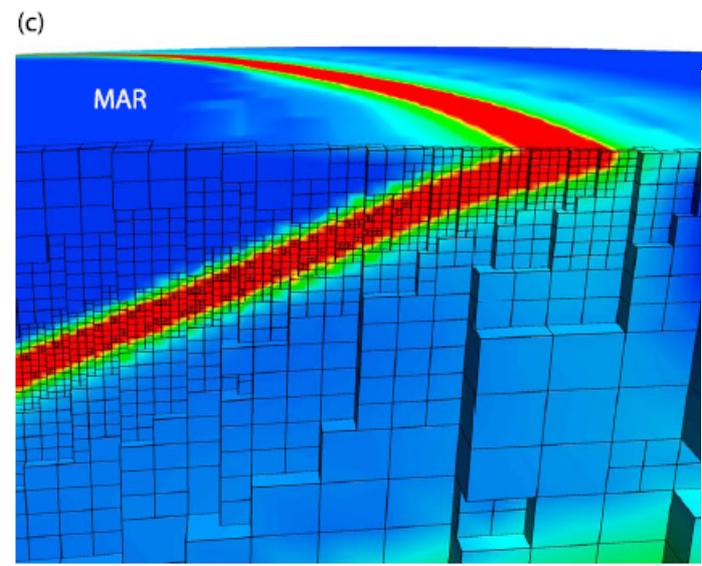
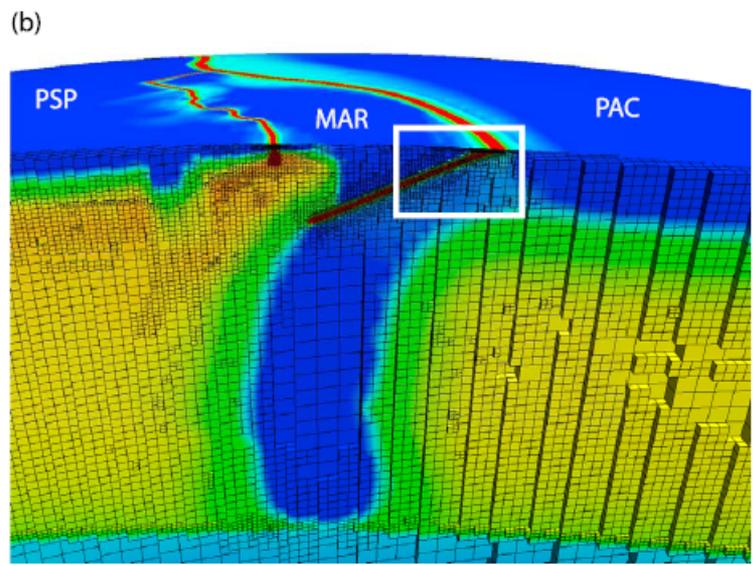
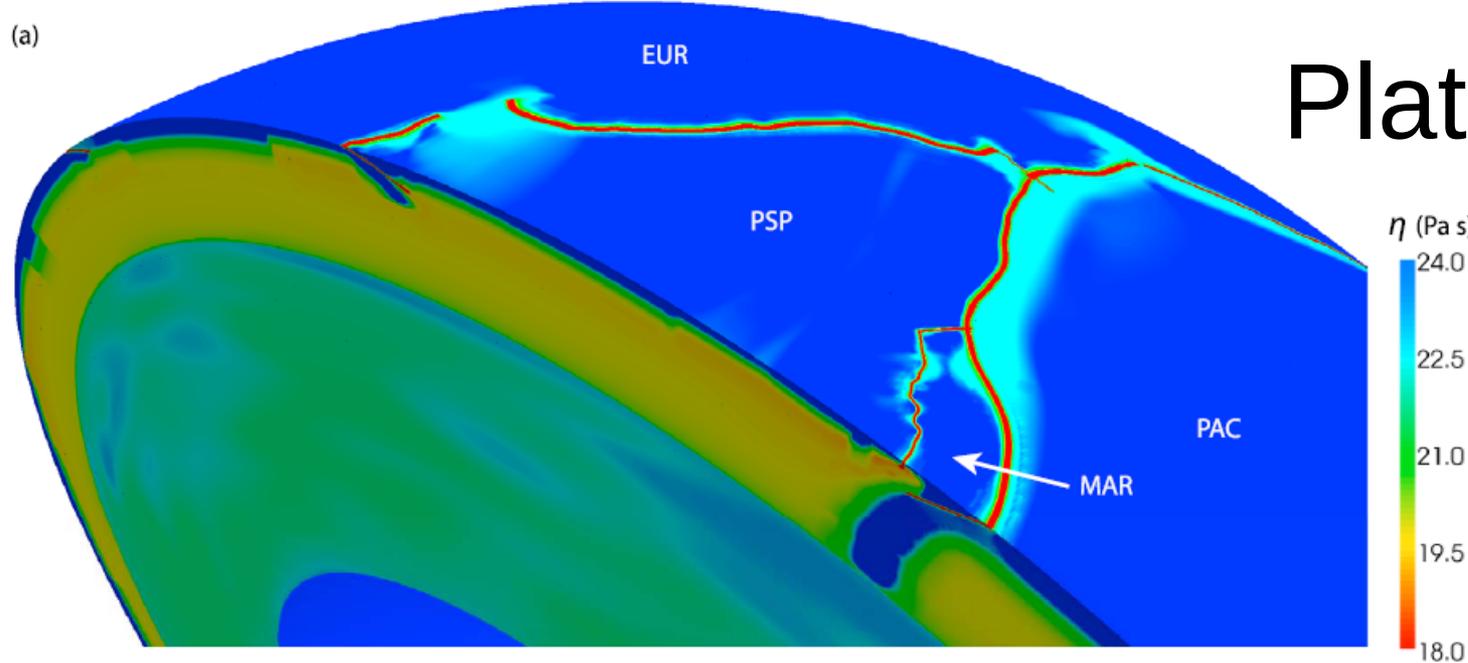
FIG. 8. Percentage of circumference of plate connected to downgoing slab. Open bar is total length, filled bar is effective length.

# How does the mantle drive the plates at present? Estimate from global circulation modeling

## Observed & Predicted Plate Motions in NNR Reference Frame



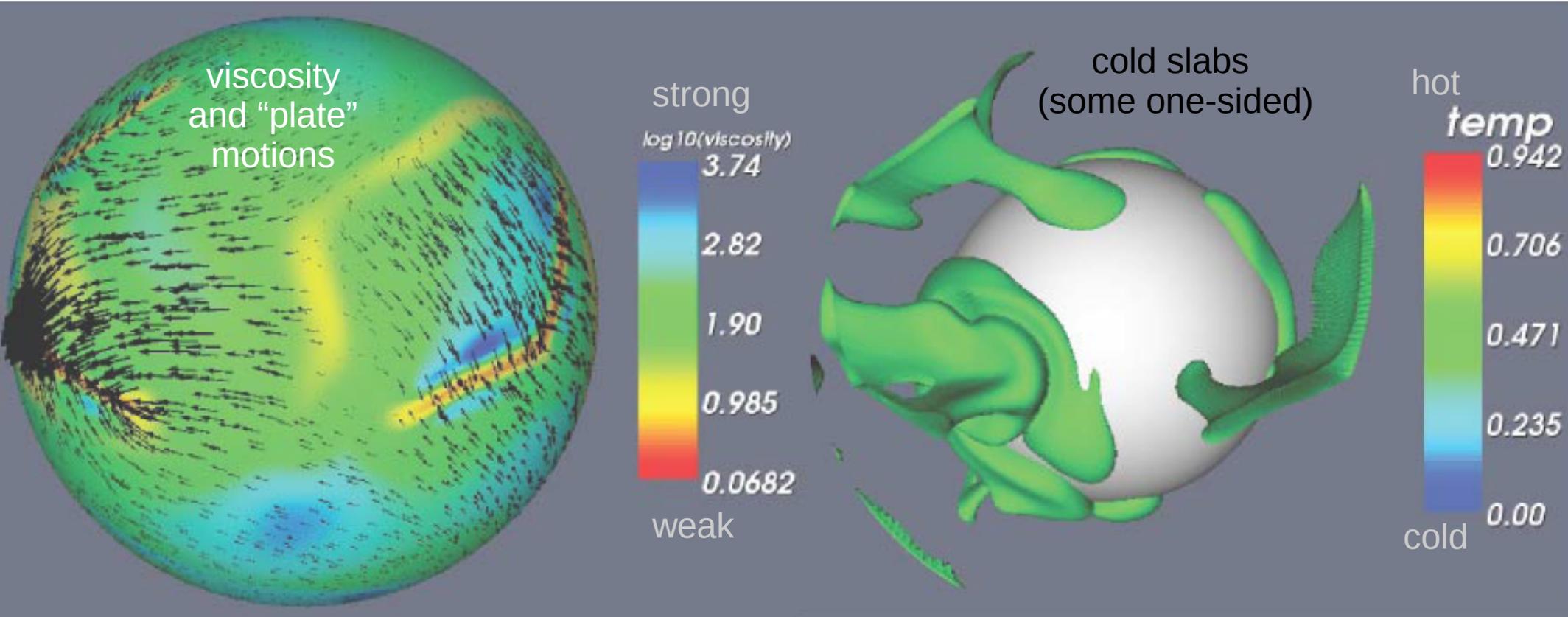
# Plate boundaries matter...



cf. Ricard and Vigny (1989), Forte (1993), King and Hager (1990), Gable et al. (1991), Han and Gurnis (1999)

Alisic et al. (2012)

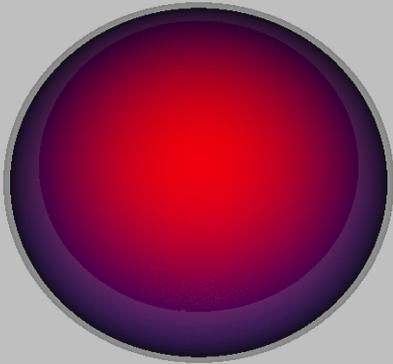
# Why do we have plate tectonics?



free-slip, global convection computation with temperature-dependent viscosity and yield stress,  $Ra > Ra_{\text{Earth}}$

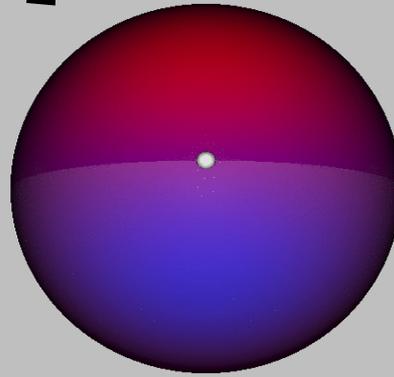
Foley & Becker (2009)

2, 0

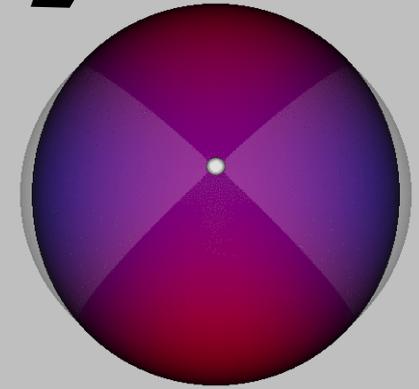


{l, m}

2, 1



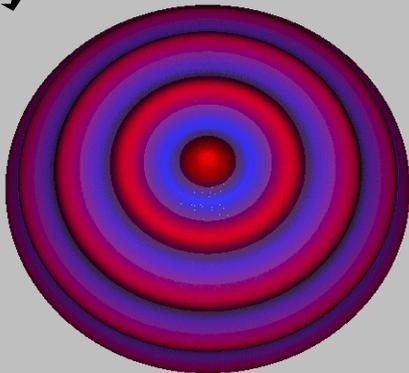
2, 2



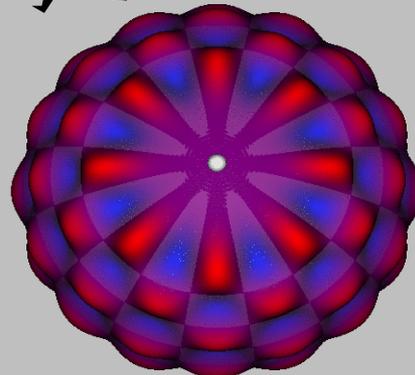
spherical harmonic basis function of degree  $l$  and order  $m$

$$\delta v(\theta, \phi) \approx \sum_{l=0}^{\ell_{\max}} \left[ a_{l0} X_{l0}(\theta) + \sqrt{2} \sum_{m=1}^{\ell} X_{lm}(\theta) \times (a_{lm} \cos m\phi + b_{lm} \sin m\phi) \right]$$

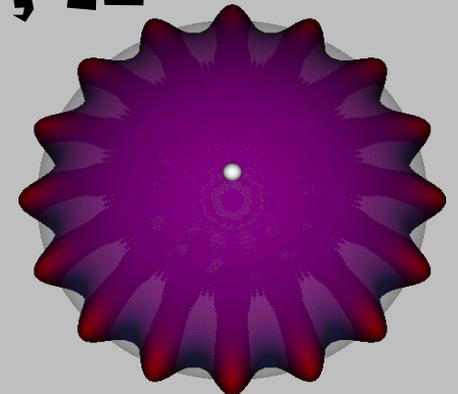
16, 0



16, 8

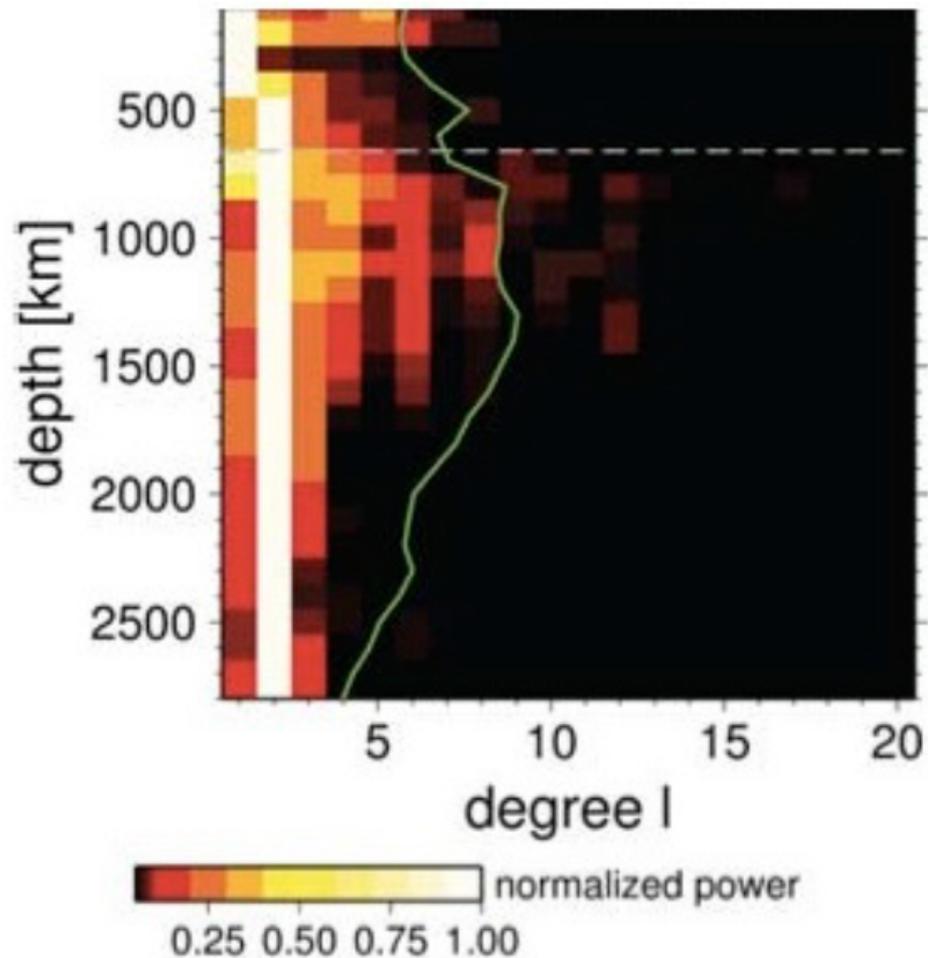


16, 16

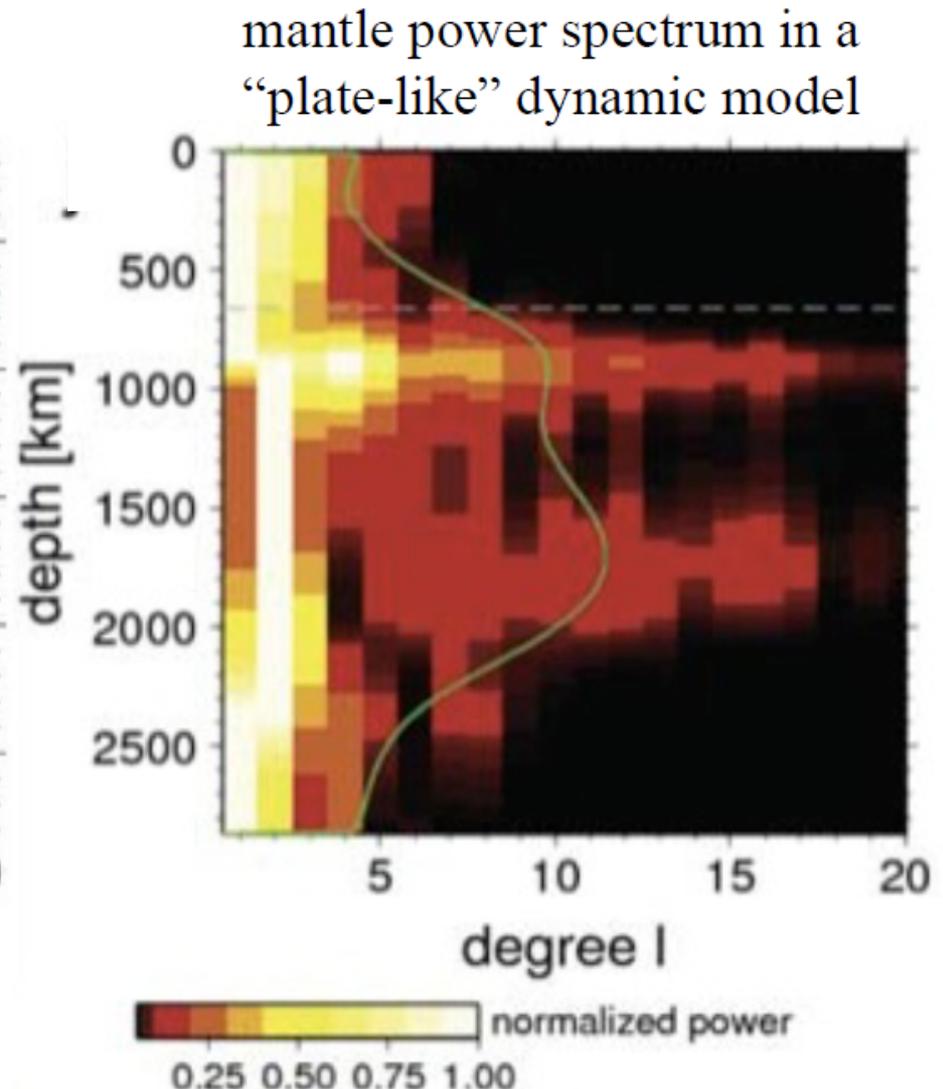
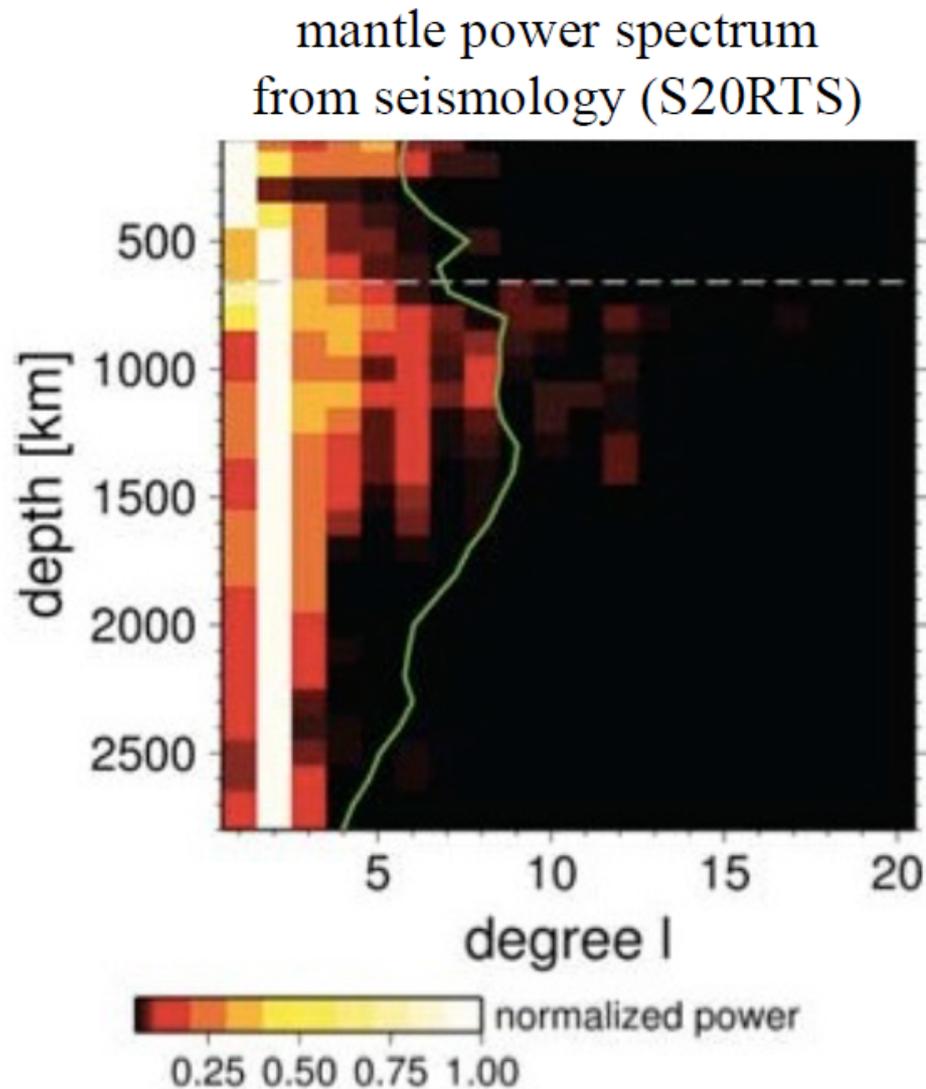


# Stochastic constraints: Mantle tomography shows long wavelength ( $l \sim 2$ ) structure

mantle power spectrum  
from seismology (S20RTS)



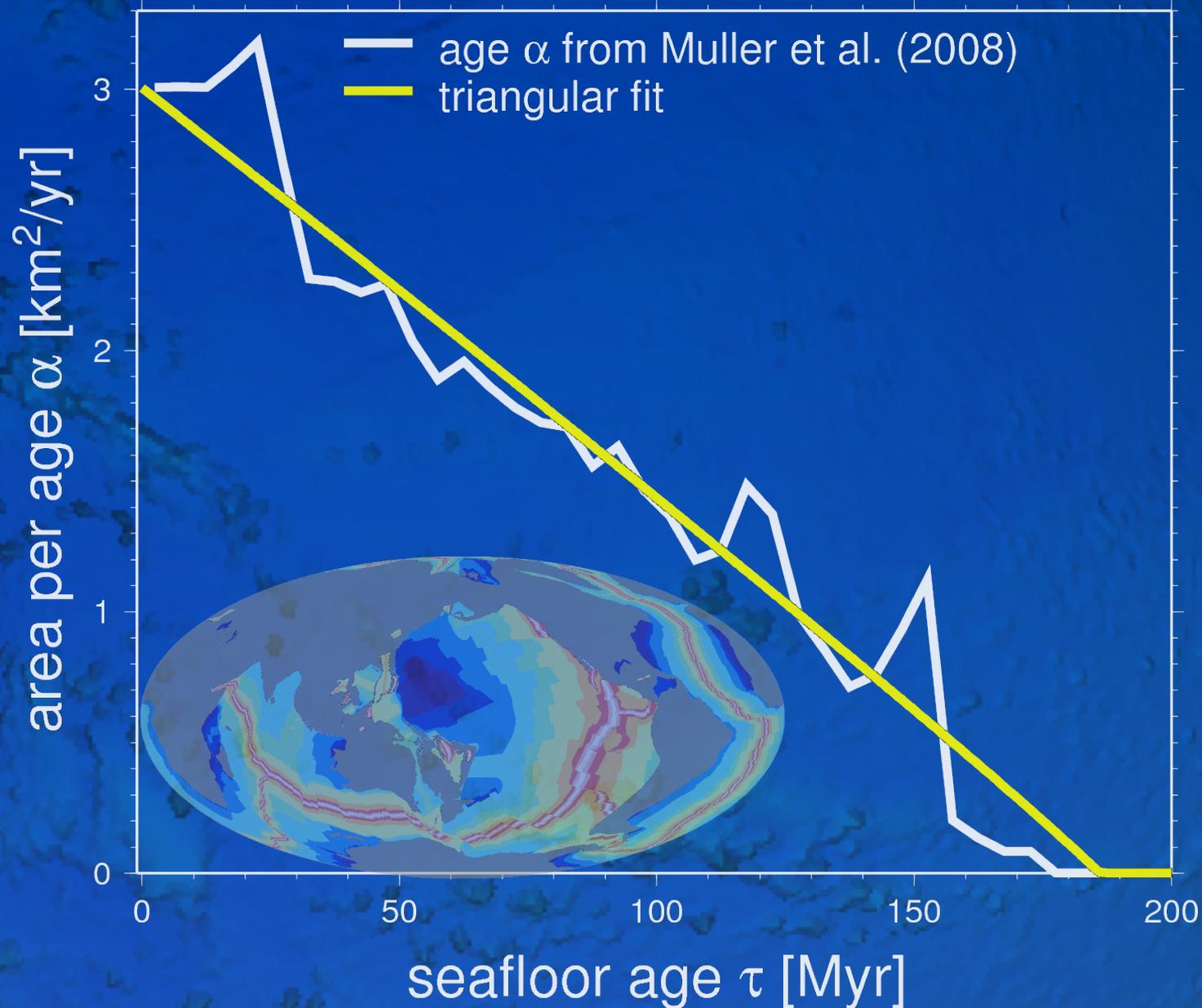
# Convection models predict this heterogeneity (because it has plates and those organize convection)



**We don't know how  
plate tectonics  
works**

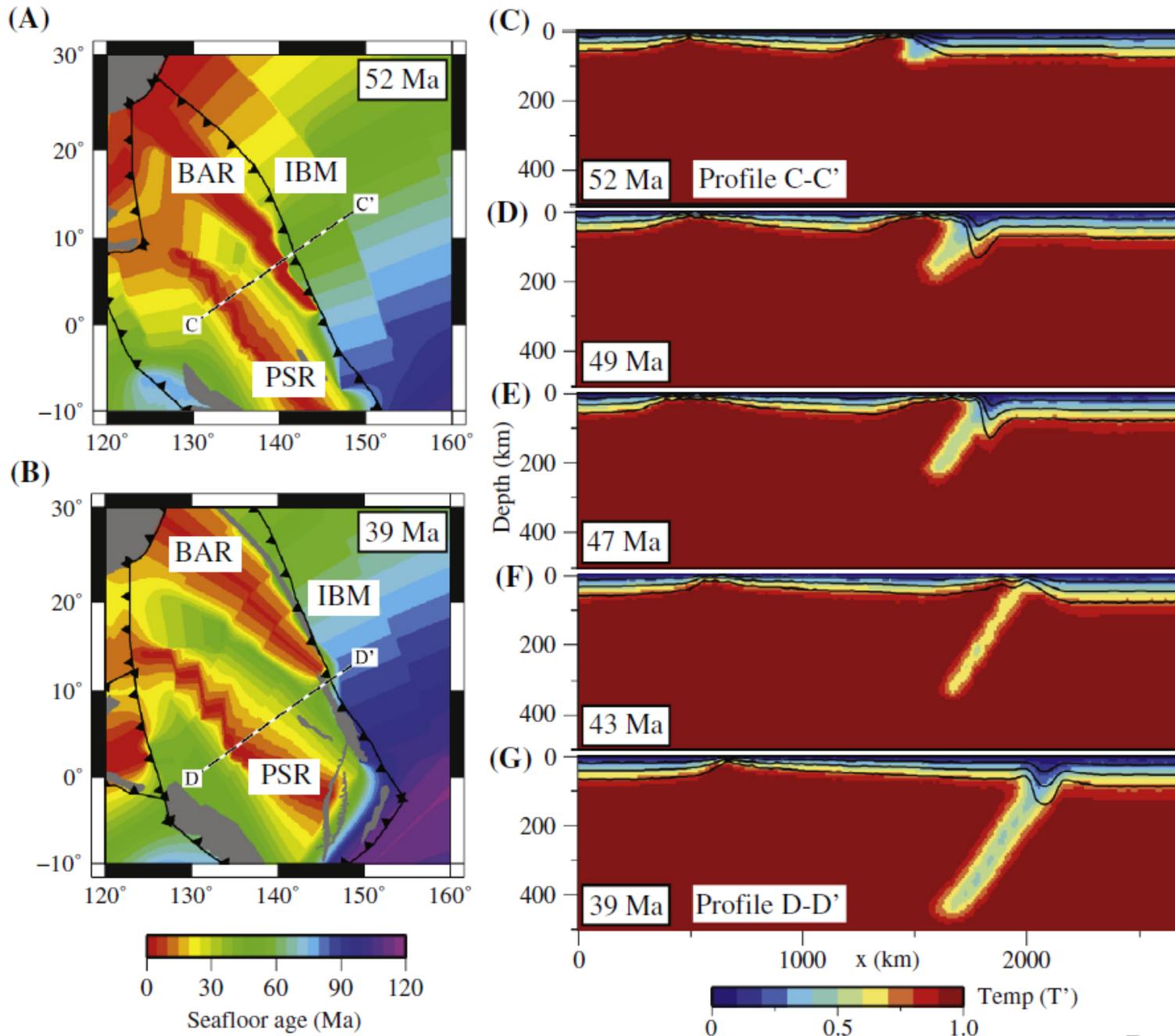
(questioning models assuming known physics)

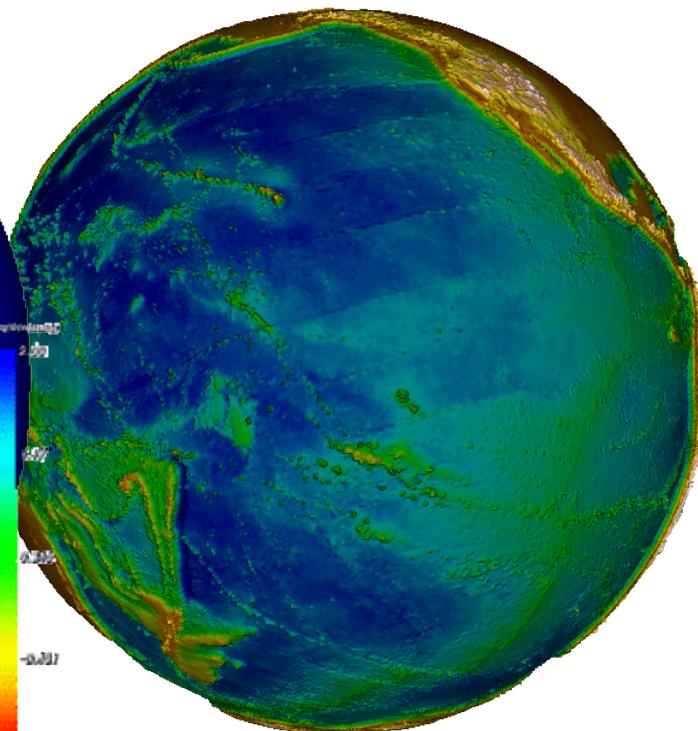
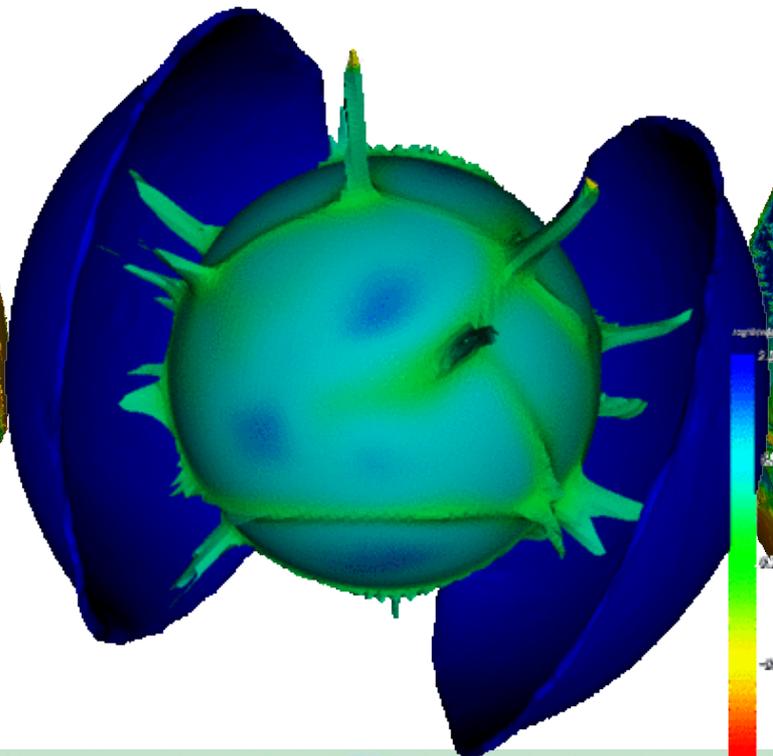
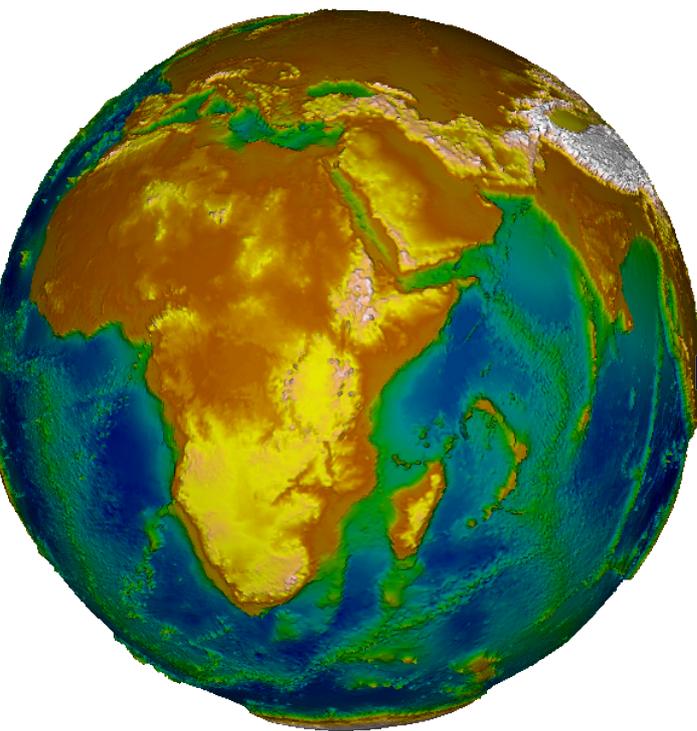
# Seafloor age distribution vs. boundary layer instability



cf. Becker et al. (2009)  
Coltice et al. (2013)

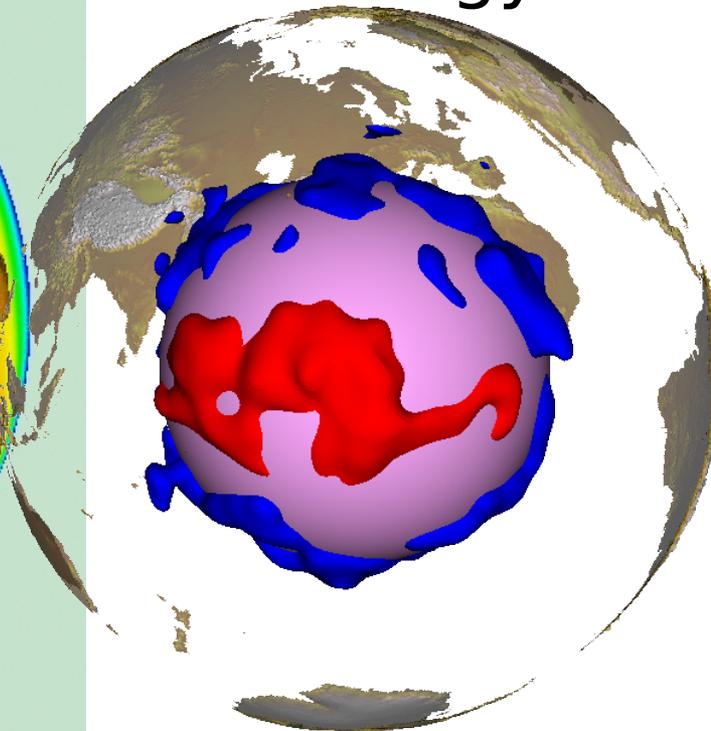
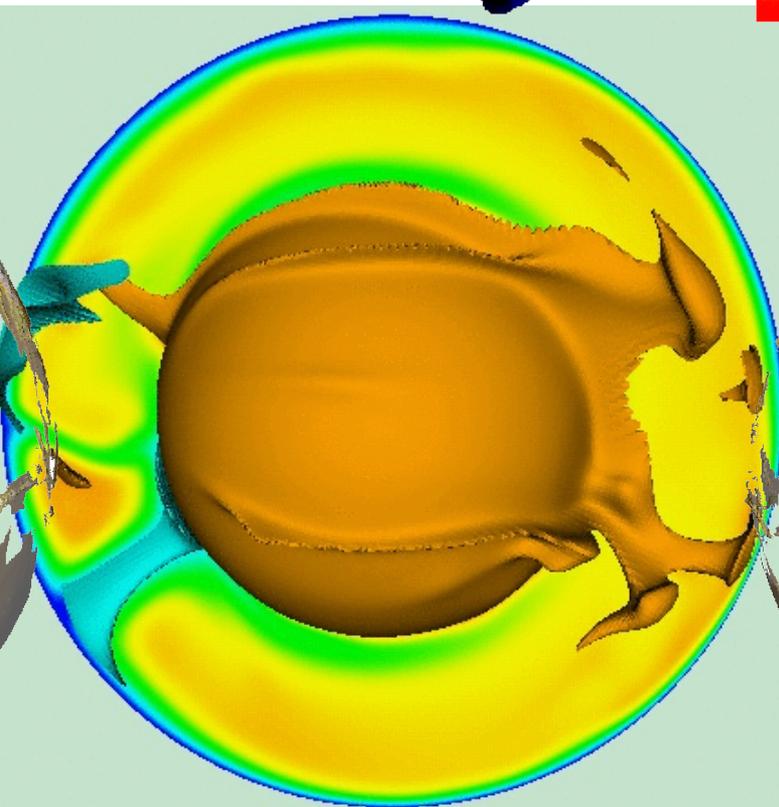
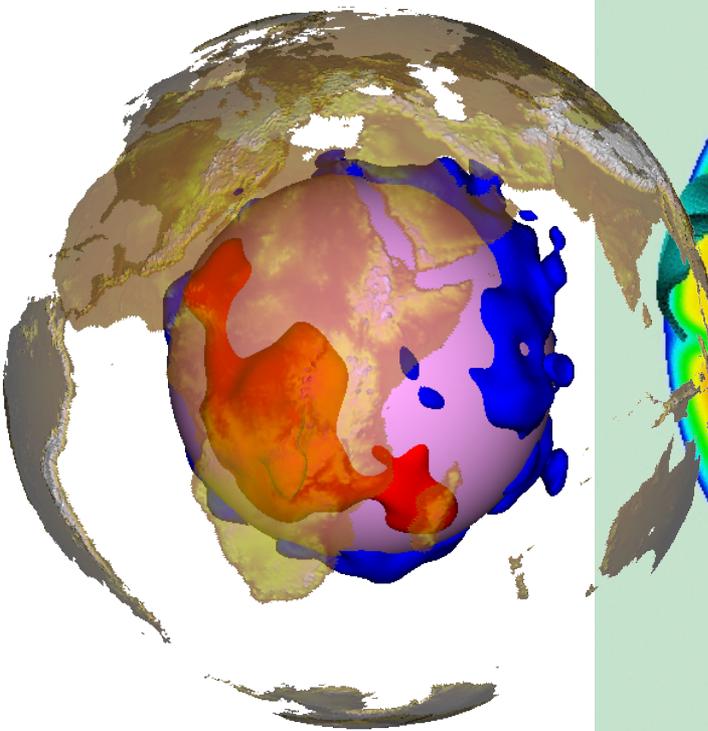
# Kinematic vs. dynamic models





composition

rheology



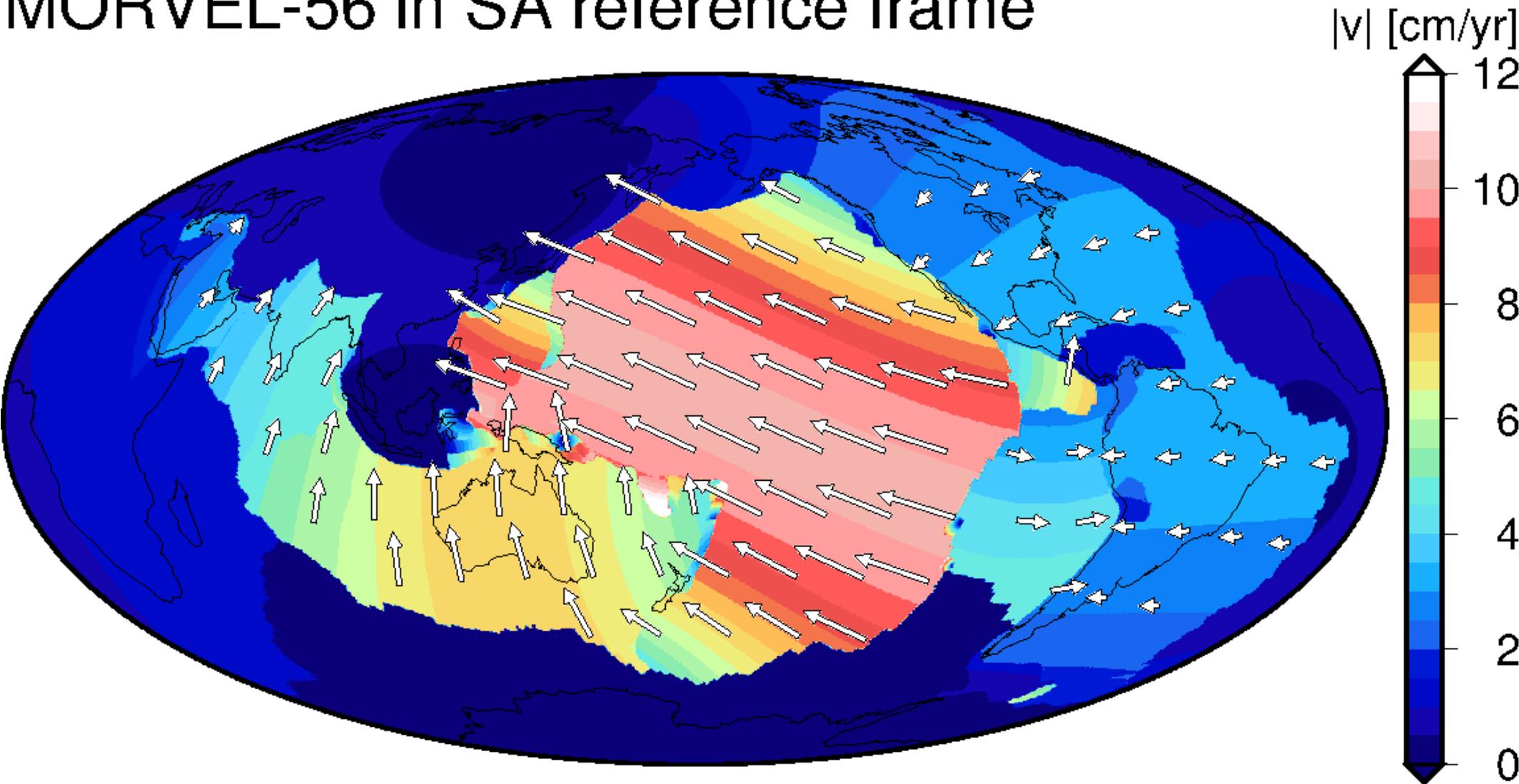
# Geophysical constraints on global mantle dynamics (applied geodynamics)

- Plate velocities
- Topography
- Geoid
- Seismic tomography
- Seismic anisotropy

# Plate velocities

(net rotation, relative plate motions, distribution of strain-rate, toroidal vs. poloidal; now and in past)

## MORVEL-56 in SA reference frame



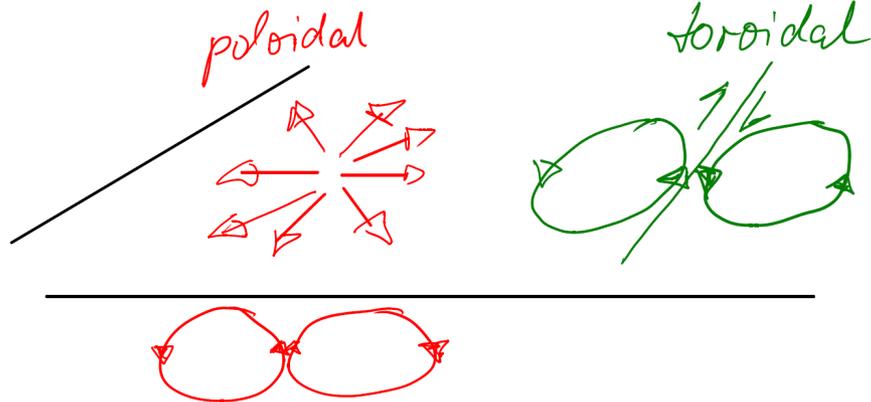
# Helmholtz decomposition

into irrotational (poloidal) and divergence free (toroidal) fields

$$\underline{v}_h \equiv \underline{v}_{h,\text{pol}} + \underline{v}_{h,\text{tor}} \equiv \underline{\nabla}_h \varphi + \underline{\nabla} \times (\psi \hat{z})$$

$$\nabla^2 \varphi = \underline{\nabla}_h \bullet \underline{v}_h; \nabla^2 \psi = \text{vort}_z(\underline{v}_h)$$

Or, in vector spherical harmonics (use *generalized* spherical harmonics for orientational,  $2\phi$ ):

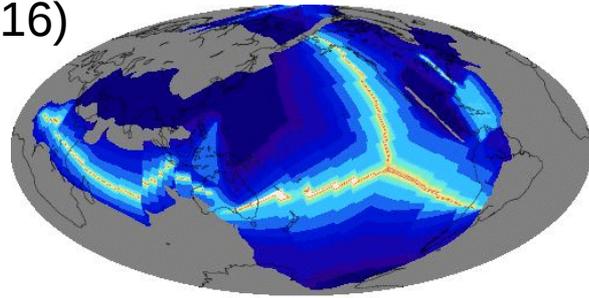


$$\mathbf{V}(\theta, \phi) = \sum_{lm} \text{Re}\{b_{lm} \mathbf{B}_{lm}(\theta, \phi) + c_{lm} \mathbf{C}_{lm}(\theta, \phi)\}$$

$$\mathbf{B}_{lm}(\theta, \phi) = \frac{1}{L} \frac{\partial P_{lm}(\theta)}{\partial \theta} e^{im\phi} \mathbf{e}_\theta + \frac{im}{L \sin \theta} P_{lm}(\theta) e^{im\phi} \mathbf{e}_\phi$$

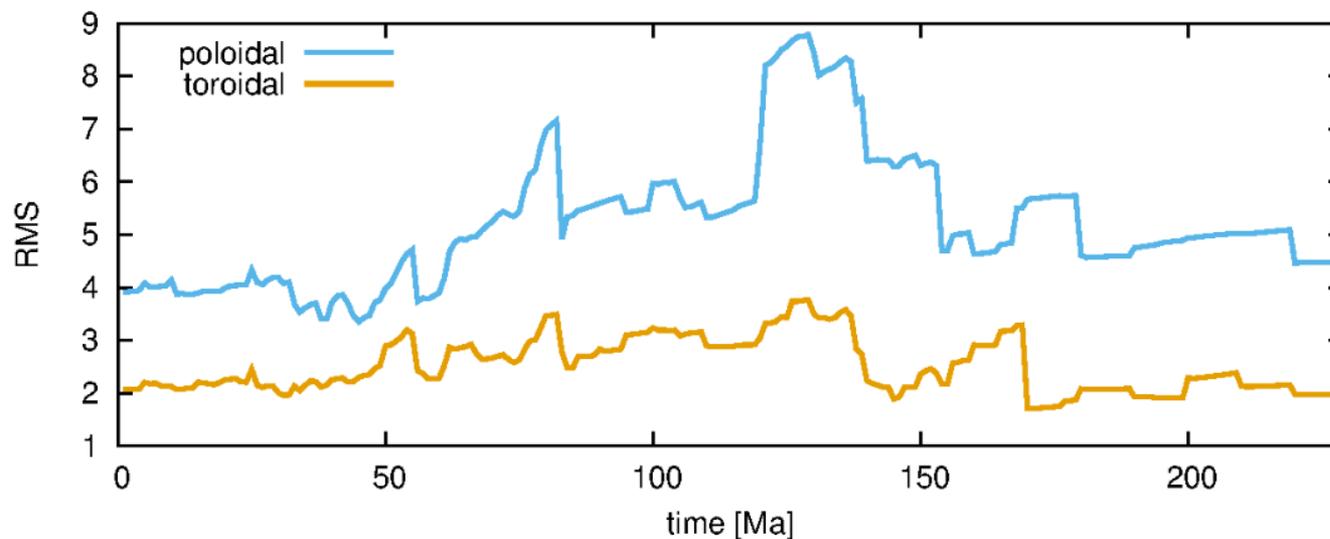
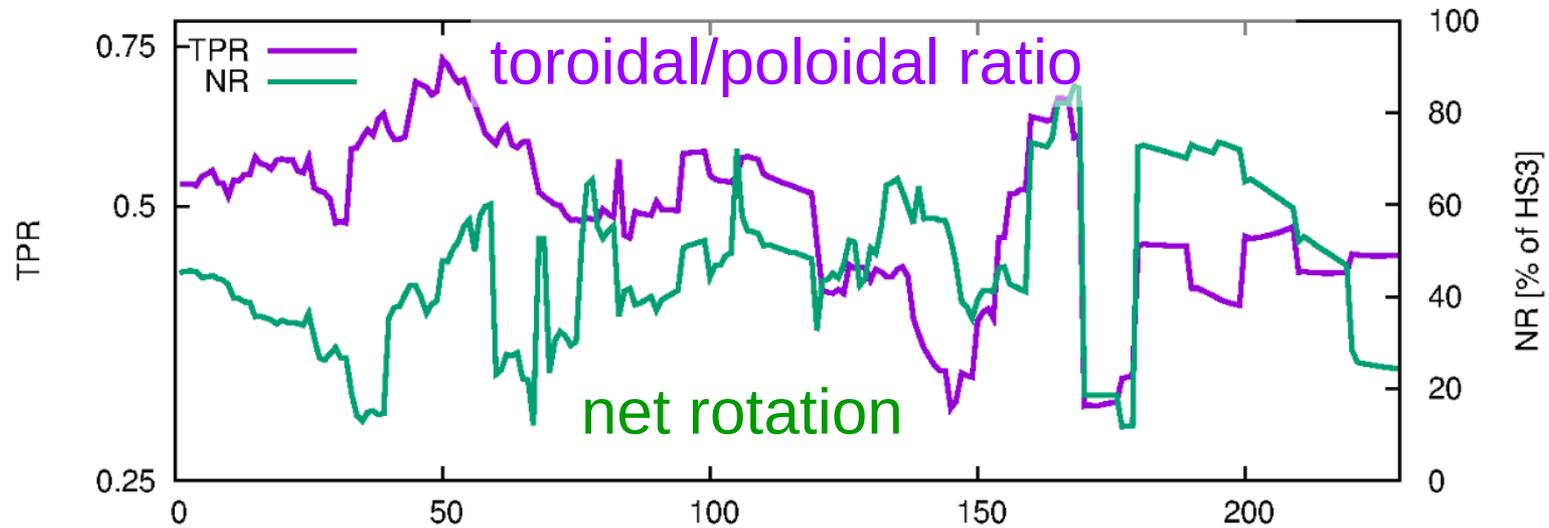
$$\mathbf{C}_{lm}(\theta, \phi) = \frac{im}{L \sin \theta} P_{lm}(\theta) e^{im\phi} \mathbf{e}_\theta - \frac{1}{L} \frac{\partial P_{lm}(\theta)}{\partial \theta} e^{im\phi} \mathbf{e}_\phi$$

Mueller et al.  
(2016)

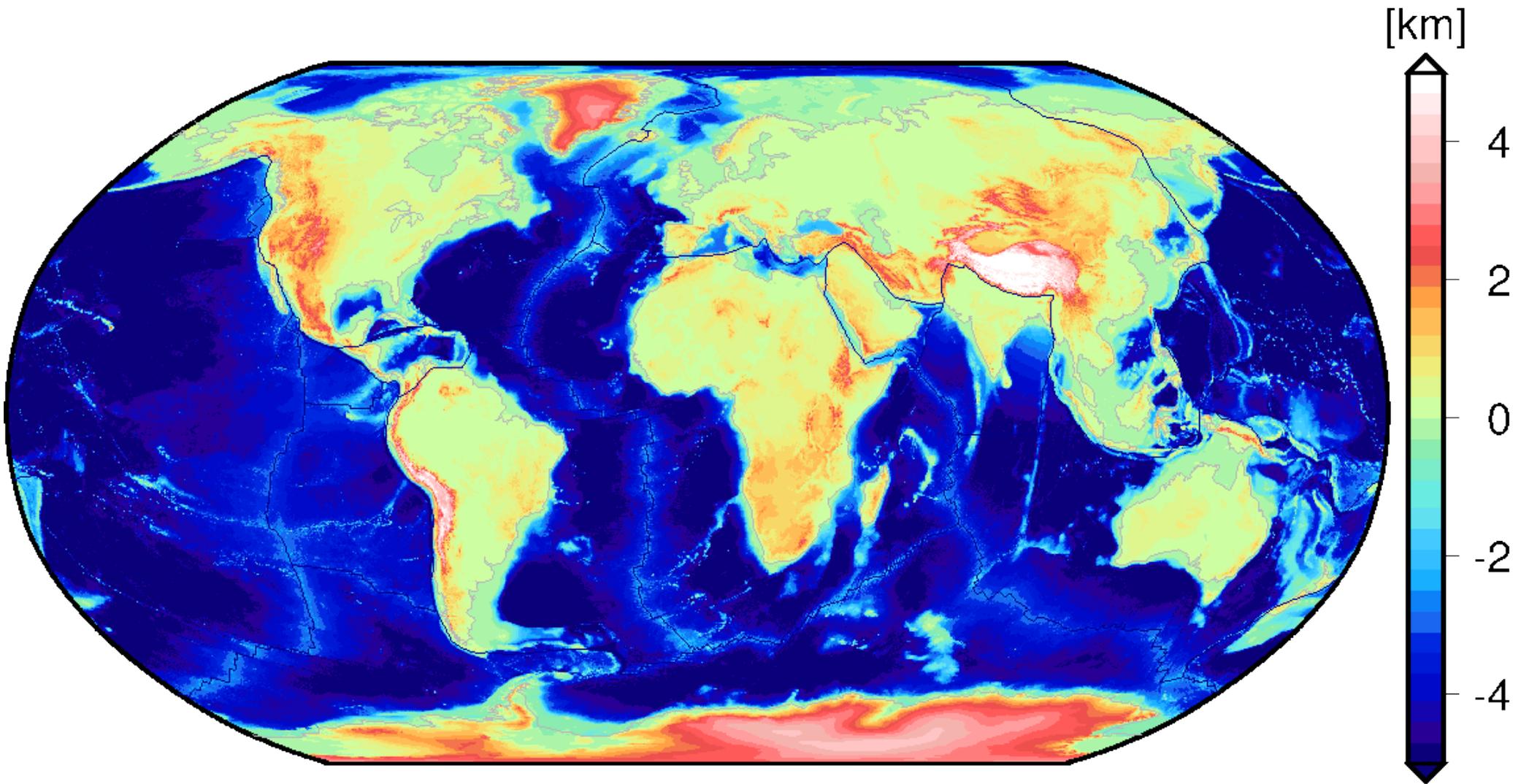


230 Ma  
0 50 100 150 200 seafloor age [Myr]

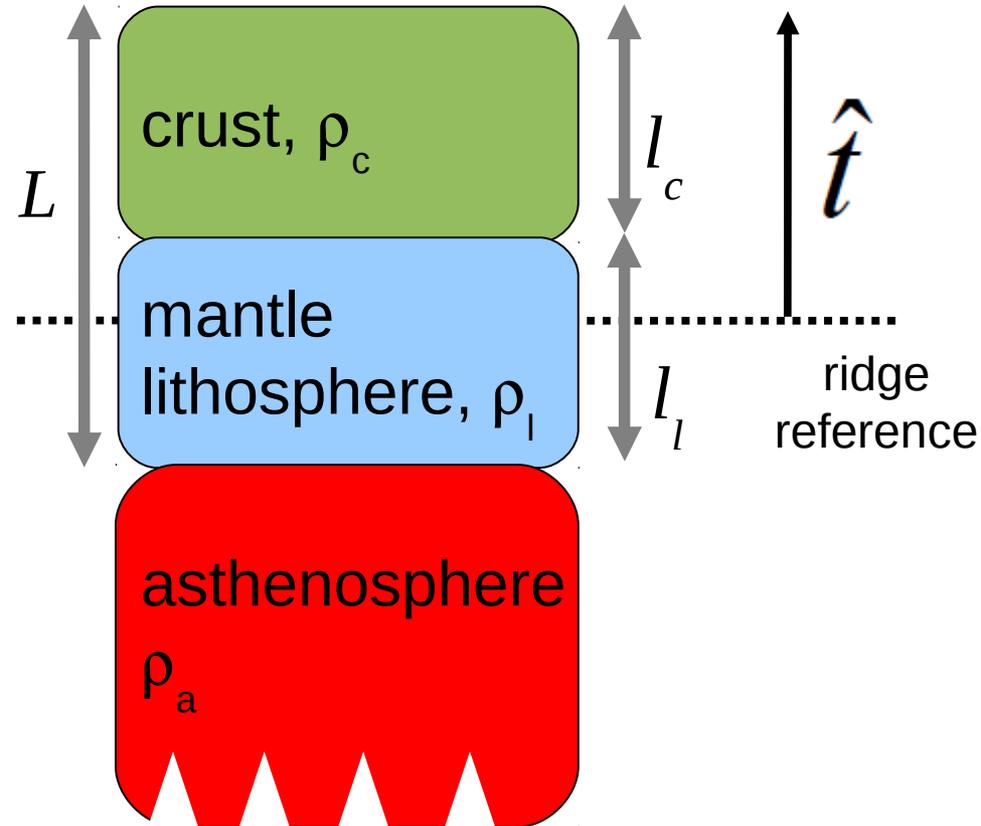
# Time-evolution of plate kinematic diagnostics



# Surface topography



# Isostasy



$$\hat{t} = f_1 l_c + f_2 l_l$$

$$f_1 = \frac{\rho_a - \rho_c}{\rho_a}, \quad f_2 = \frac{\rho_a - \rho_l}{\rho_a}$$

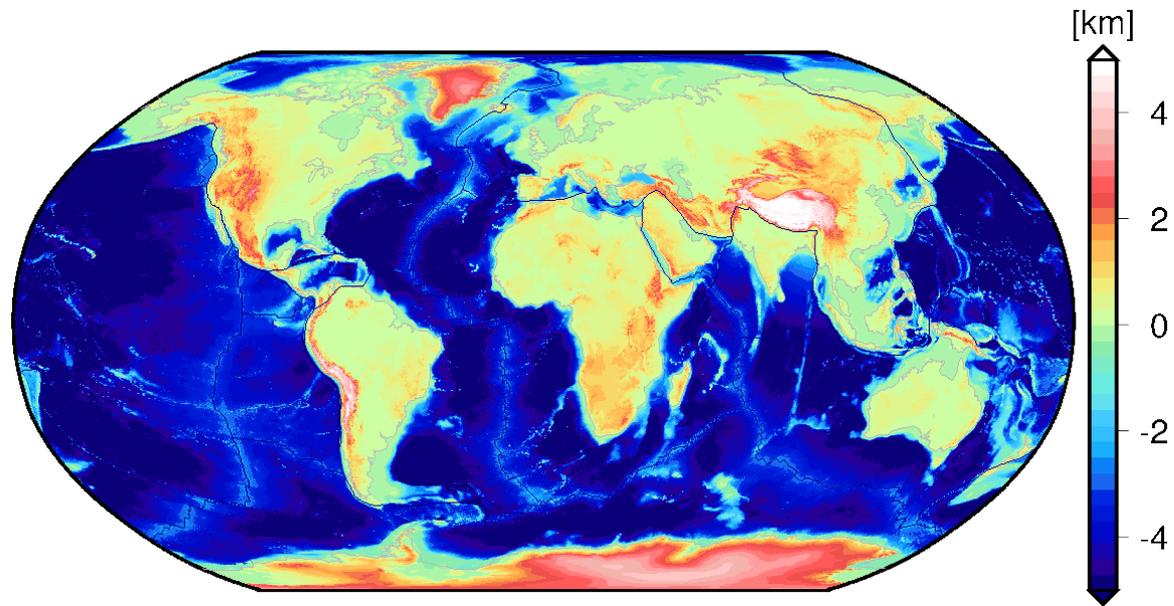
$$f_1 \approx 0.12 \quad f_2 \approx -0.01$$

Airy :  $\rho$  const. ( $\sim$ continents)

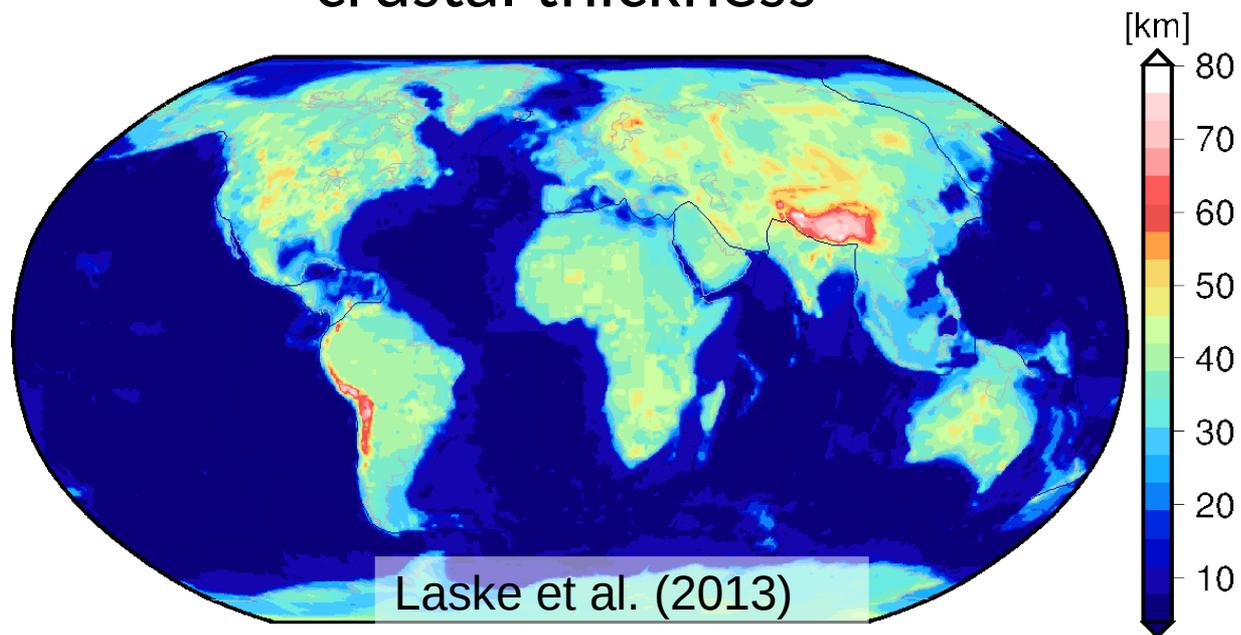
Pratt:  $l$  const. ( $\sim$ oceans)

$\rho = \text{const. @ compensation level}$

# actual topography

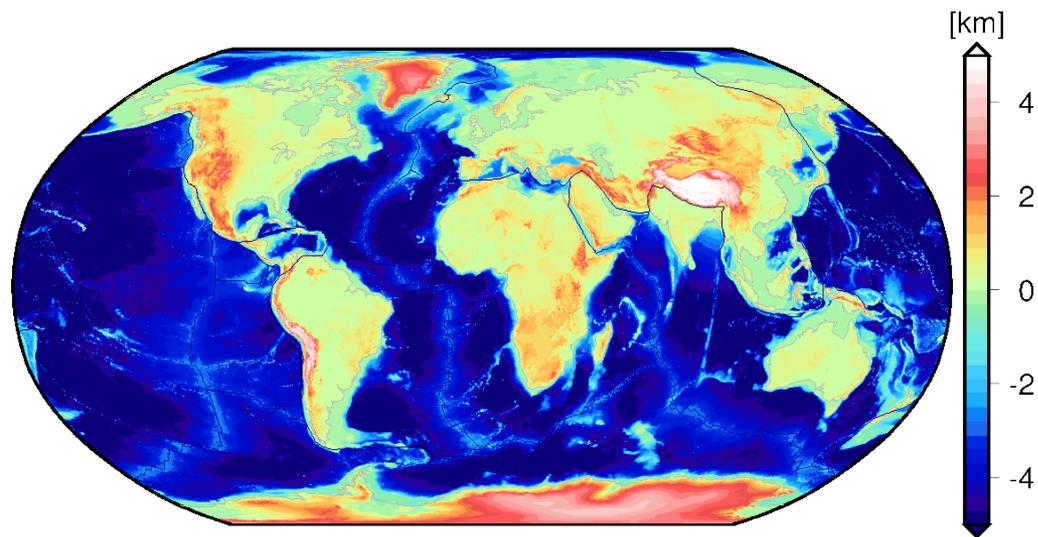


# crustal thickness



*Note:*  
very uneven  
coverage.  
active source  
best, RF next  
best thing

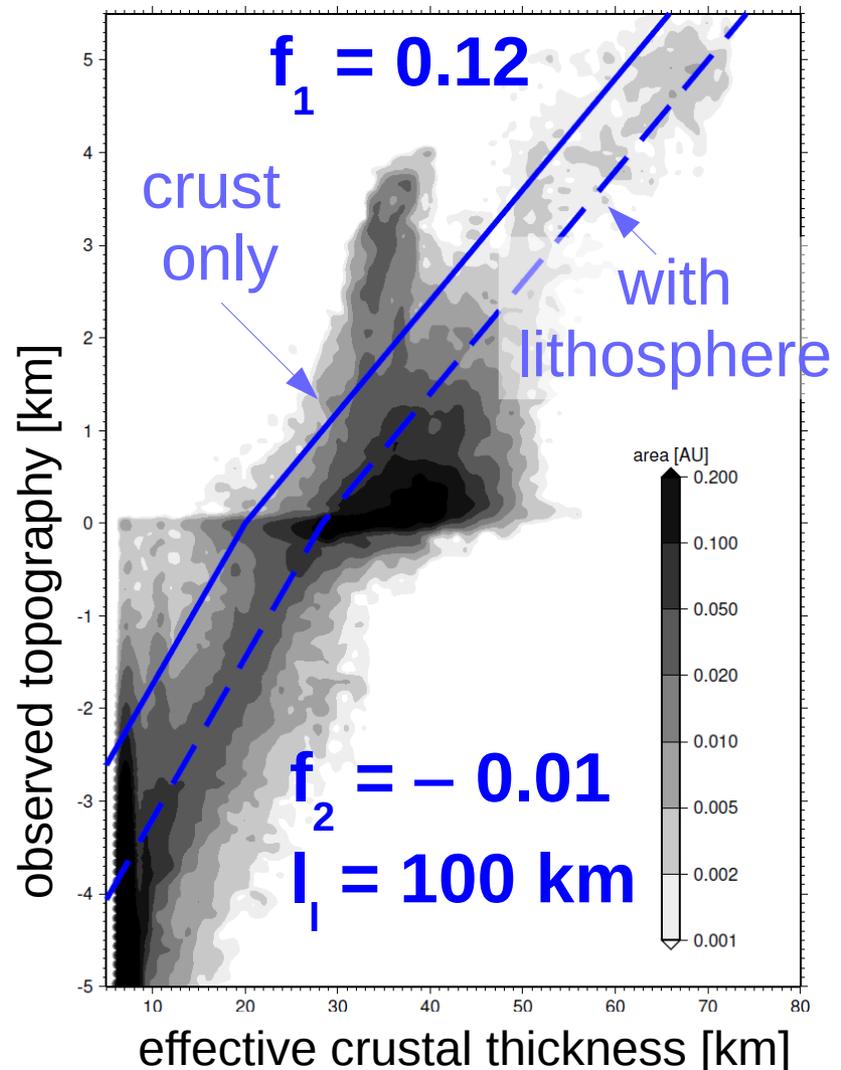
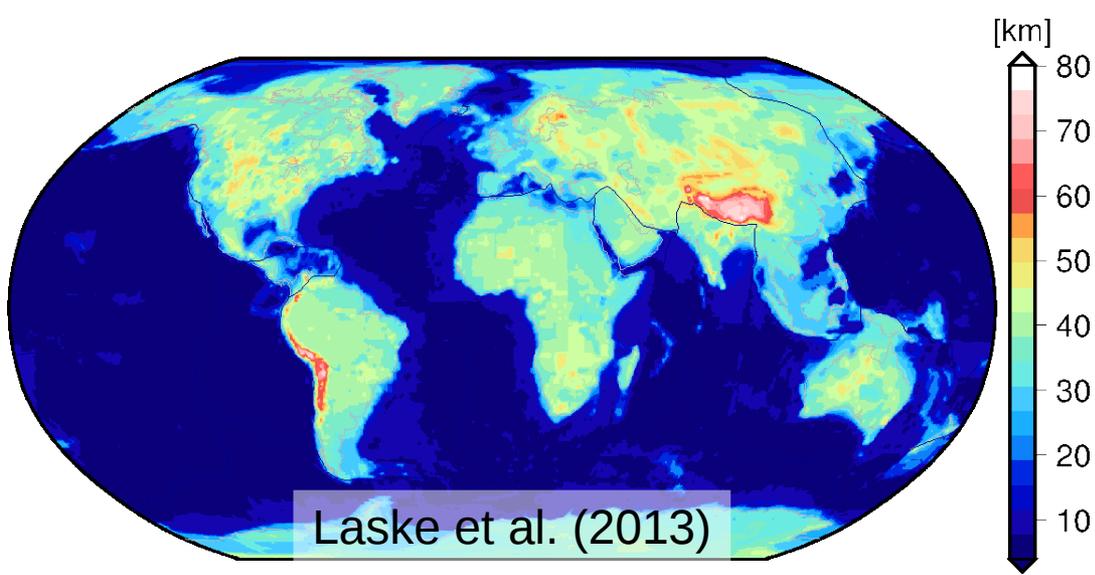
# actual topography



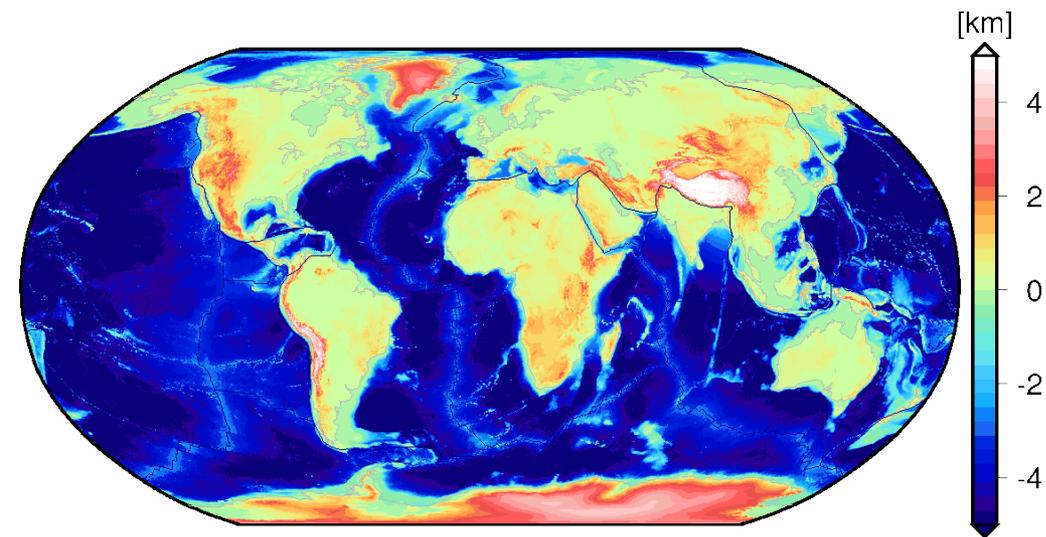
$$\hat{t} = f_1 l_c + f_2 l_l$$

$$f_1 = \frac{\rho_a - \rho_c}{\rho_a} \quad f_2 = \frac{\rho_a - \rho_l}{\rho_a}$$

# crustal thickness from CRUST1.0



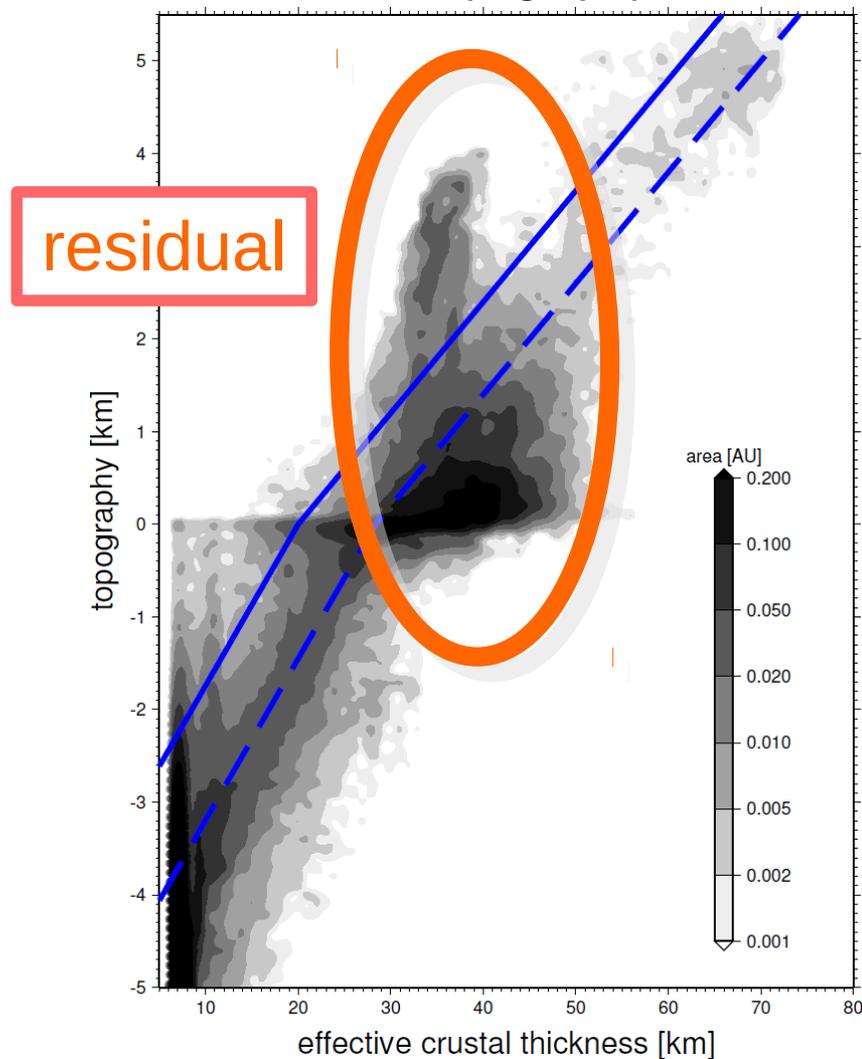
# actual topography



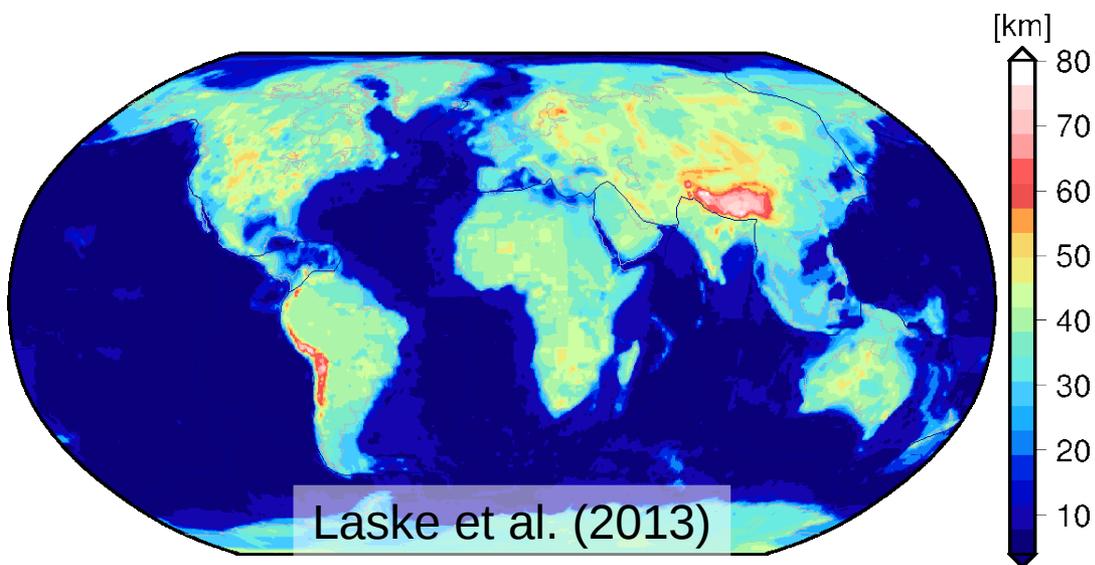
$$\hat{t} = f_1 l_c + f_2 l_l$$

$$f_1 = \frac{\rho_a - \rho_c}{\rho_a} \quad f_2 = \frac{\rho_a - \rho_l}{\rho_a}$$

## thickness vs. topography

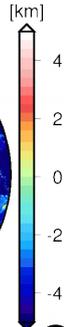
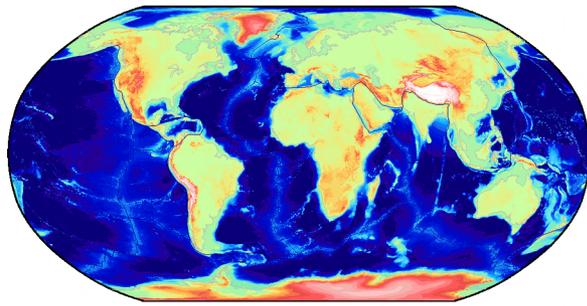


# crustal thickness from CRUST1.0



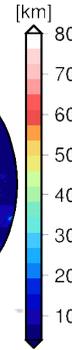
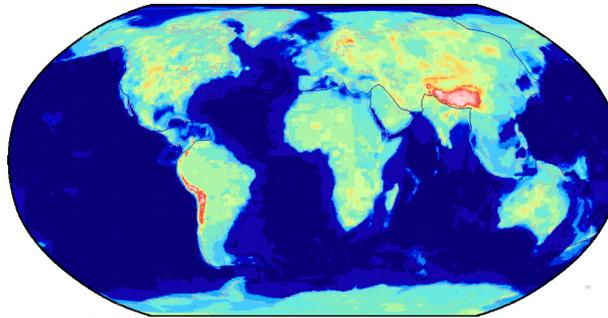
Laske et al. (2013)

topography



crustal model

−

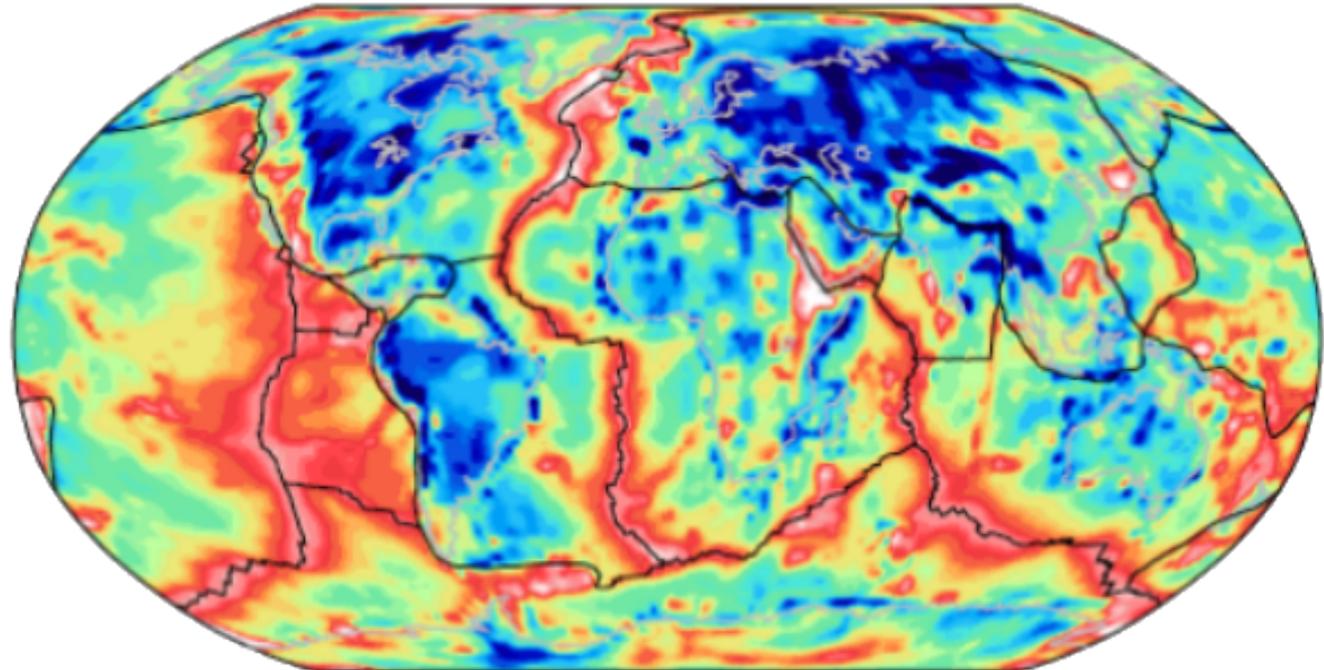


=

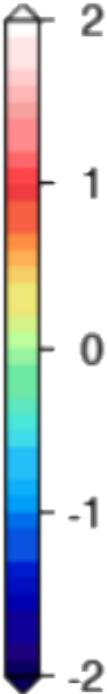
# Global crustal residual topography

*Note: Like oh so many things, “model”, not “data”*

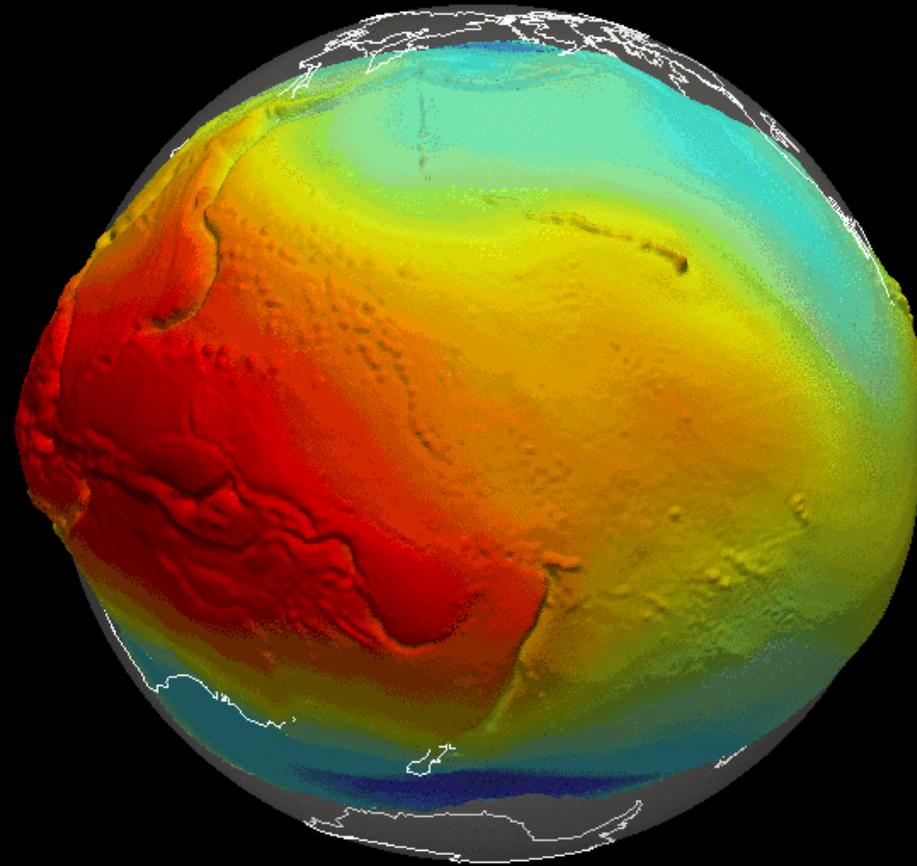
Non-isostatic, ~Airy residual topography



[km]

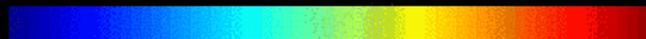


# Geoid anomalies



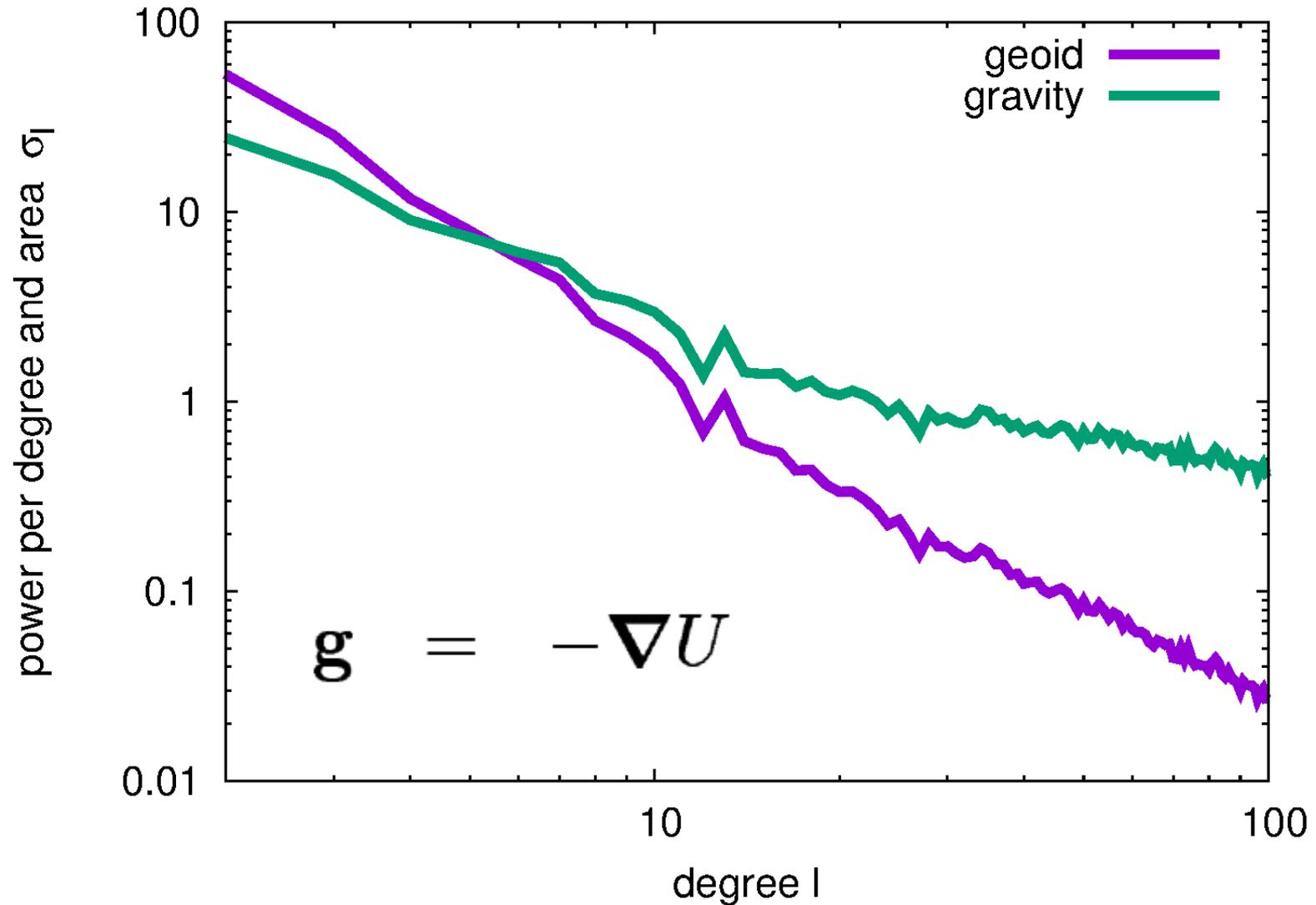
geoid anomaly (m)

-100   -50   0   50   100



(corrected for  
*hydrostatic shape*)

# Potential field constraints



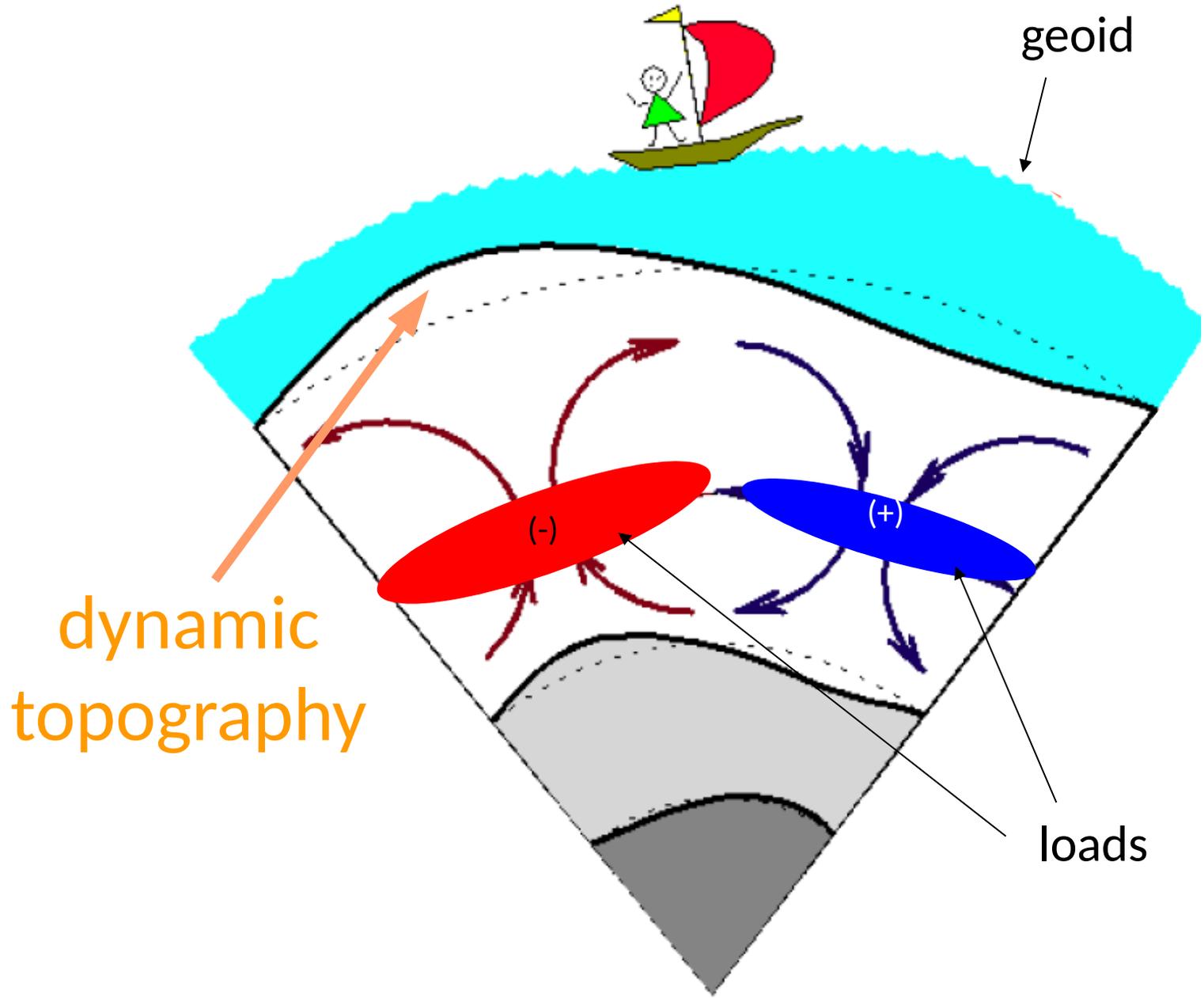
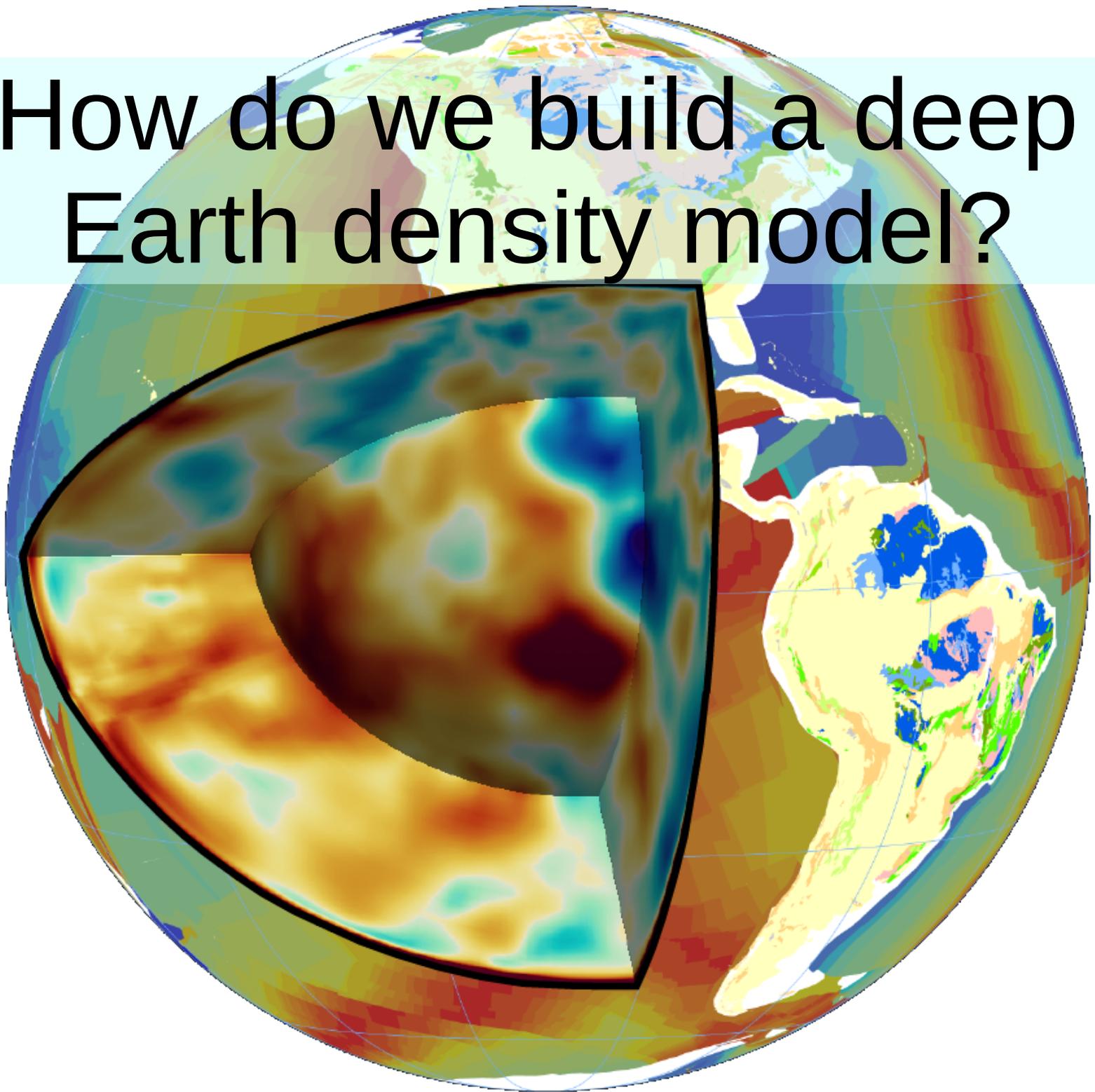
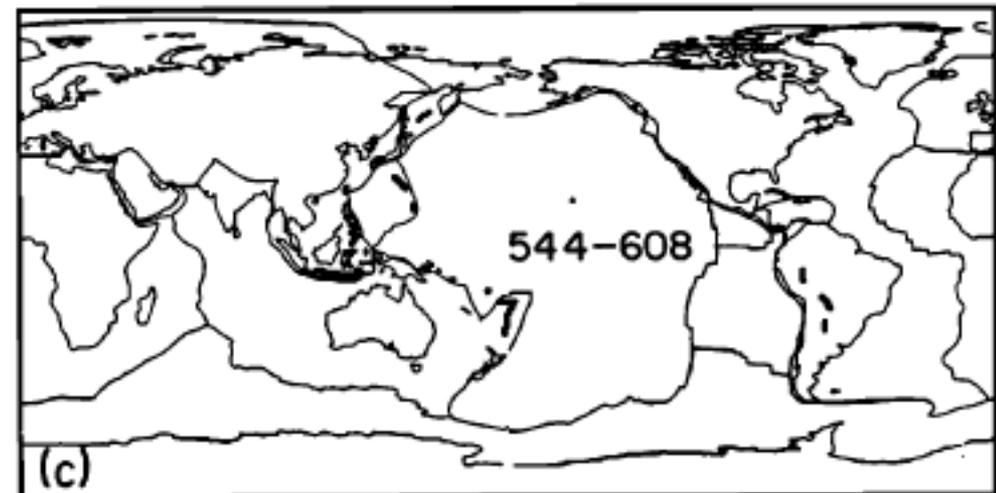
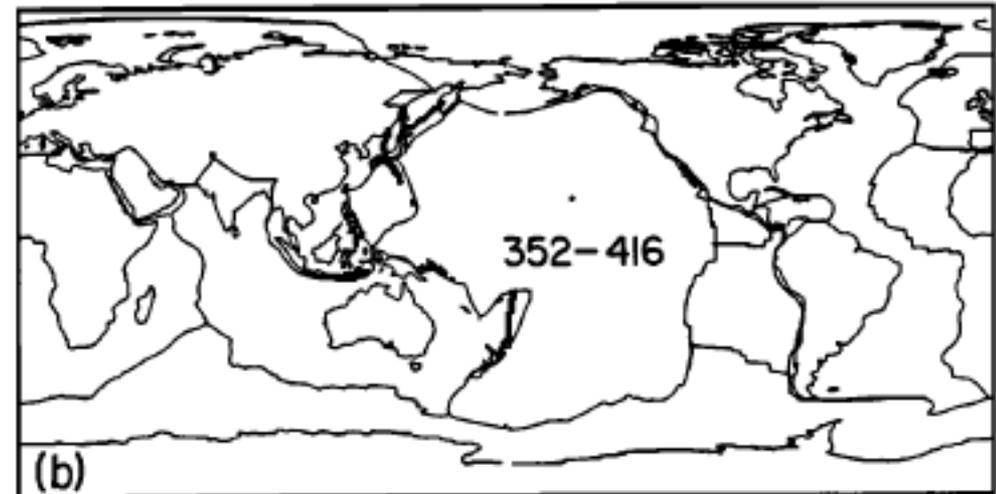
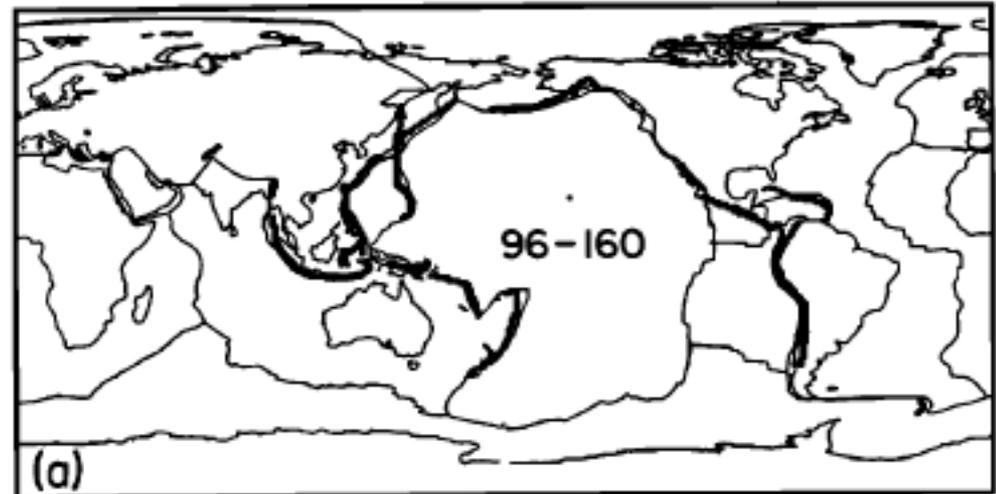


Figure from Y. Ricard

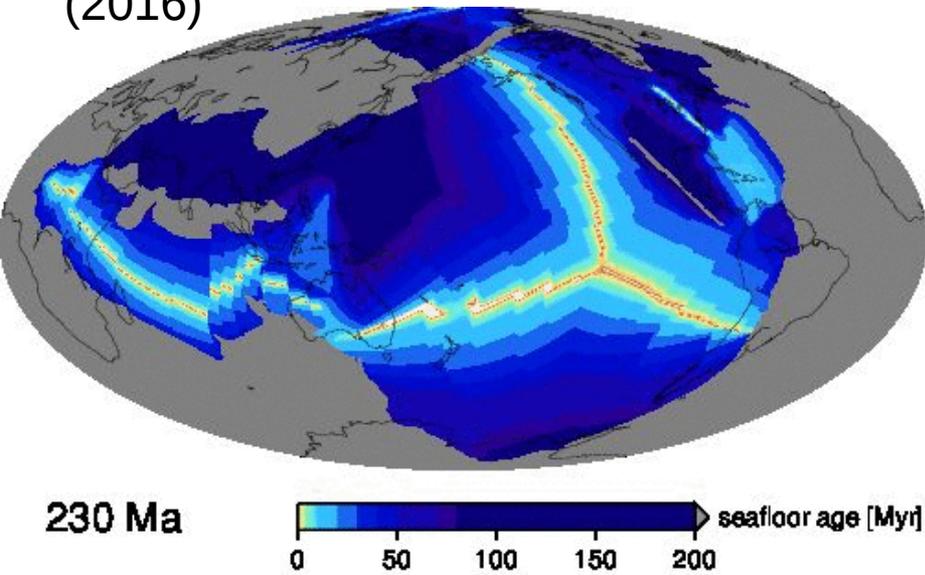
How do we build a deep  
Earth density model?



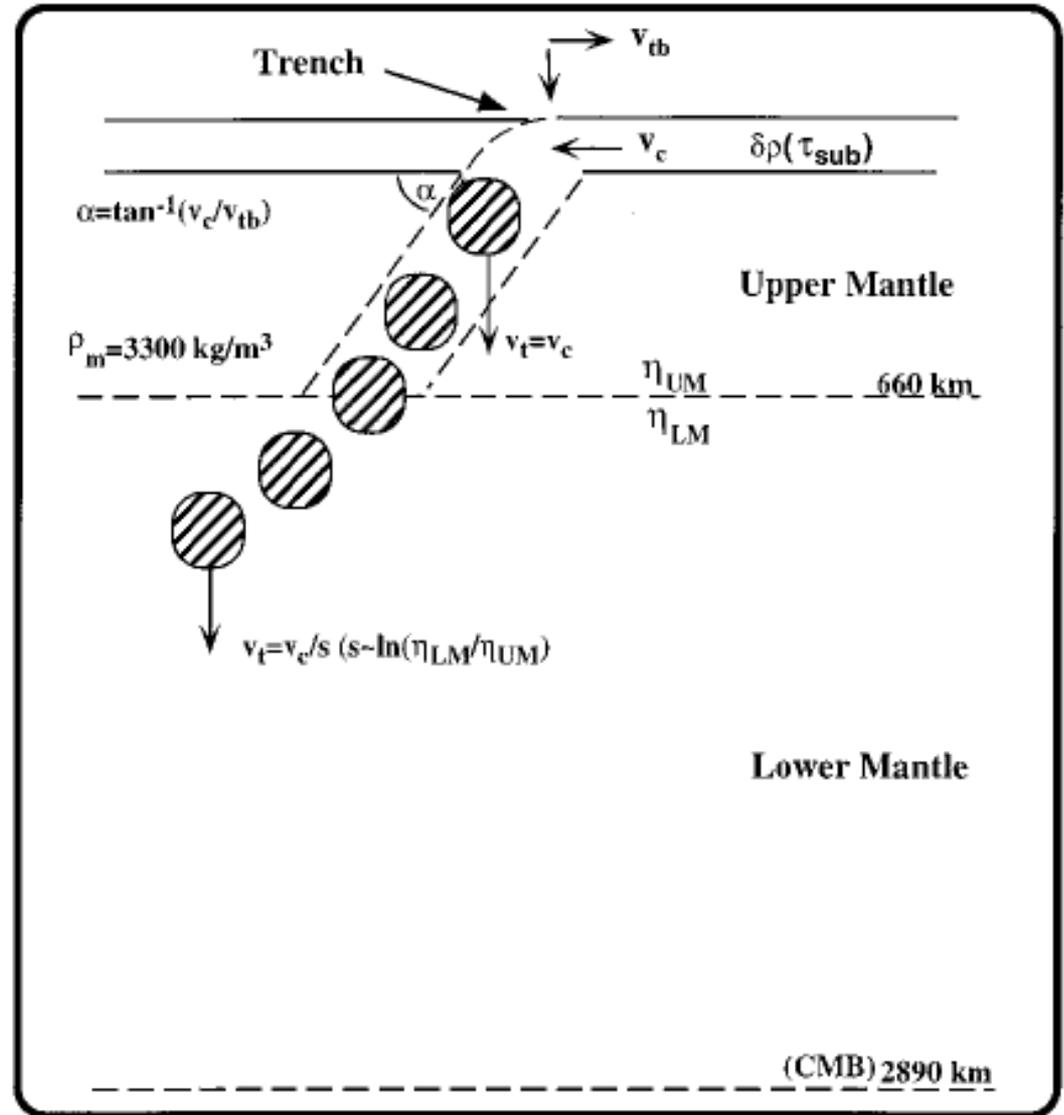
# Upper mantle slabs from Wadati-Benioff zones



Mueller et al.  
(2016)



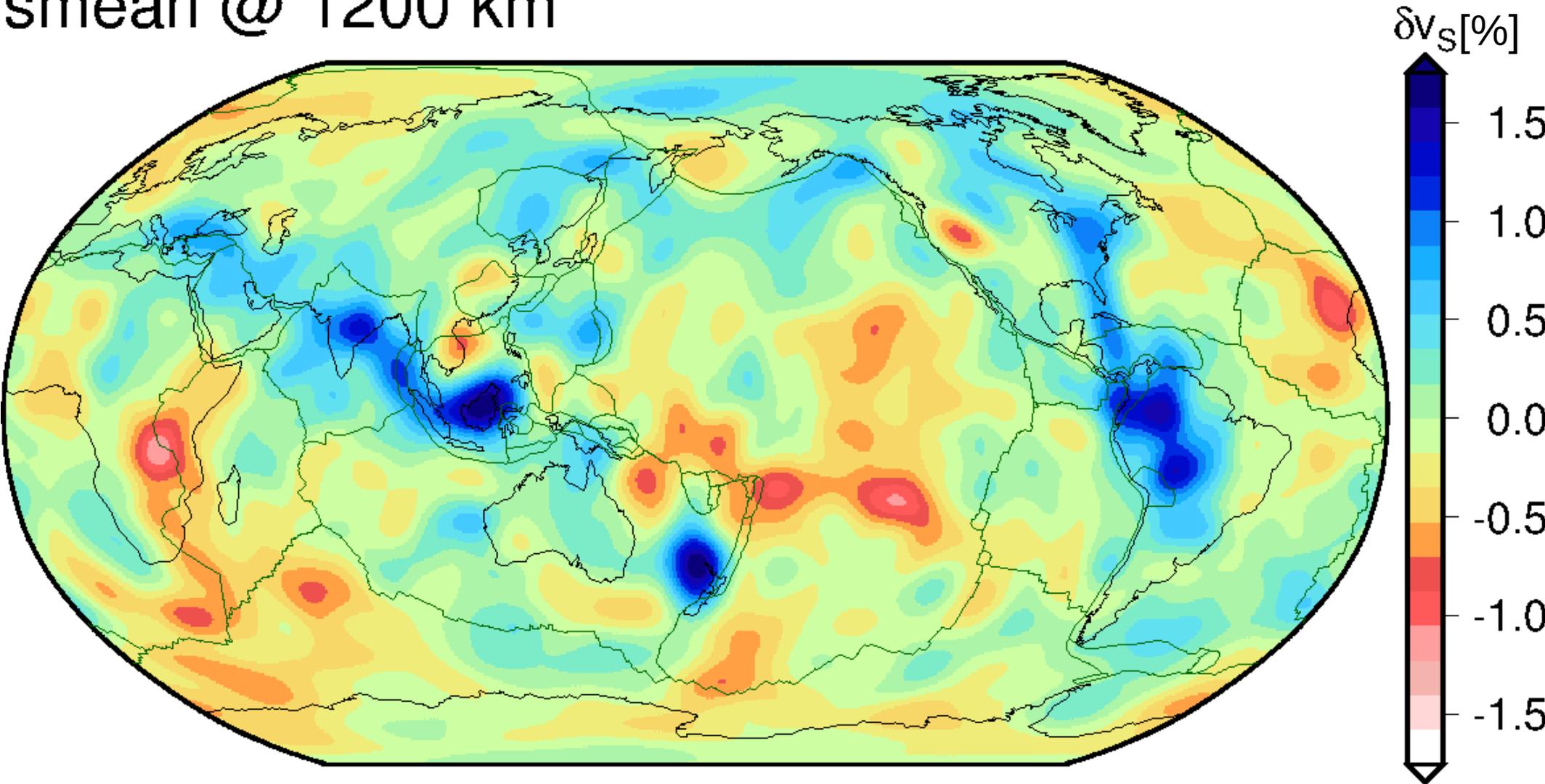
# Stokeslets link subduction history to mantle structure



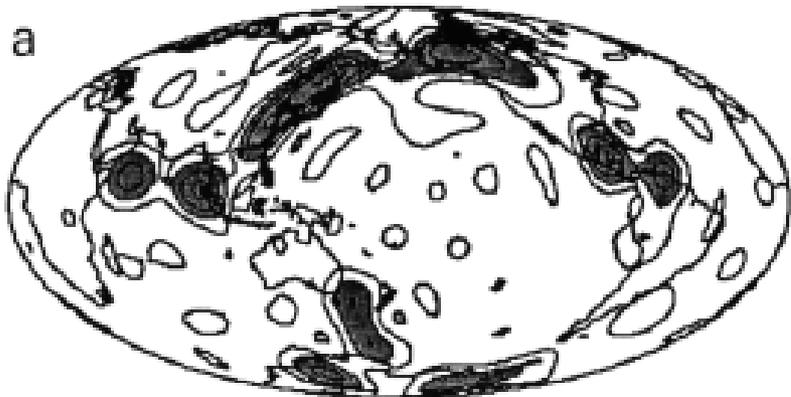
Ricard et al. (1993); Lithgow-Bertelloni et al. (1993, 1998)

# Seismic tomography shows slabs in lower mantle

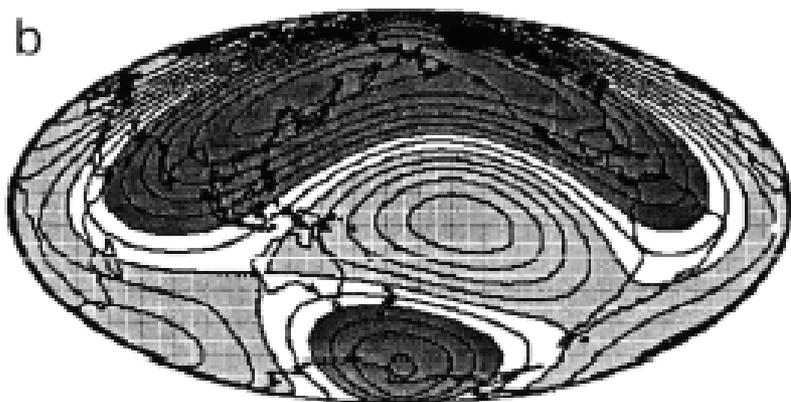
smean @ 1200 km



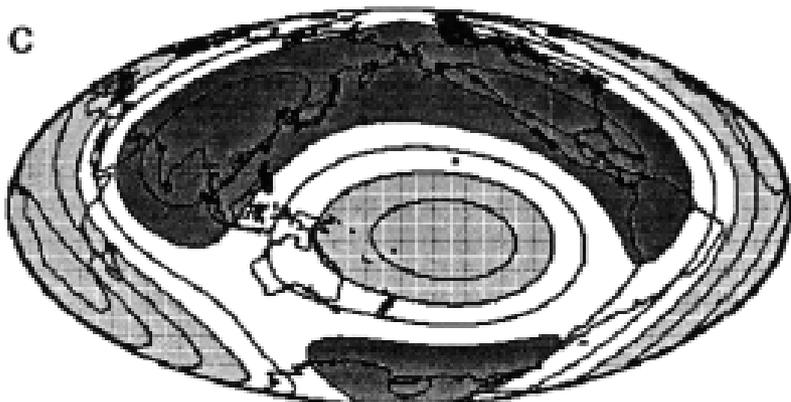
SLABS (DEPTH 2000 KM, DEGREES 1-15)



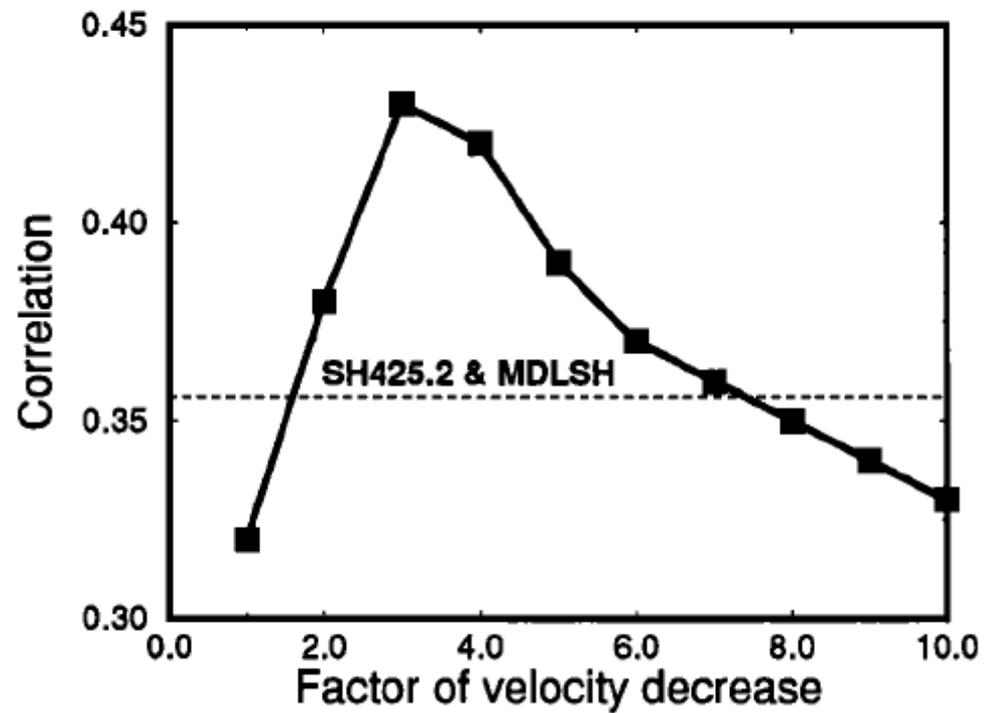
SLABS (DEPTH 2000 KM, DEGREES 1-3)



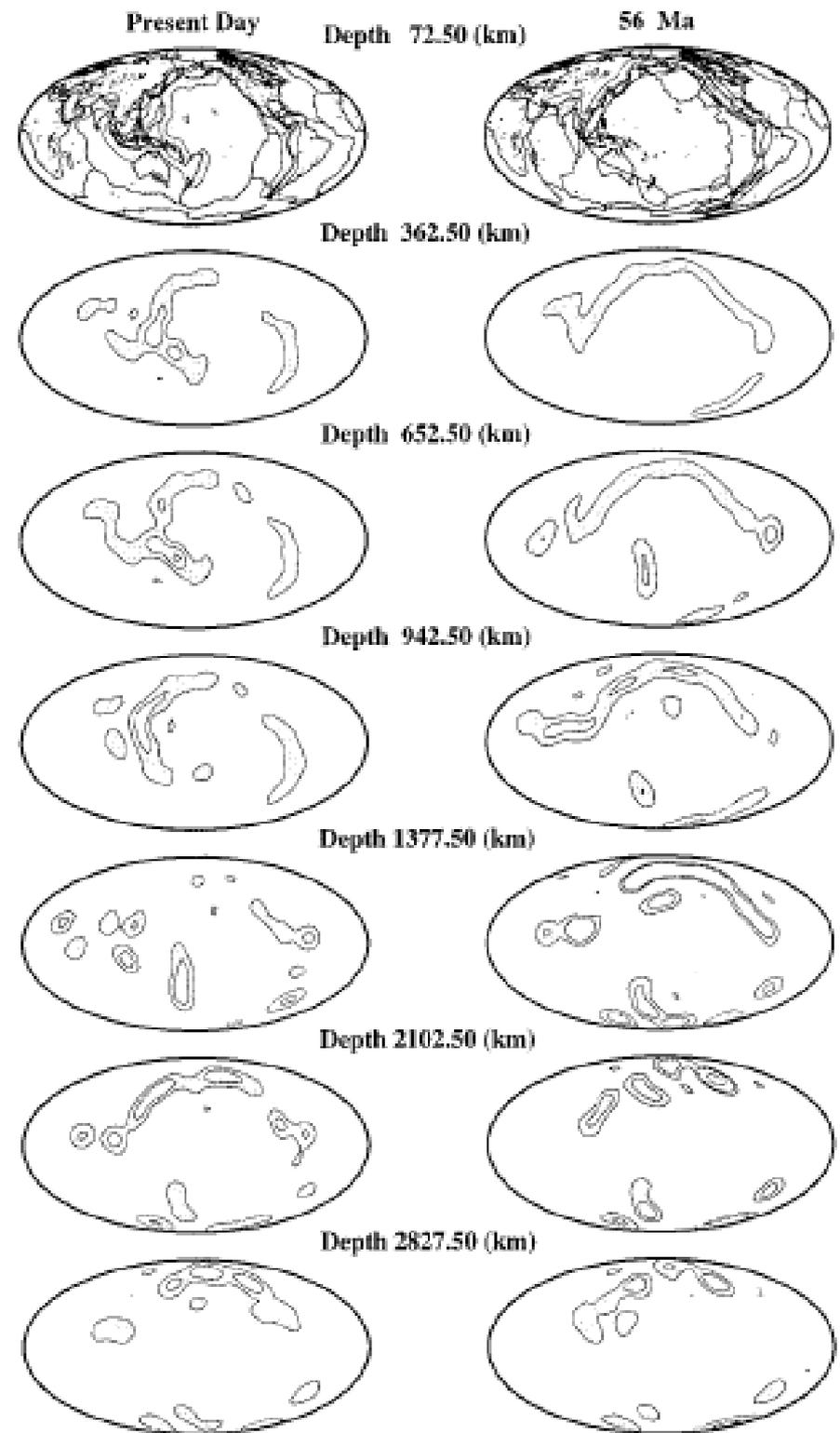
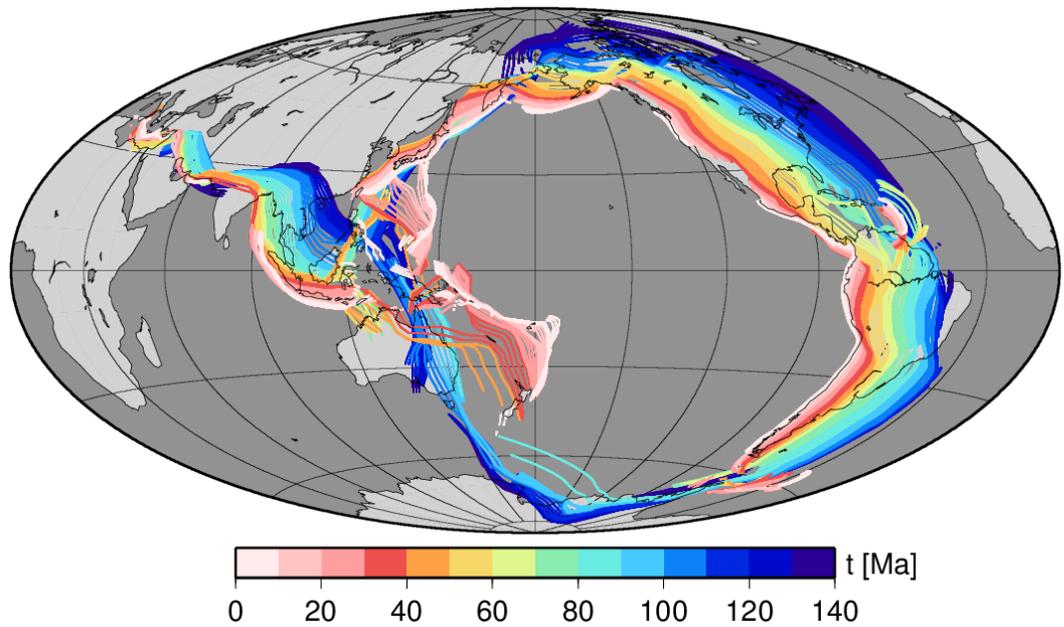
SH425.2 (DEPTH 2000 KM, DEGREES 1-3)



CORRELATION SLABS/SH425.2

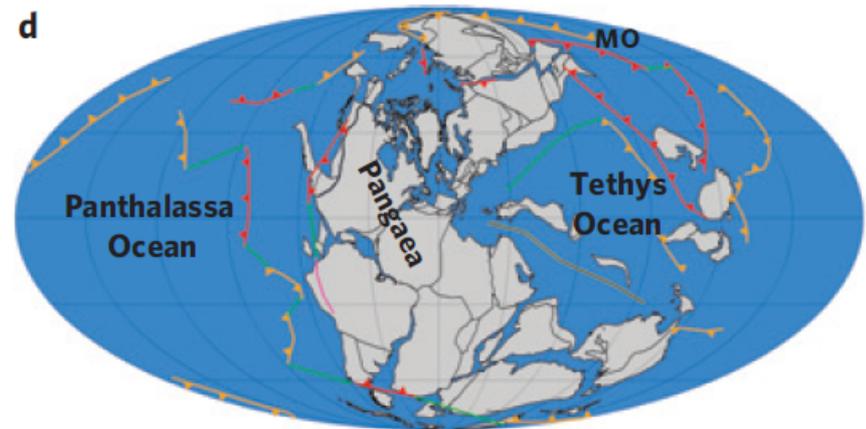
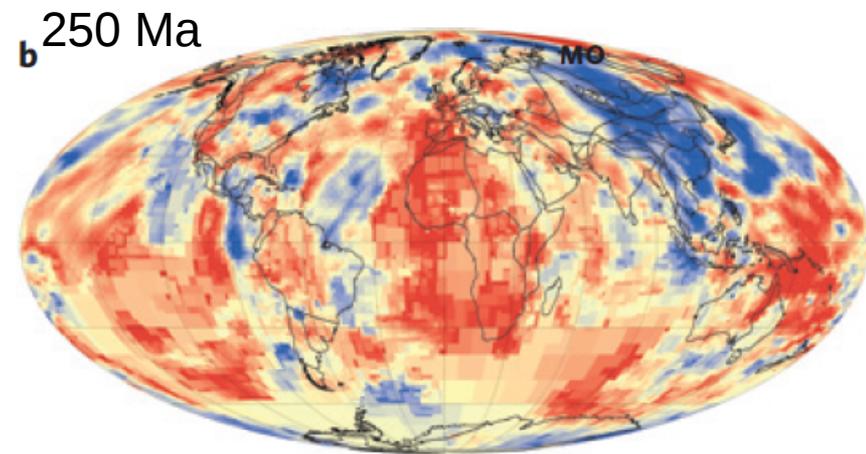
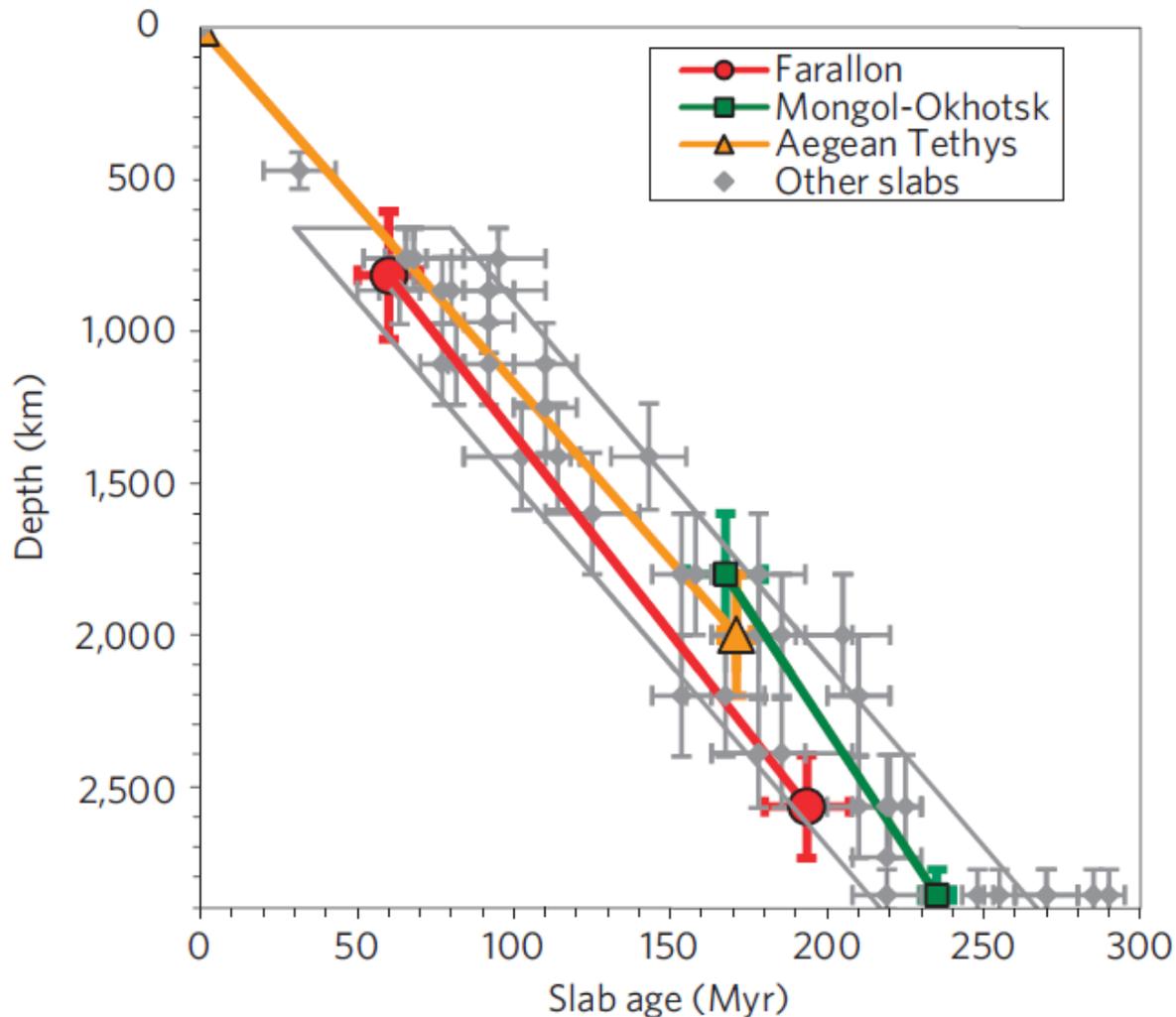


# Mantle structure as $f(t)$ from subduction



Ricard et al. (1993); Lithgow-Bertelloni & Richards (1998);  
Steinberger (2000); Spasojević et al. (2009); Steinberger and Torsvik (2010)  
Steinberger et al. (2014); Bower et al. (2015)

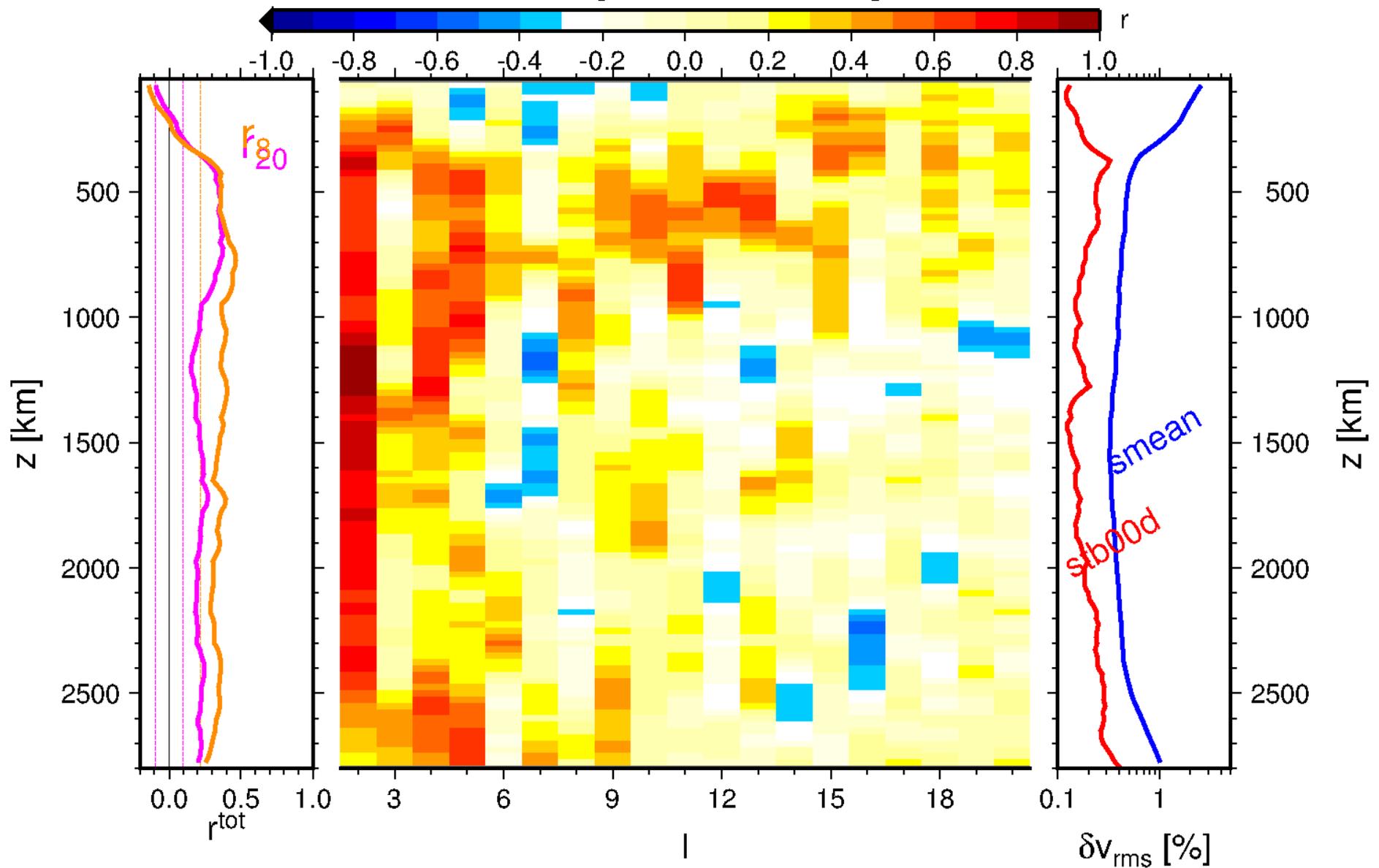
# Possible reference frame (but sinking rates uncertain)



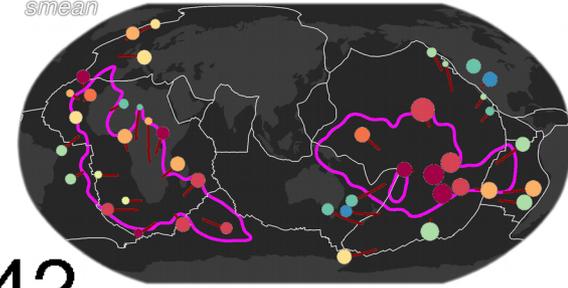
# Correlation:

## Advection slabs vs. tomography

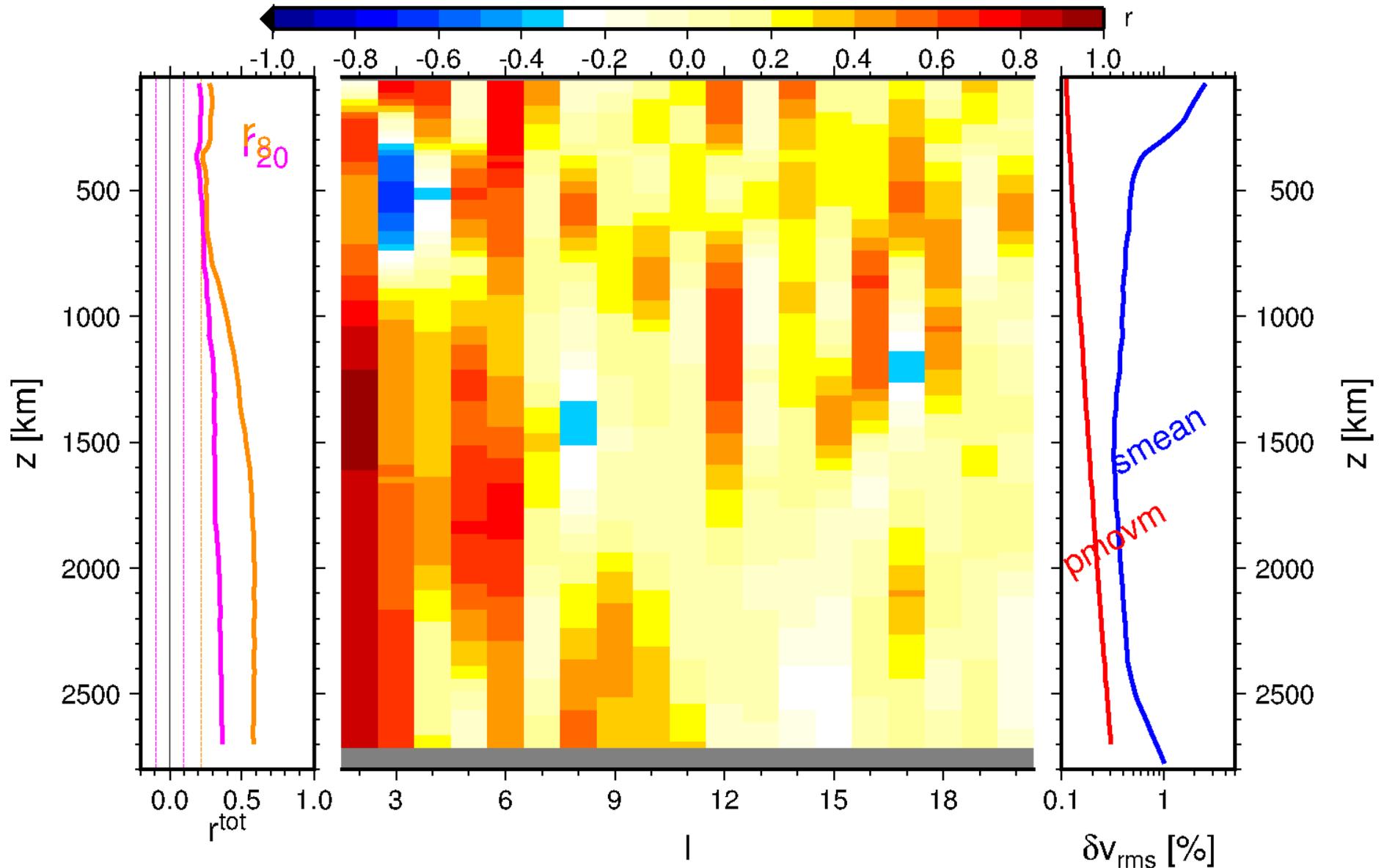
stb00d vs. smean,  $\langle r_{20} \rangle = 0.21$ ,  $\langle r_8 \rangle = 0.30$



# Advected plumes vs. tomography (Steinberger flow model conduits)



pmovm vs. smean,  $\langle r_{20} \rangle = 0.28$ ,  $\langle r_8 \rangle = 0.42$

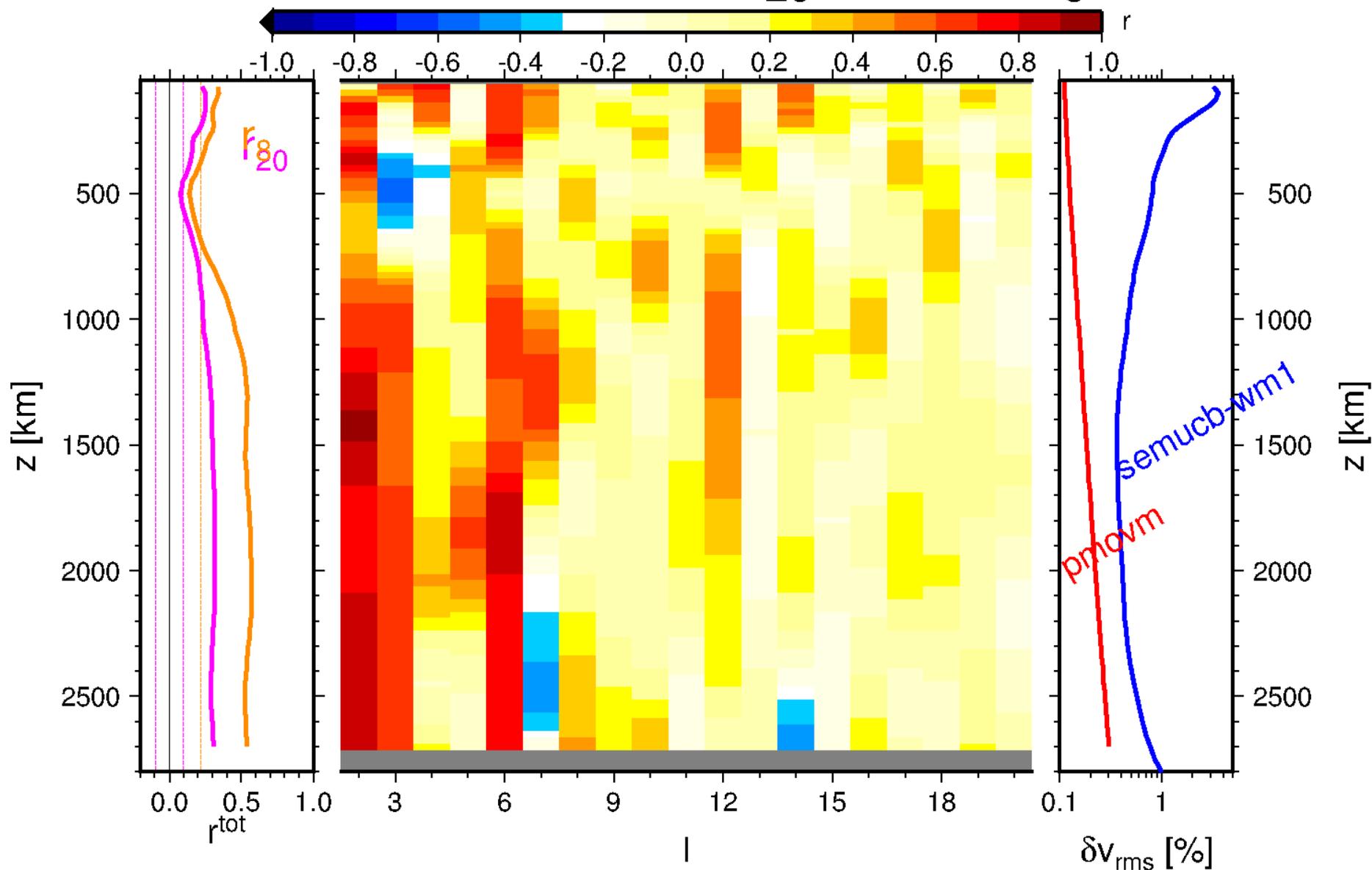


Note: Correlation with *moving* plumes **better** than with slabs

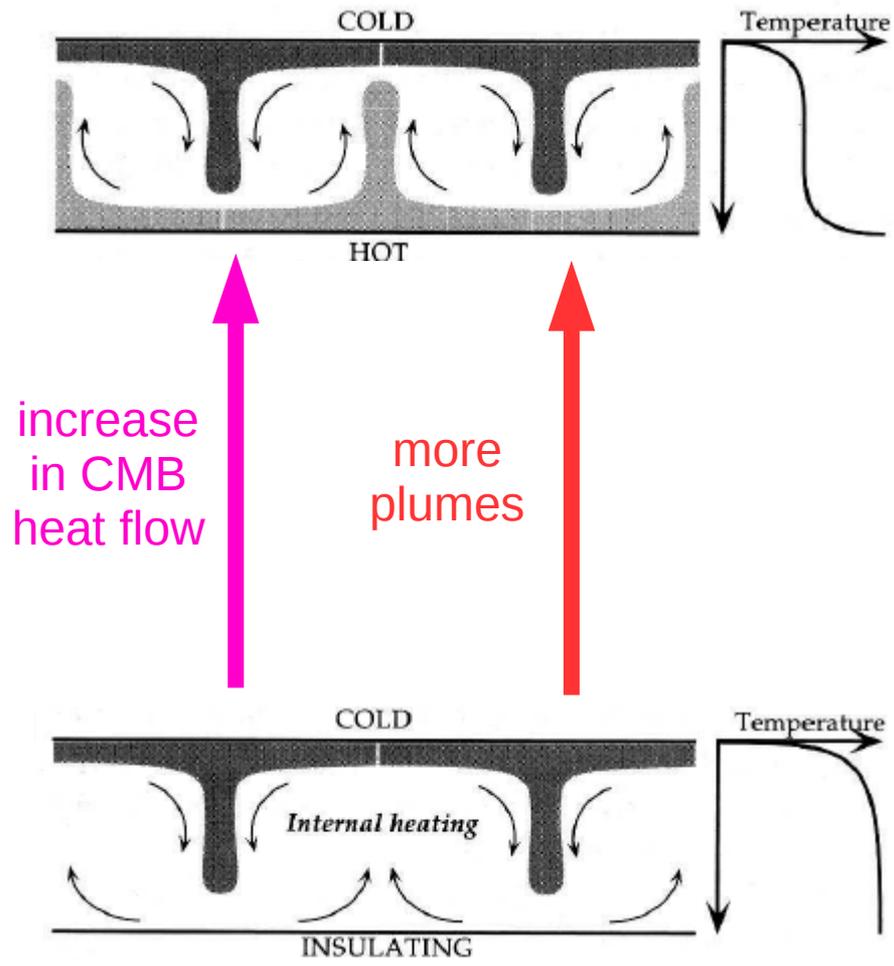
# Advected plumes vs. tomography

(Steinberger flow model conduits)

pmovm vs. semucb-wm1,  $\langle r_{20} \rangle = 0.24$ ,  $\langle r_8 \rangle = 0.41$

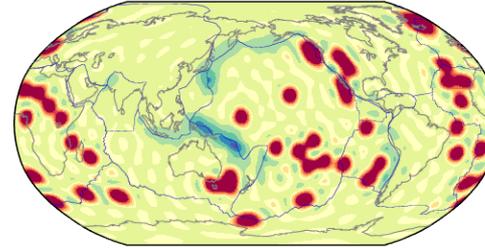


# Building a model of Earth from slabs and plumes

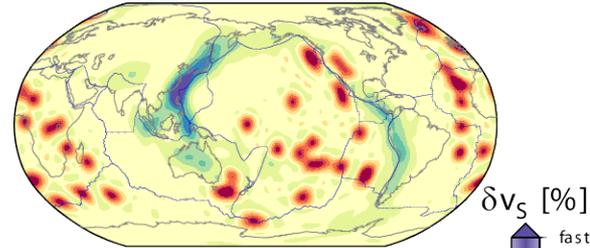


## *Input: Slabs and plumes*

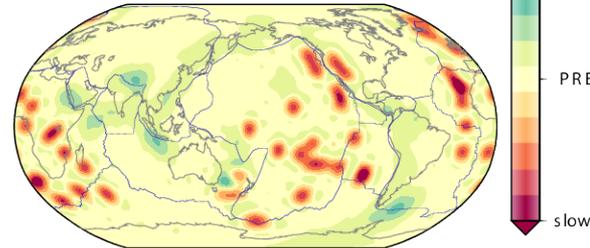
z = 500 km, input



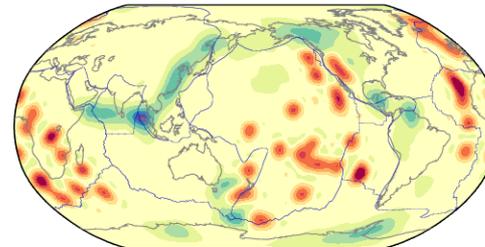
z = 1000 km, input



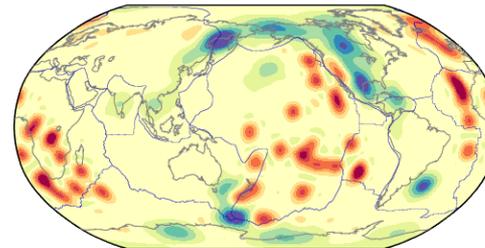
z = 1500 km, input



z = 2000 km, input

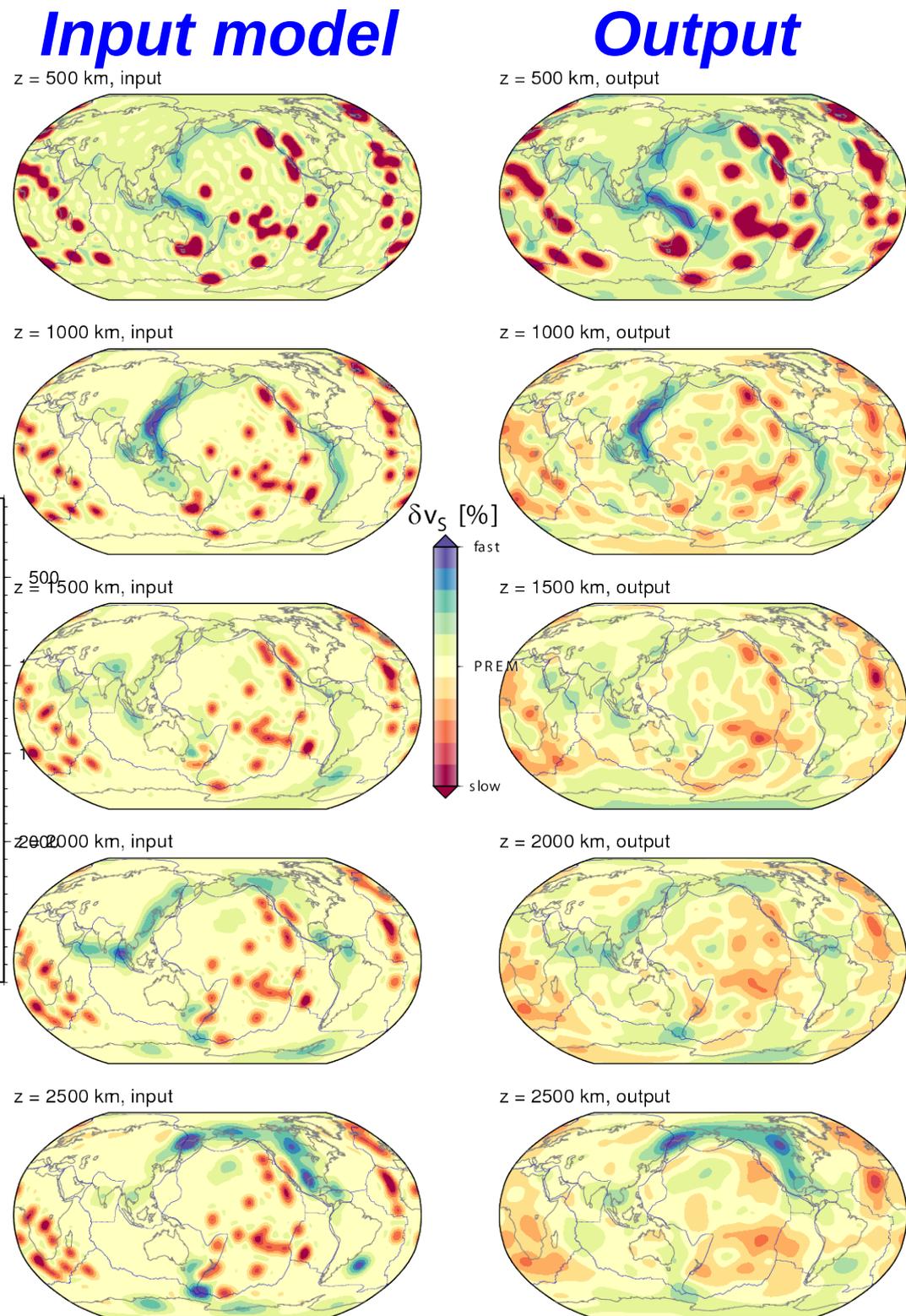
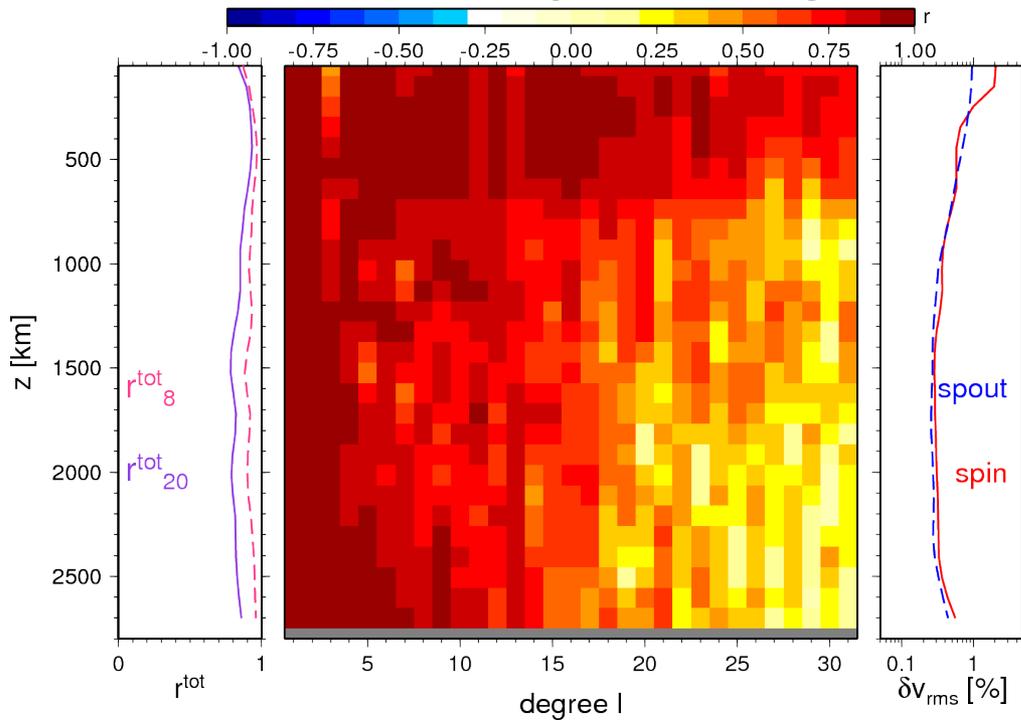


z = 2500 km, input



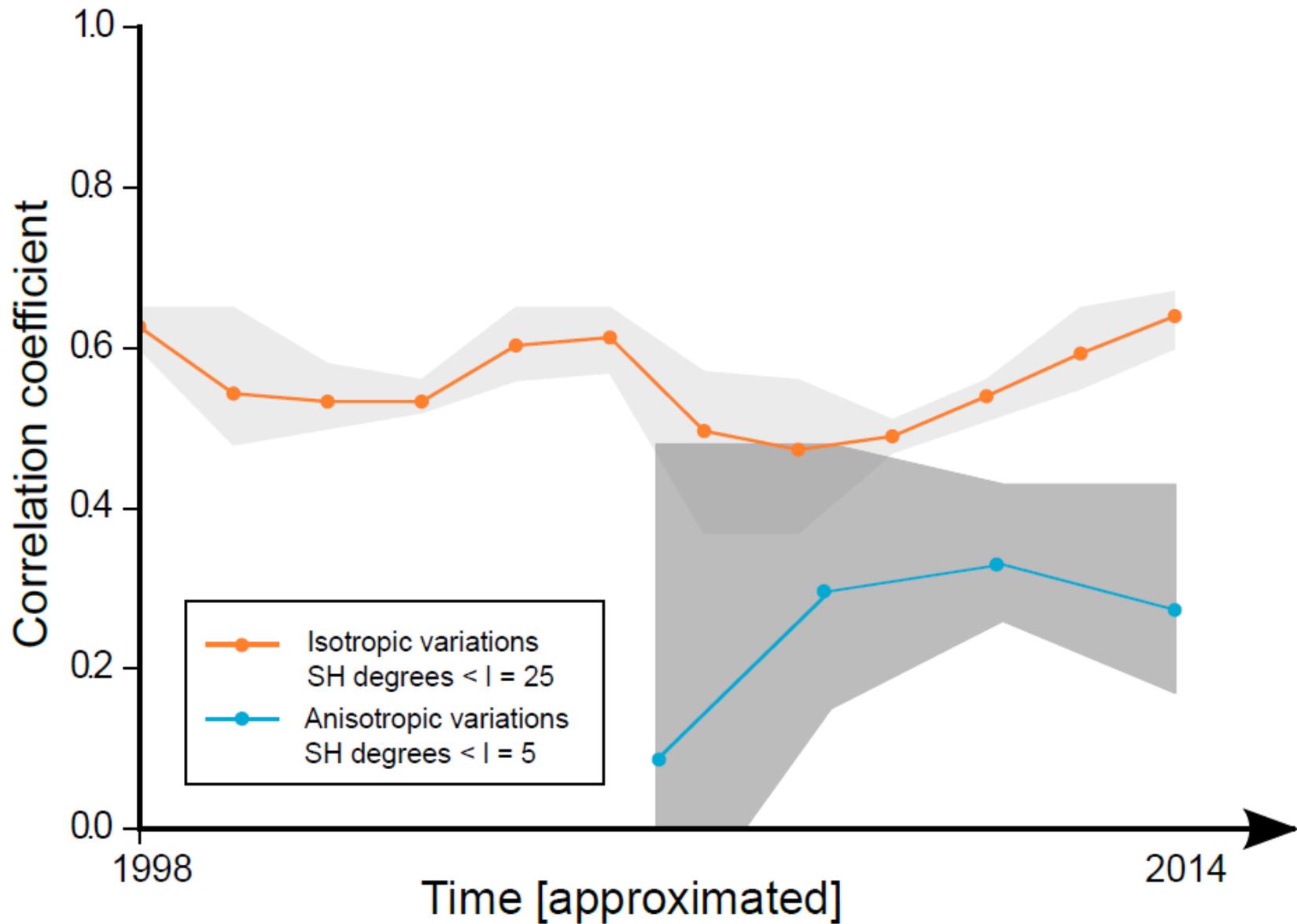
# Resolving power of tomography

spin - spout,  $\langle r_{20} \rangle = 0.85$ ,  $\langle r_8 \rangle = 0.93$



*Note:* can resolve structure up to  $\sim 15$  in wave theoretical framework

# Inter tomography-model correlation



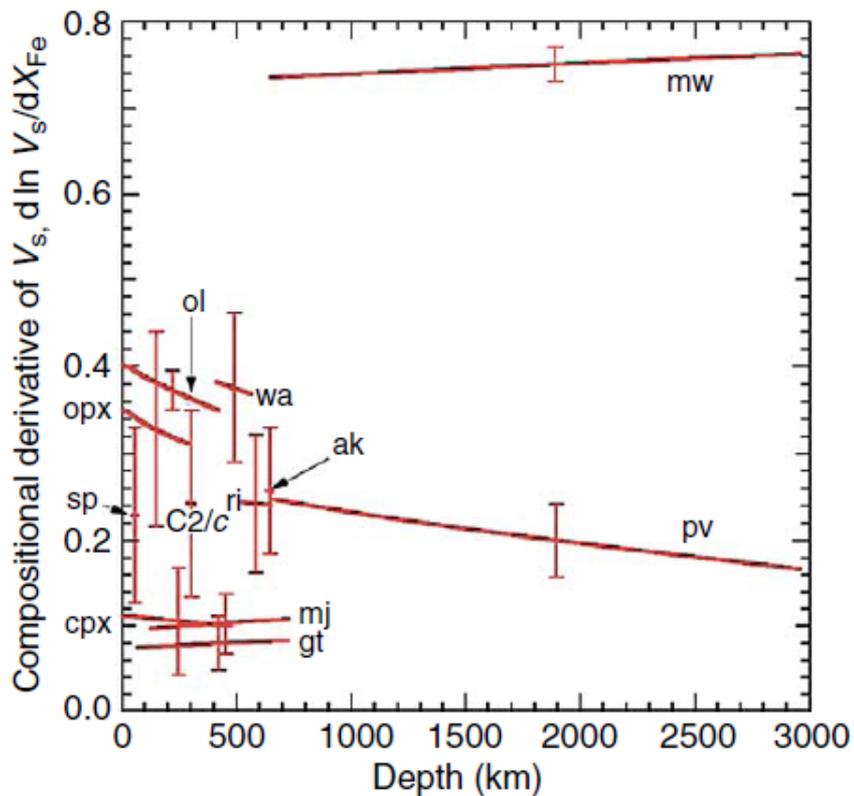
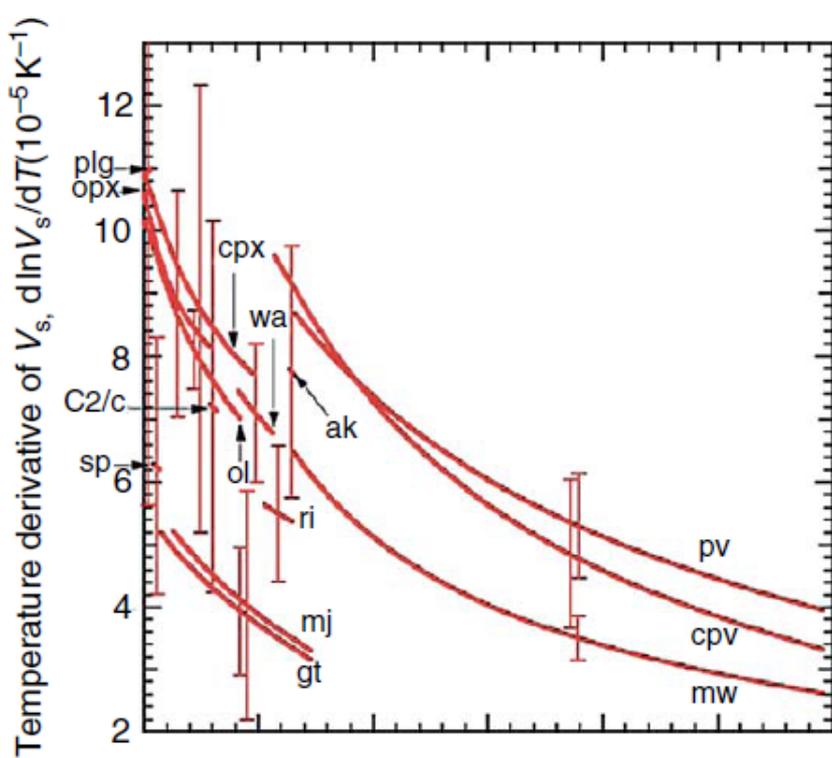
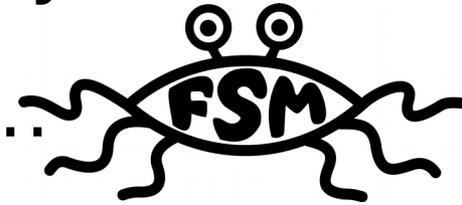
Note: data *and* theory matter

Auer (2016)

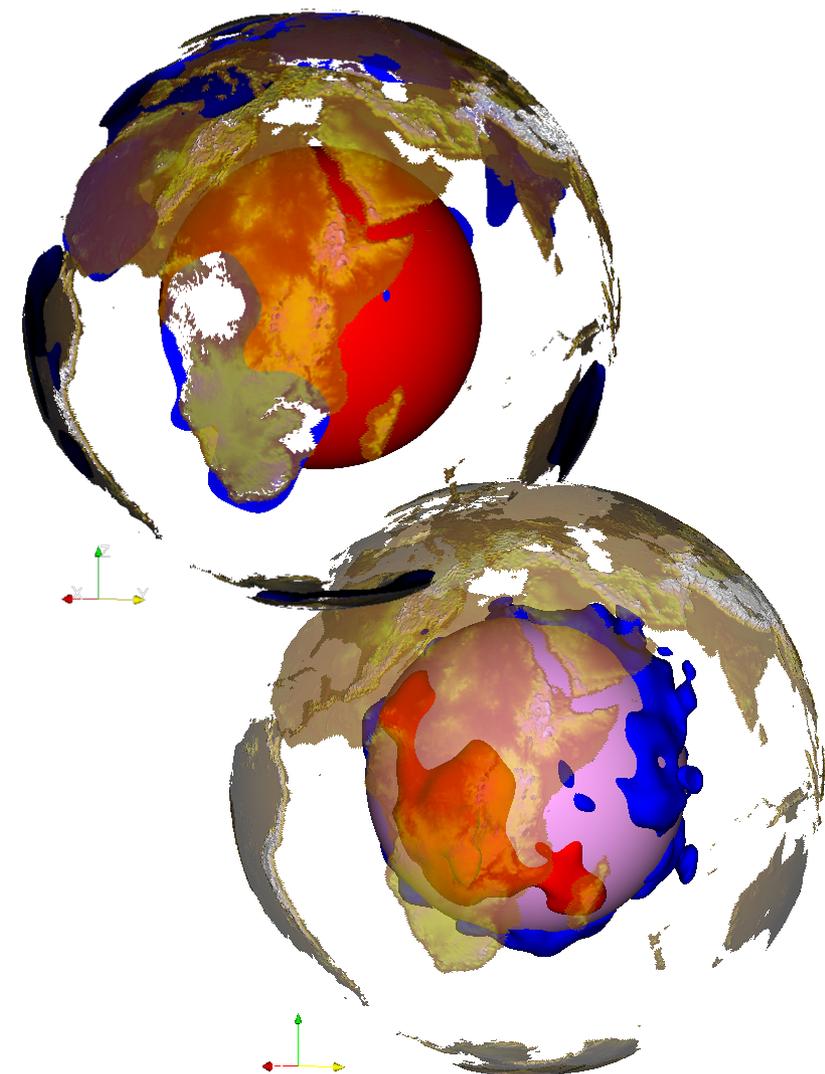
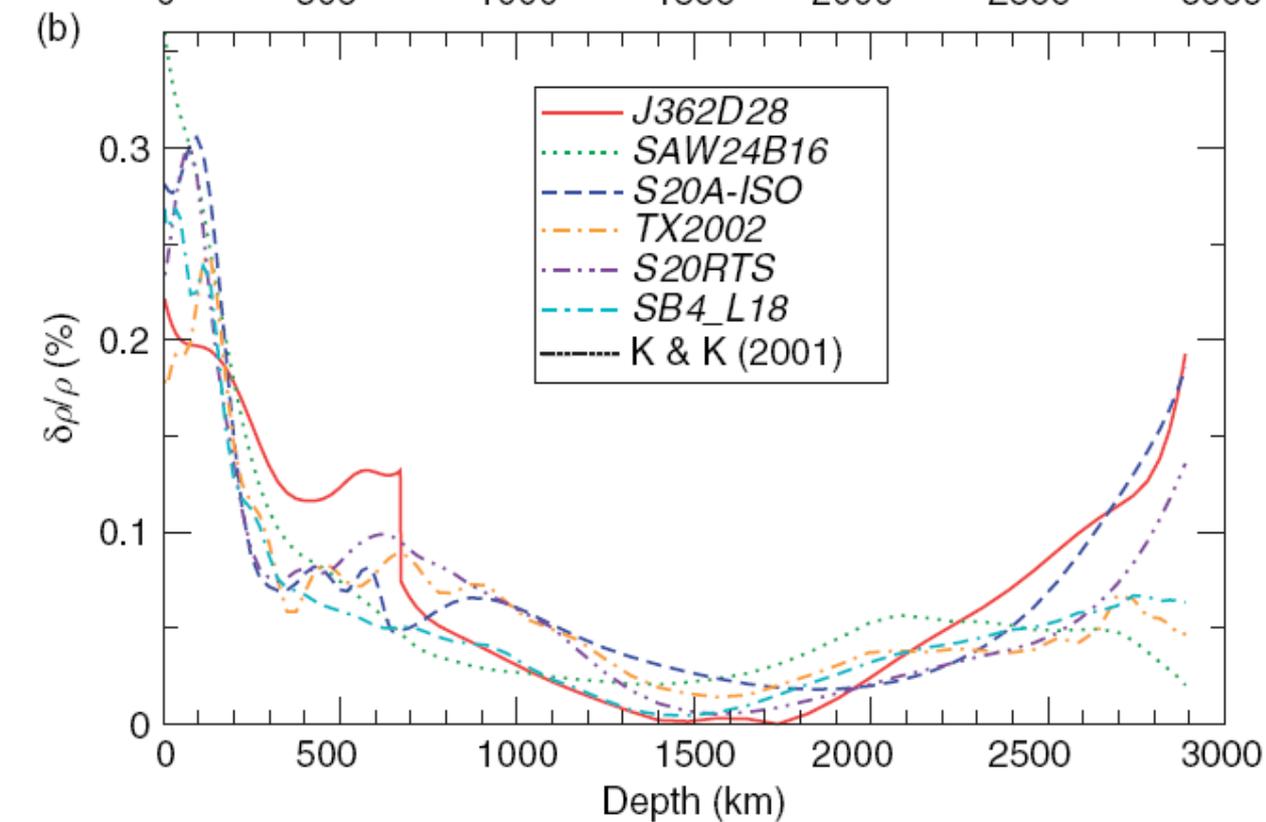
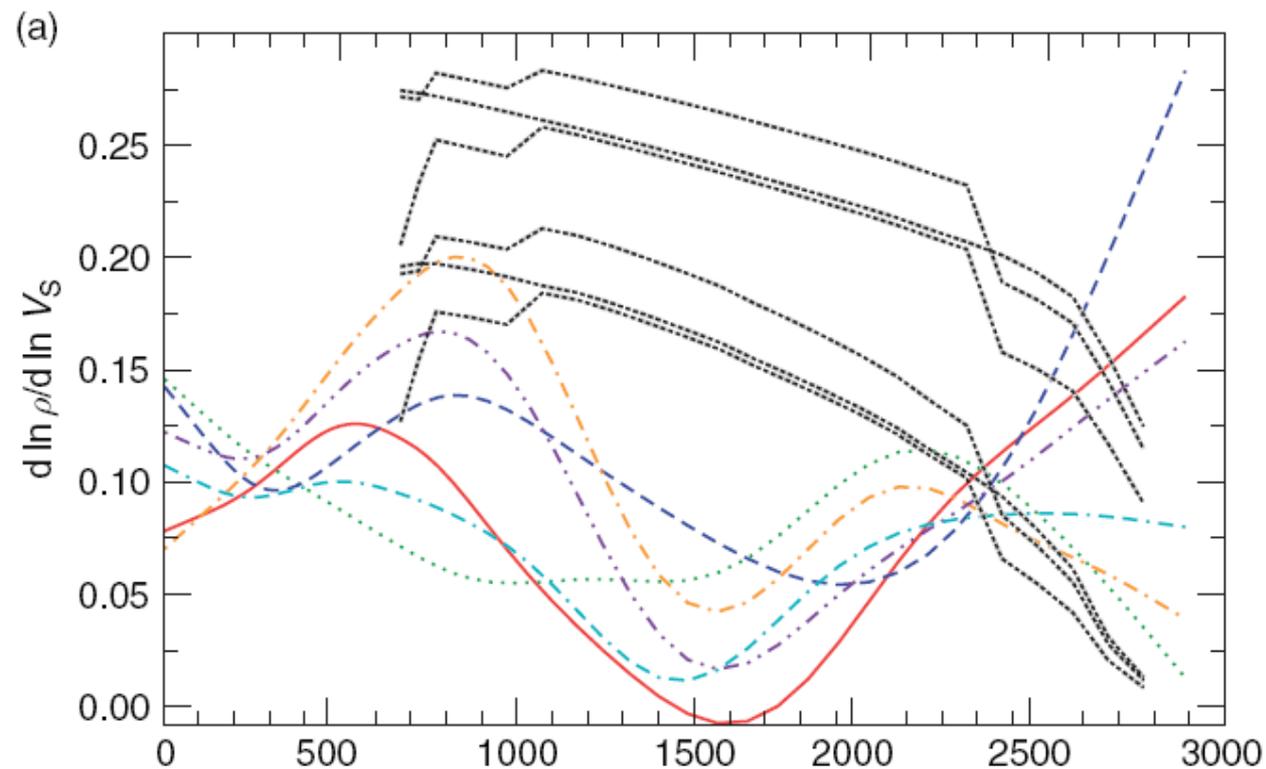
# How to convert tomography to density anomalies?

I. Take it from mineral physics and

composition from...



## II. Best-fit scaling and composition as inverse problems

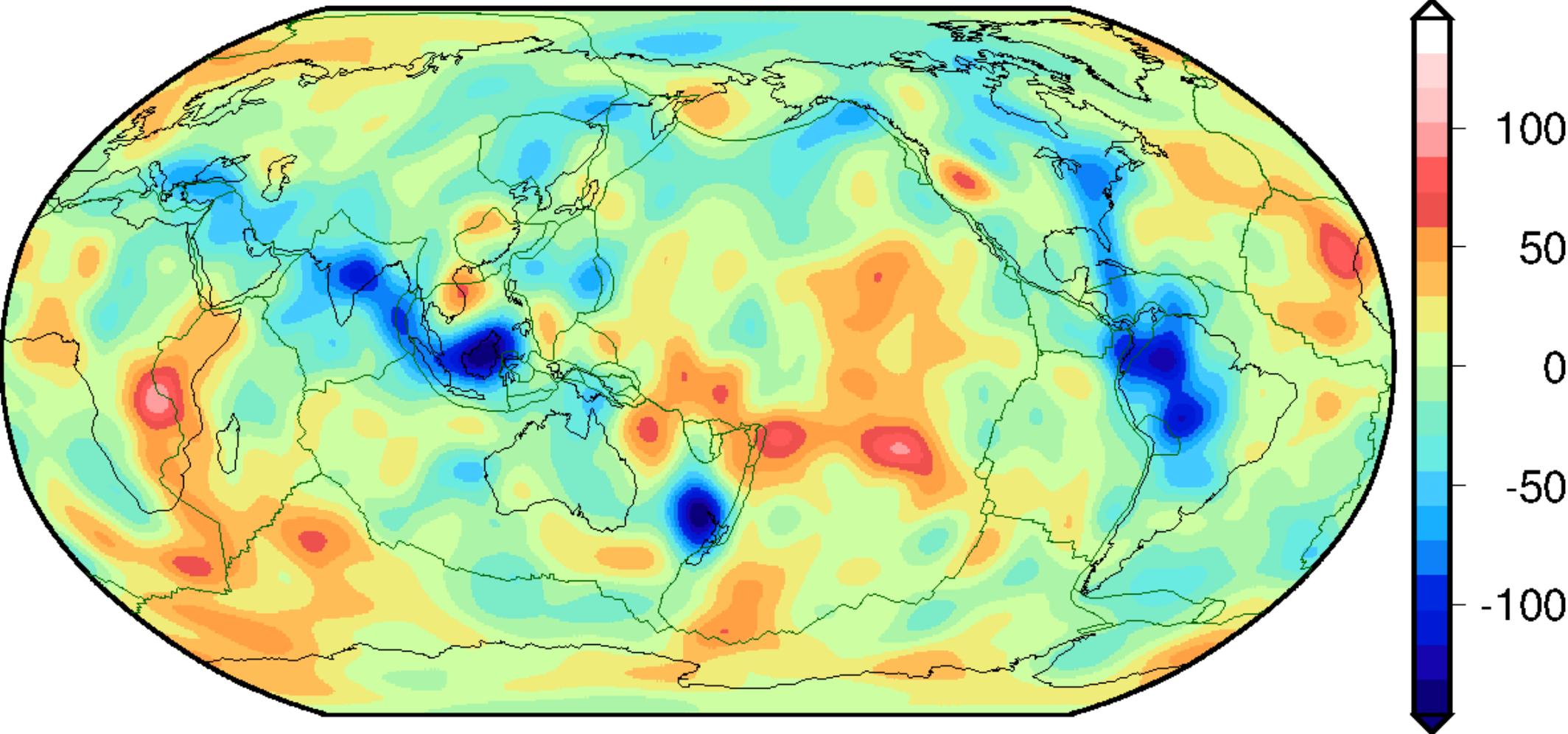


cf. Simmons et al. (2008, 2010),  
Soldati et al. (2014), Forte et al. (2015)

Besides cratons and piles:

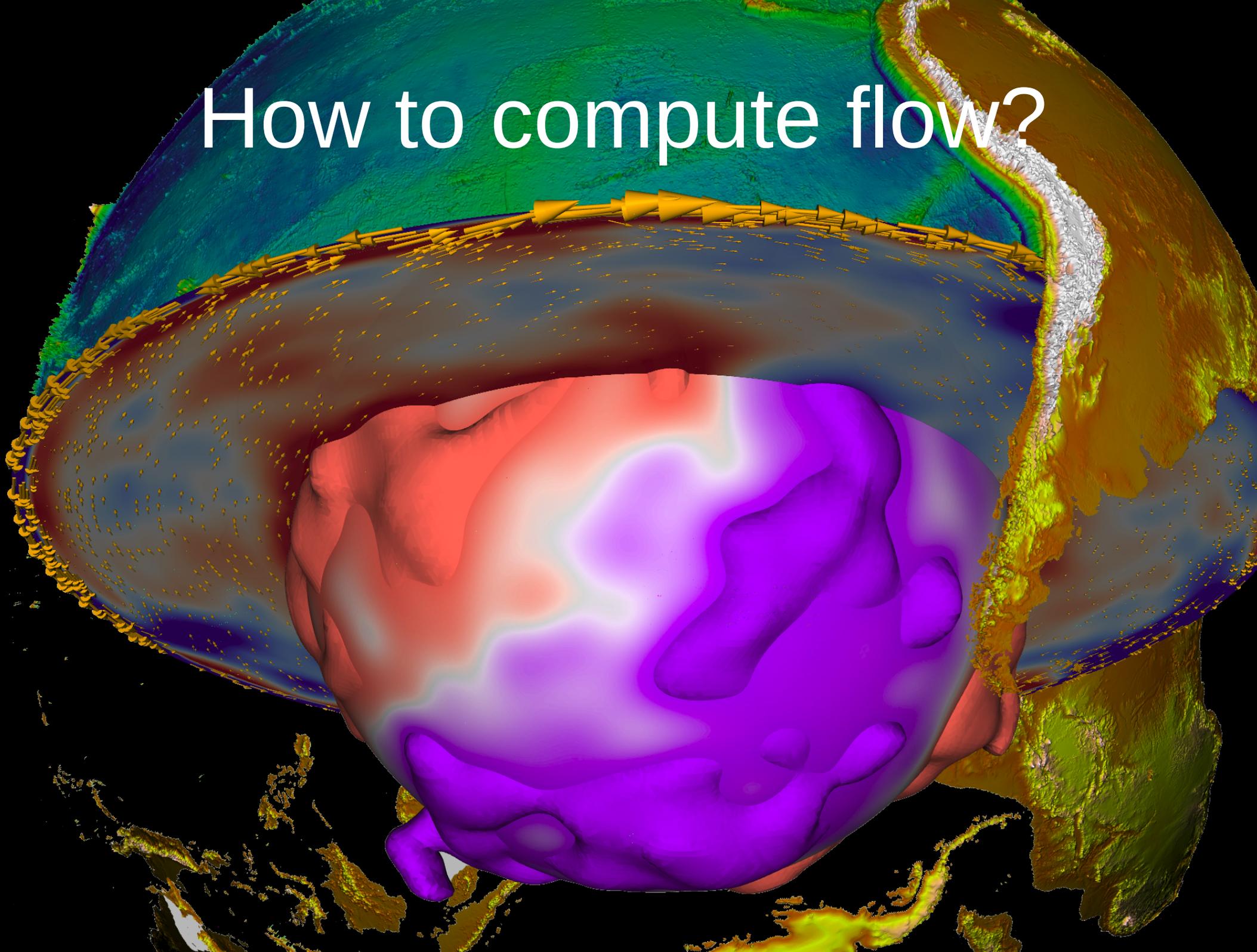
$$d \ln \rho / d \ln v_s = 0.2$$

$\Delta T$  @ 1200 km



- Use mineral physics to convert velocity into temperature (density) anomalies

# How to compute flow?



# Mantle circulation

- × Treat mantle and lithosphere as a fluid
- × Infinite Prandtl number (no inertia) approximation
  - × Navier-Stokes turns into Stokes equation
- × Instantaneous solution for given density and boundary conditions
  - × Can solve in  $< \sim 1$  s for spherical Earth without lateral viscosity variations (LVVs)

force balance  
(conservation of momentum)

$$\frac{\partial \sigma_{ij}}{\partial x_j} = - Ra \Delta T \delta_{ir}$$

constitutive equation  
(rheology)

$$\sigma_{ij} = -P \delta_{ij} + 2\eta \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

viscous drag

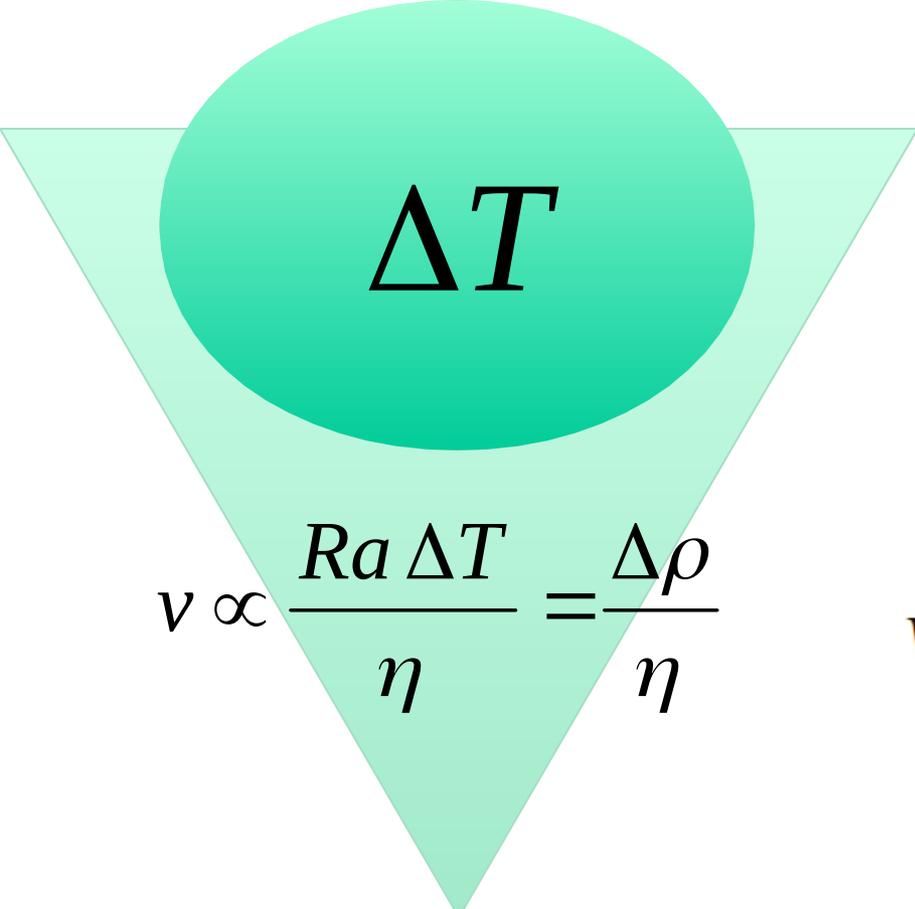
stress tensor

dynamic pressure

Newtonian viscosity

strain-rate

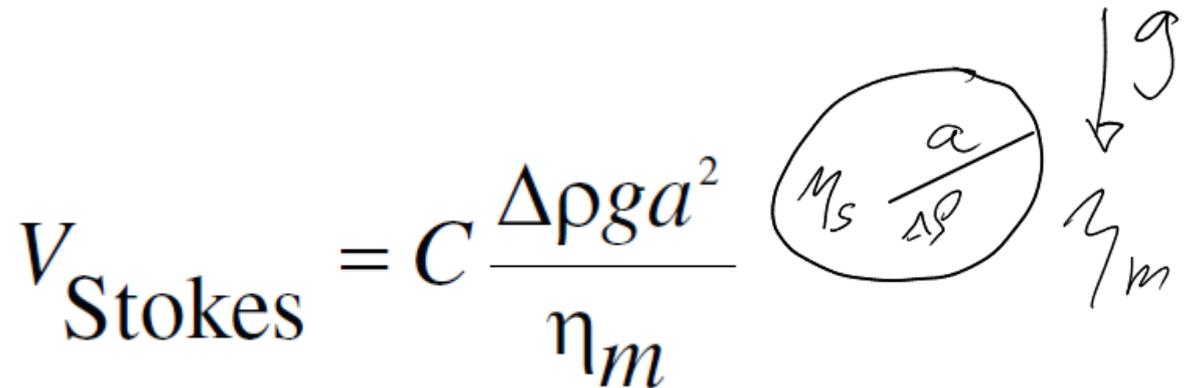
thermal buoyancy


$$\Delta T$$

$$v \propto \frac{Ra \Delta T}{\eta} = \frac{\Delta \rho}{\eta}$$

# Stokes sphere

$$V_{\text{Stokes}} = C \frac{\Delta \rho g a^2}{\eta_m}$$



$$C = \frac{2 + 2\eta'}{6 + 9\eta'} \quad \text{where} \quad \eta' = \frac{\eta_s}{\eta_m}$$

*Asides:*

→ A needle, as opposed to sphere, will sink with  $0.5 \dots 2 v_{\text{Stokes}}$

→ Pe # = ratio of diffusive to convective time scale =  $t_d / t_c$

$t_c = a^2 / \kappa$ ,  $t_c = a / v_{\text{Stokes}}$ ,  $\Delta \rho = \Delta T \alpha \rho_0$ , then Pe → Ra (with  $a$  instead of  $L$ )

# Mantle circulation

- × Thermo-chemical heterogeneity and complex rheologies make things interesting
- × Finite element methods best suited for lateral viscosity variations (we can now solve all of this, at < km resolution without approximations)



force balance  
(conservation of momentum)

$$\frac{\partial \sigma_{ij}}{\partial x_j} = - Ra \Delta T \delta_{ir} - Ra_C \Delta C \delta_{ir}$$

chemical buoyancy

constitutive law  
(rheology)

$$\sigma_{ij} = -P \delta_{ij} + 2\eta(\sigma, T, d, H_2O, \varepsilon) \mathbf{e}_{ij}$$

non-Newtonian viscosity with memory

# Mantle convection

- × Energy equation introduces time-dependence
- × Coupling between velocity and temperature introduces non-linearity
  - × Can time reverse advection, but not diffusion

force balance  
(conservation of momentum)

$$\frac{\partial \sigma_{ij}}{\partial x_j} = - Ra \Delta T \delta_{ir} - Ra_C \Delta C \delta_{ir}$$

constitutive law  
(rheology)

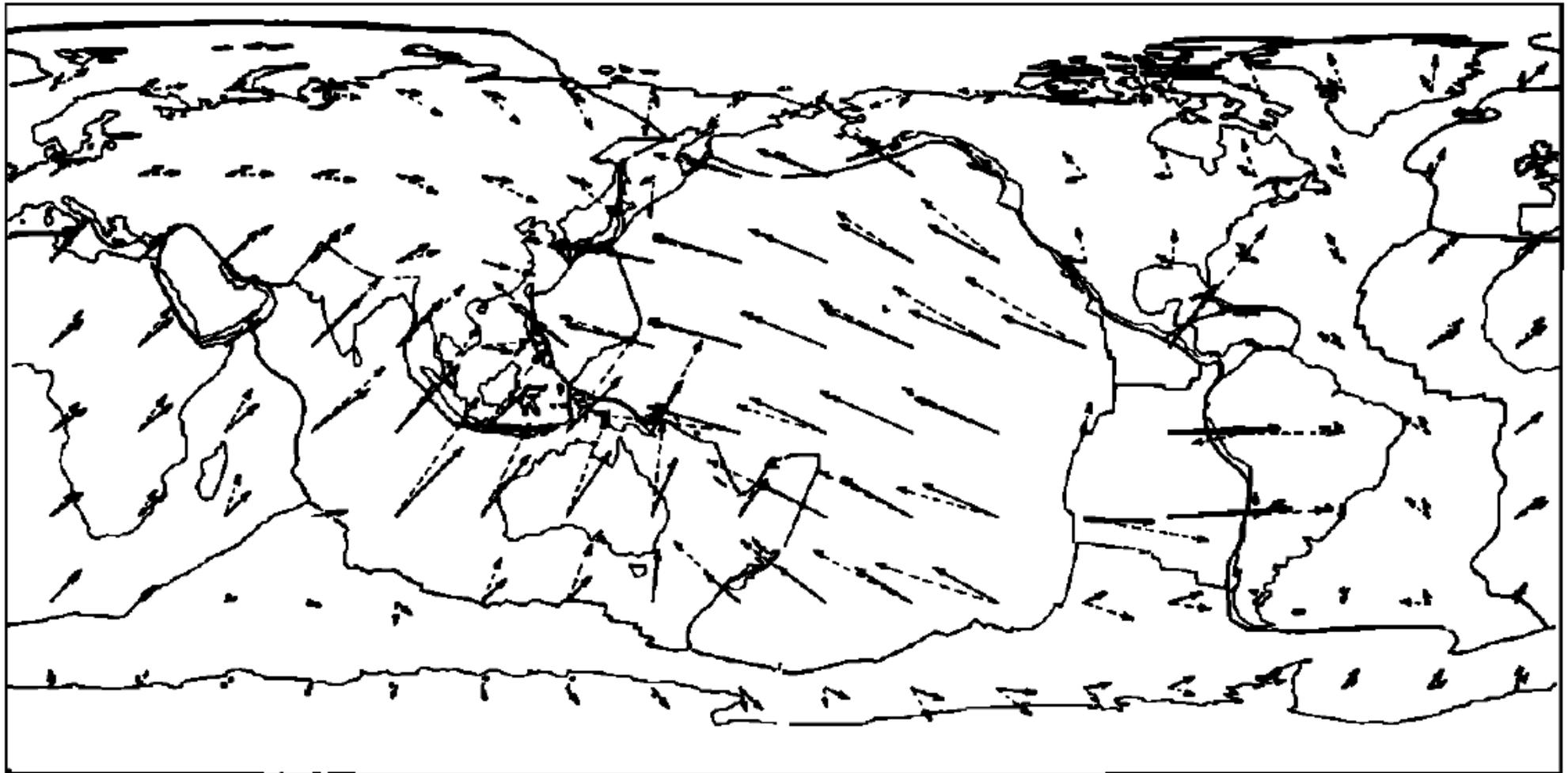
$$\sigma_{ij} = -P \delta_{ij} + 2\eta(\sigma, T, d, H_2O, \varepsilon) \epsilon_{ij}$$

conservation of energy

$$\frac{\partial T}{\partial t} = \underbrace{-u_i \frac{\partial T}{\partial x_i}}_{\text{advection}} + \underbrace{\kappa \frac{\partial^2 T}{\partial x^2}}_{\text{diffusion}} + \underbrace{H}_{\text{heat production}}$$

# Global circulation models

show that plate-motion induced shear cannot always be guessed from surface motions

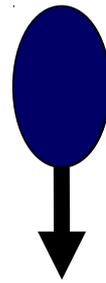


SHEAR STRESS AT BASE OF LITHOSPHERE

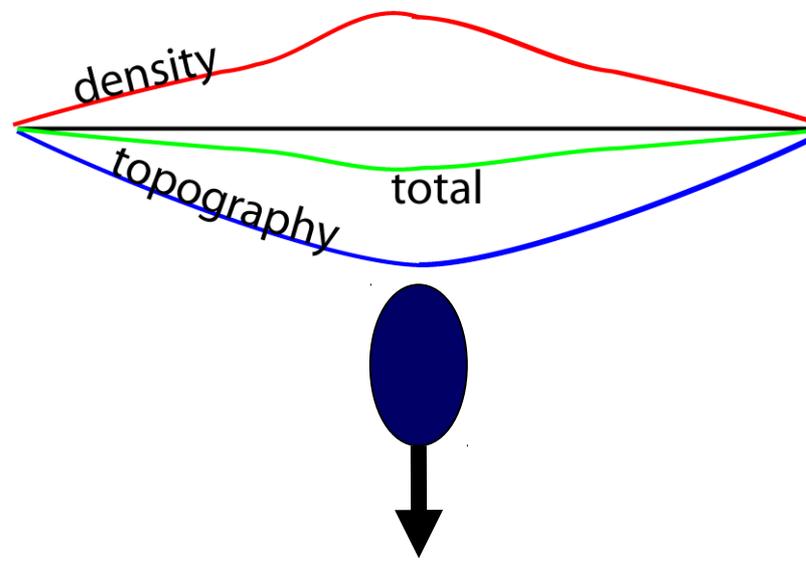
—→ Simple Drag

- - -→ Flow Model

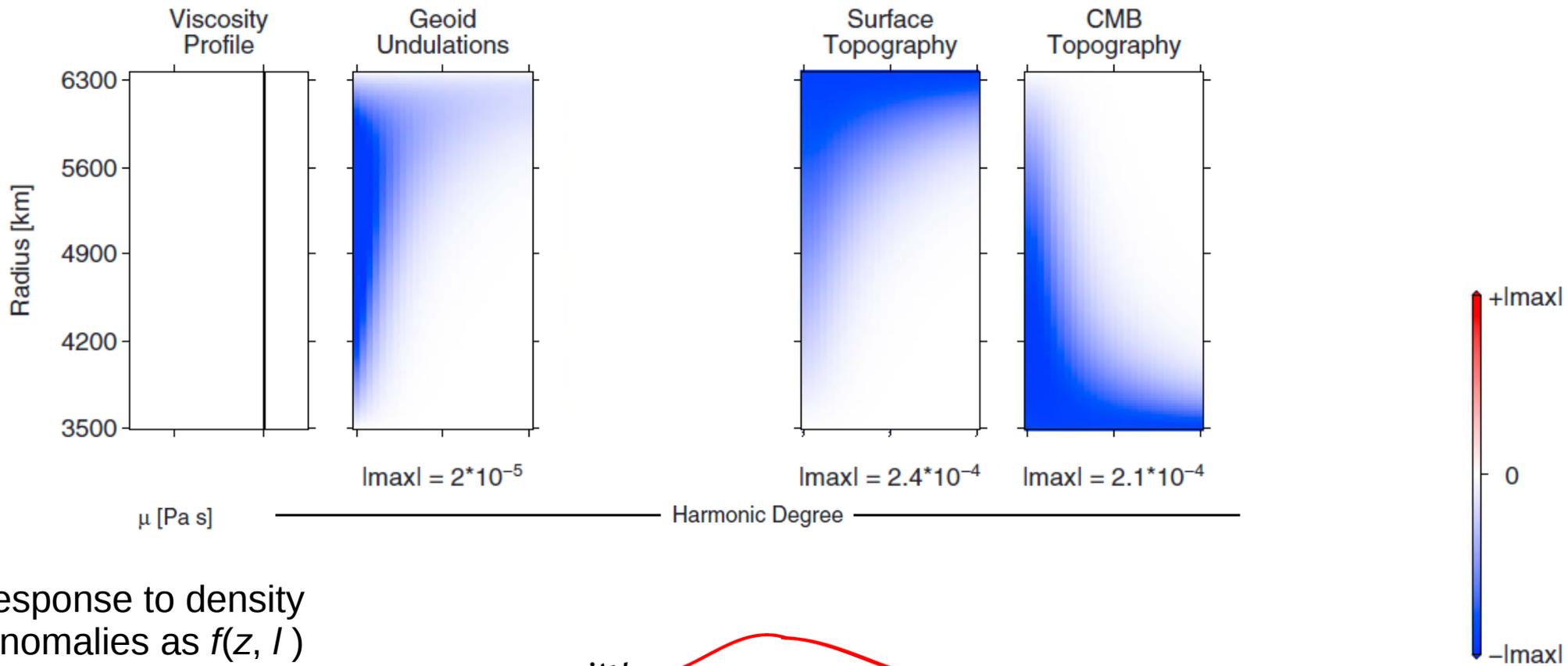
# Let's fit the geoid: Static effect of slablet



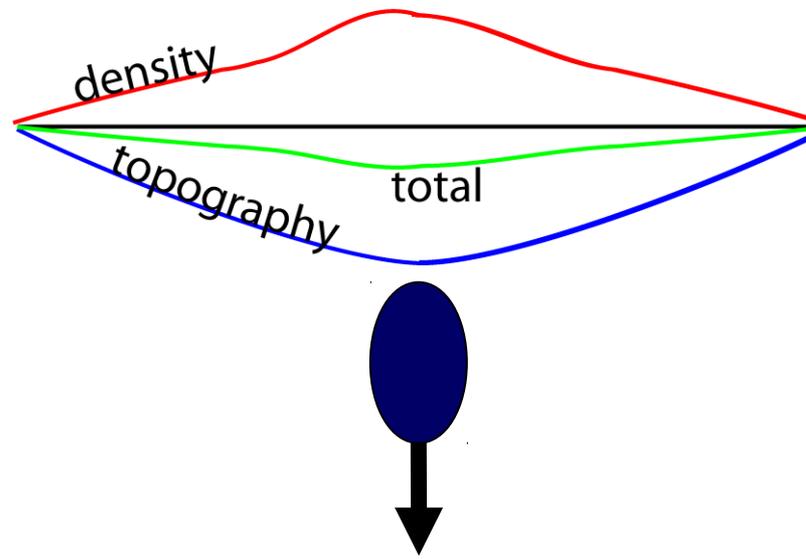
# Combined static and dynamic geoid effect of slablet



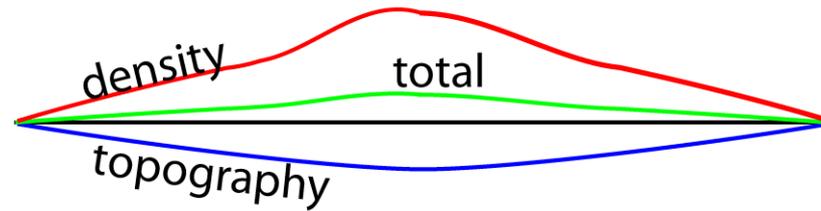
# Kernels for homogeneous mantle



response to density anomalies as  $f(z, l)$

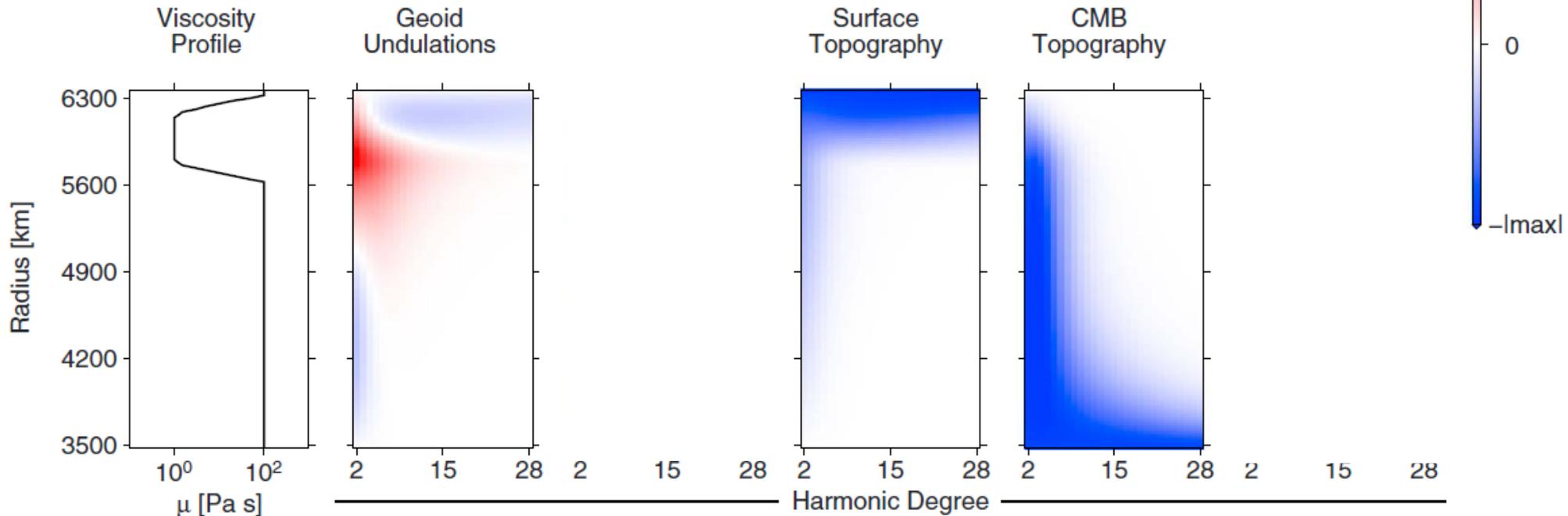
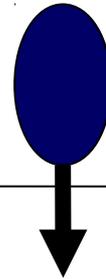


# Kernels for layered viscosity mantle



Layer 1

Layer 2

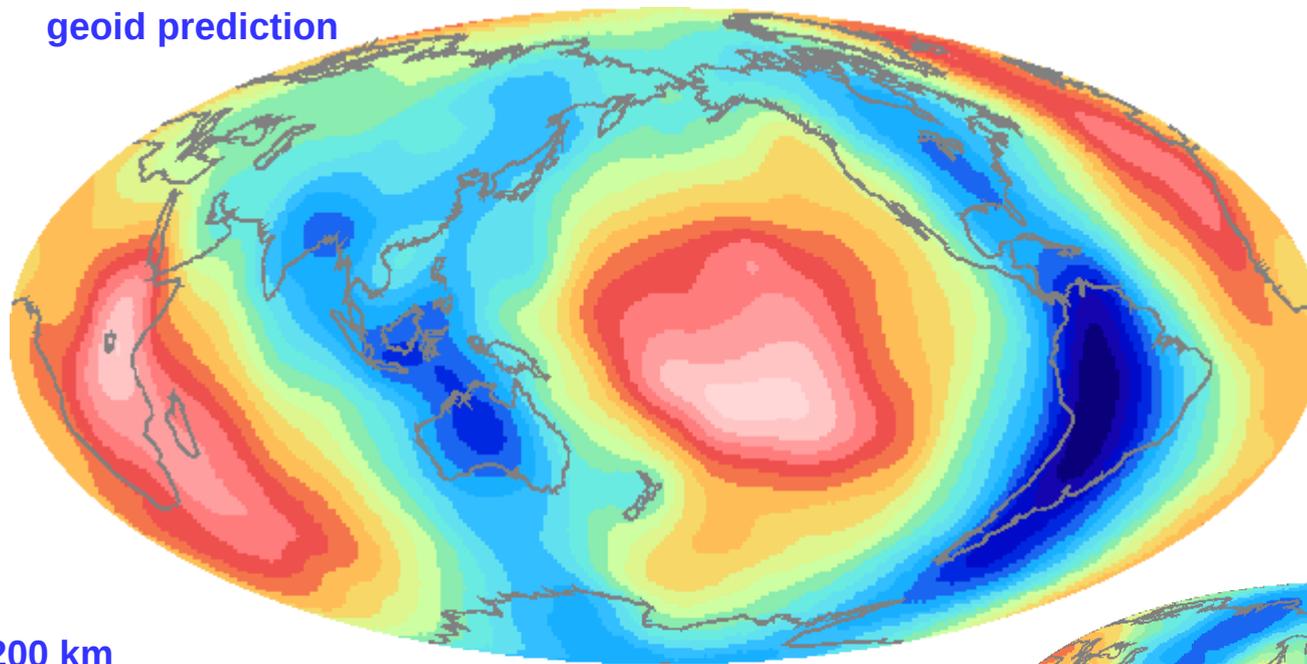


# Geoid for tomography driven flow

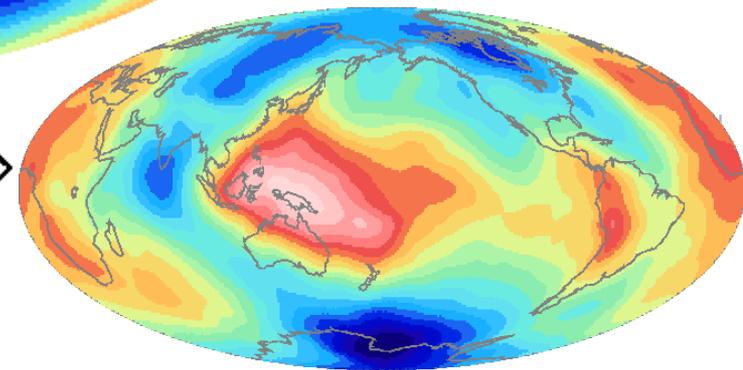
- Isoviscous – free slip surface boundary condition

geoid prediction

Constant  
 $d \ln \rho / d \ln v_s$

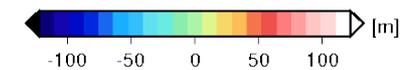
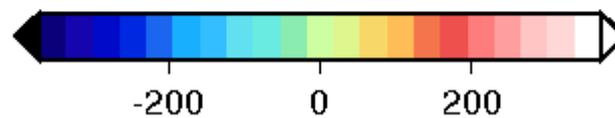
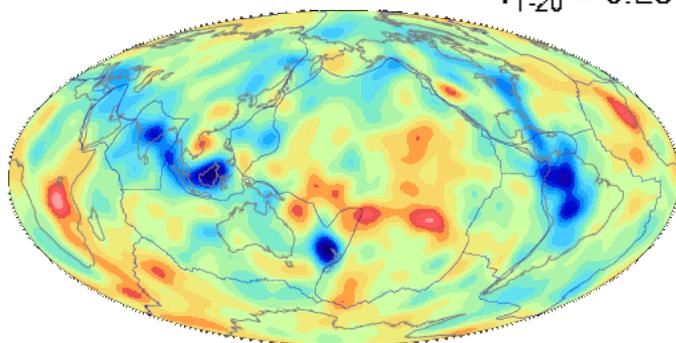


observed geoid



tomography @ 1200 km

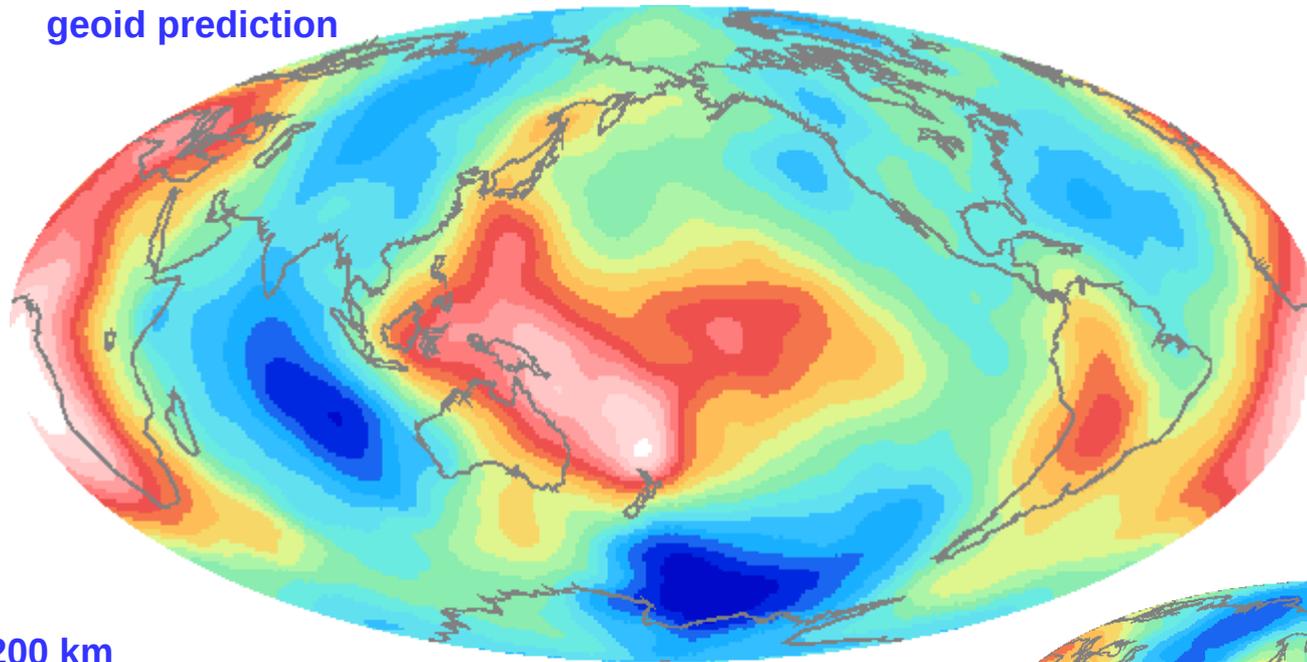
$r_{1-20} = 0.28$



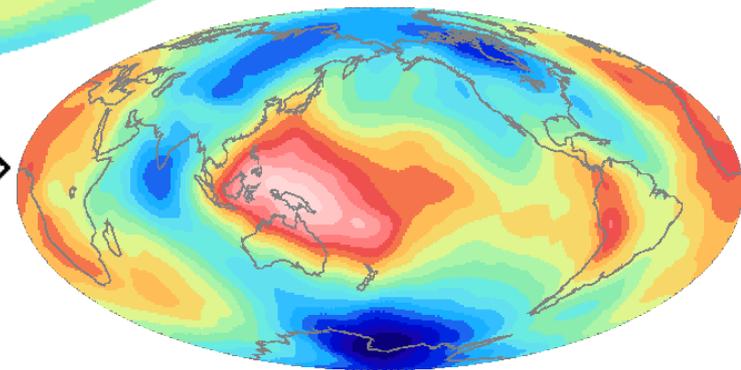
# Geoid for tomography driven flow

- Four layer model with viscosity increase in lower mantle

geoid prediction

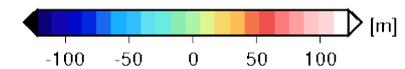
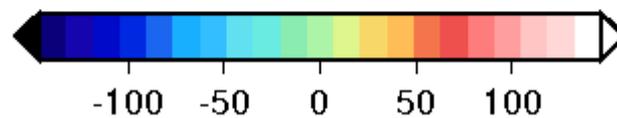
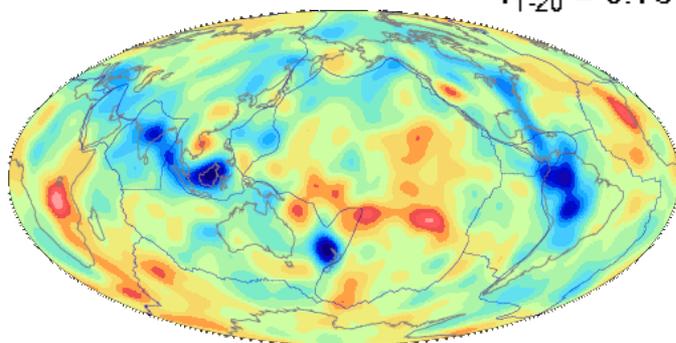


observed geoid



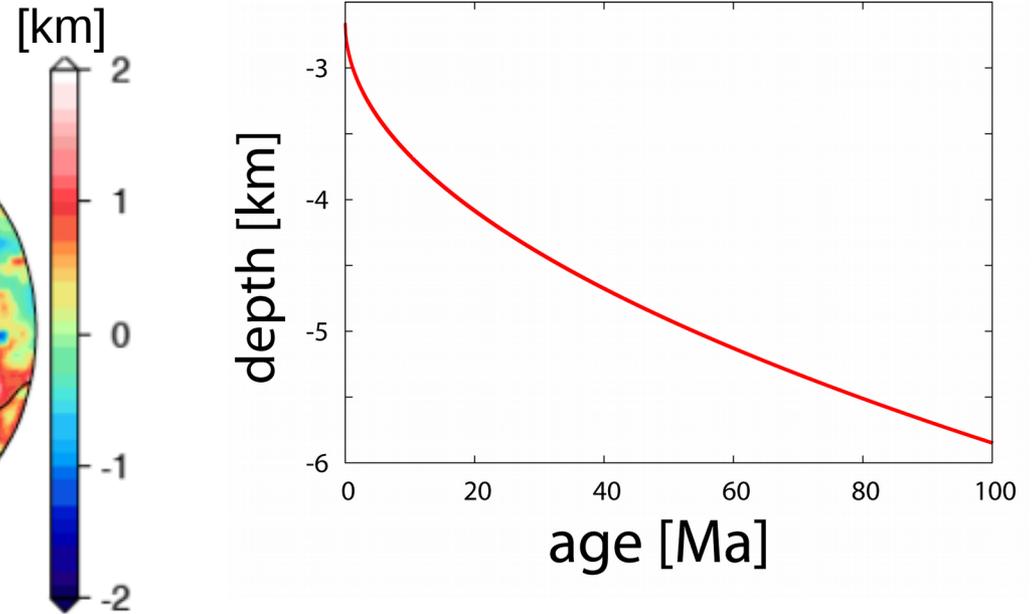
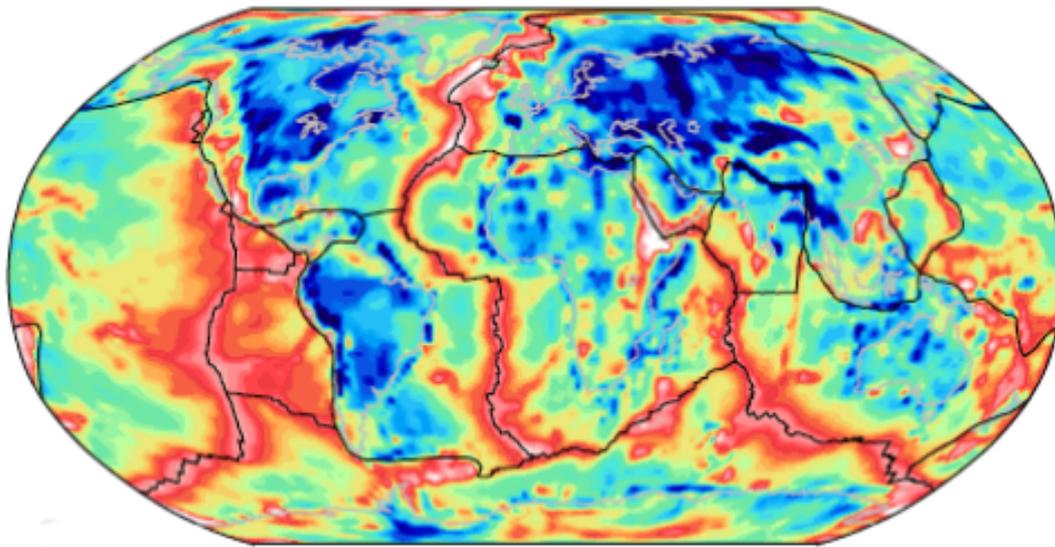
tomography @ 1200 km

$r_{1-20} = 0.79$



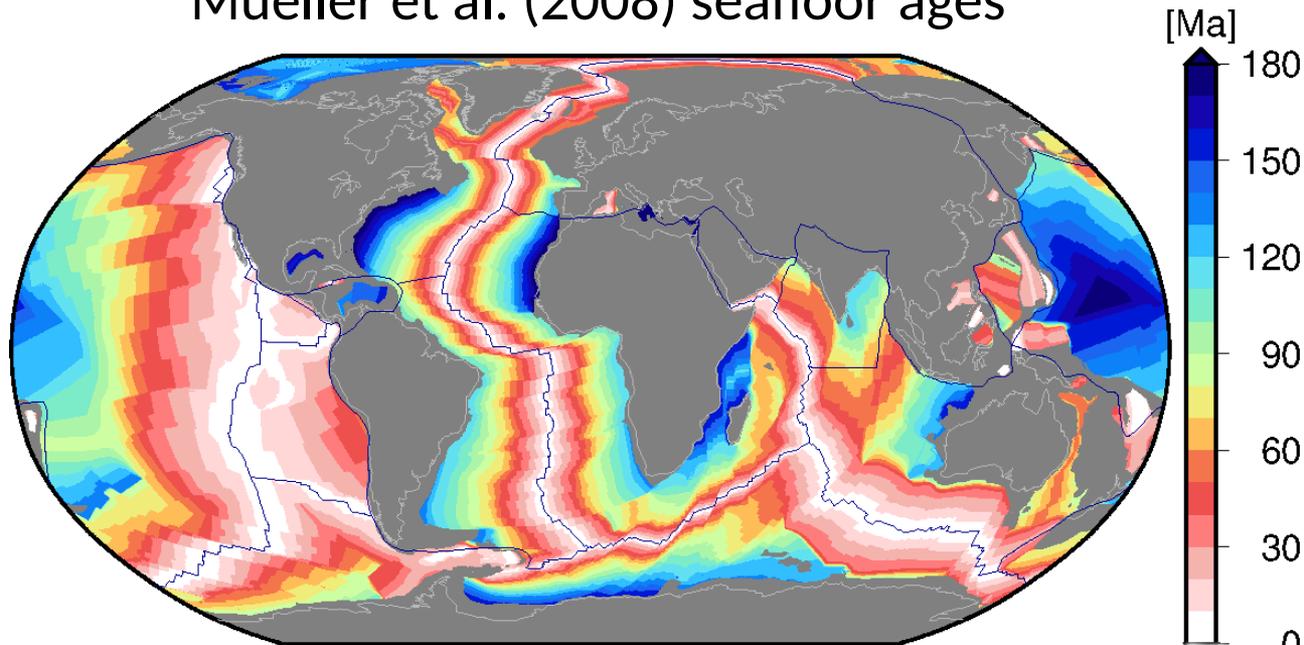
# “Airy” residual surface topography: mainly due to half-space cooling

Non-isostatic residual

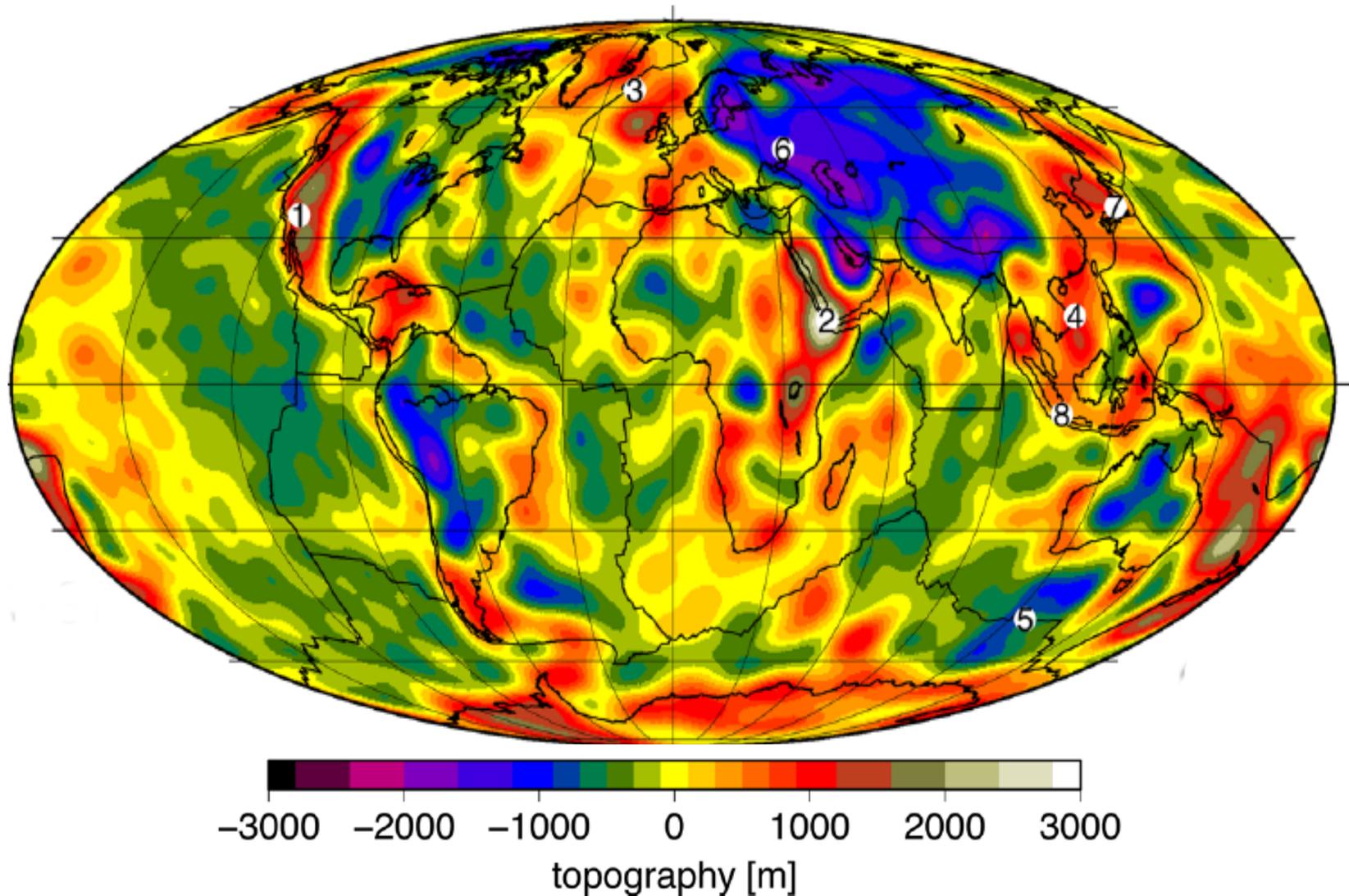


Mueller et al. (2008) seafloor ages

→ the outstanding performance  
of this geodynamic “model”  
complicates attribution of  
anomalous topography

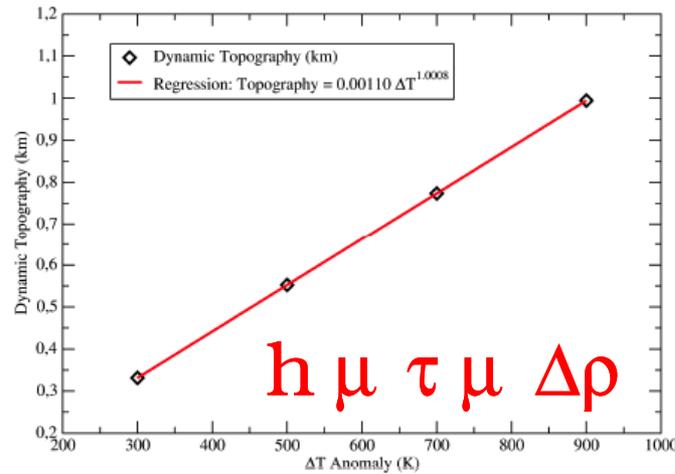
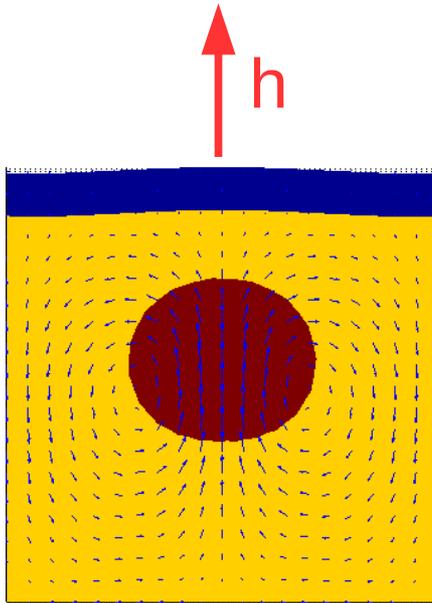


# Non half-space cooling residual topography



Topography corrected for Airy isostasy, half-space cooling, and lithospheric model (Steinberger, 2016)

# Dynamic topography physics: “plume” case



→ dynamic topography  $h \propto \mu \Delta\rho$  with density (temperature anomaly),  $\Delta\rho$  (here =  $\Delta T$ )

Stokes' sphere  
velocity/stress:

$$v_{\text{Stokes}} \propto \mu \Delta\rho / \eta$$

$$\tau \propto \mu \Delta\rho$$

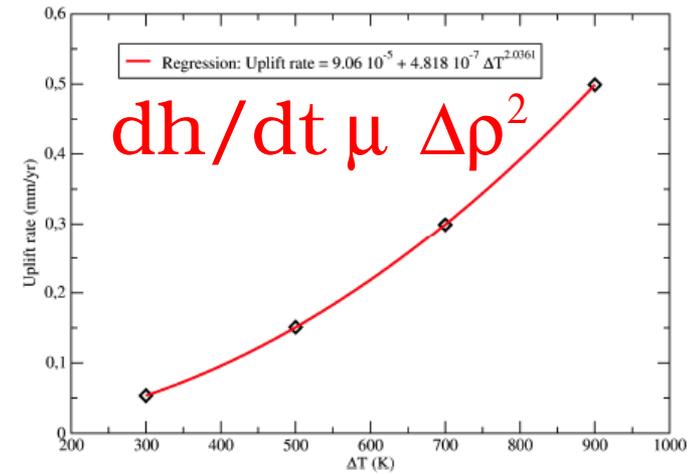
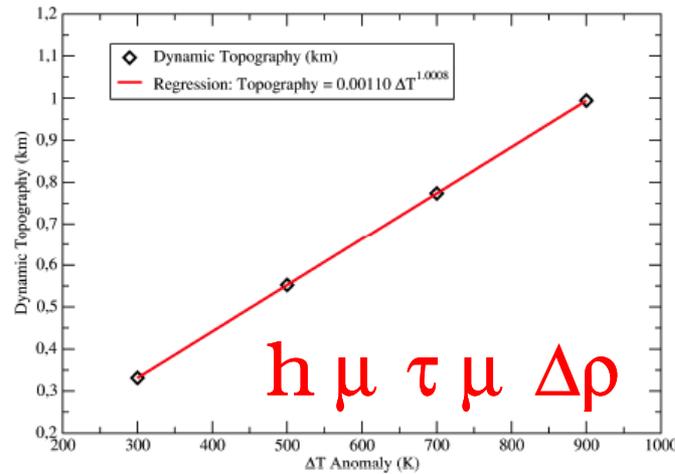
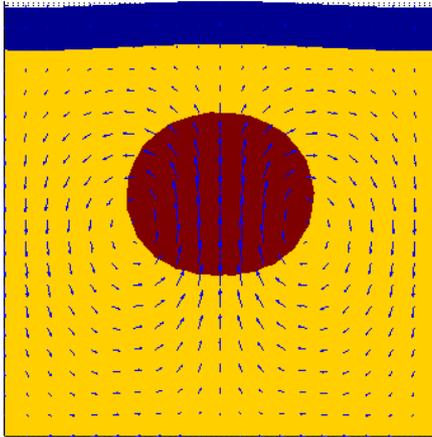
Notes:

→ Often, we infer equivalent topography from  $\sigma_{zz}$  pushing on a free slip surface

→ this works remarkably well in most cases ( $\lambda \ll L$ )

# Dynamic topography physics: “plume” case

$h, dh/dt$



→ dynamic topography  $h \propto \mu$  with density (temperature anomaly),  $\Delta\rho$  (here =  $\Delta T$ )

$$v \propto \mu \Delta\rho/\eta$$

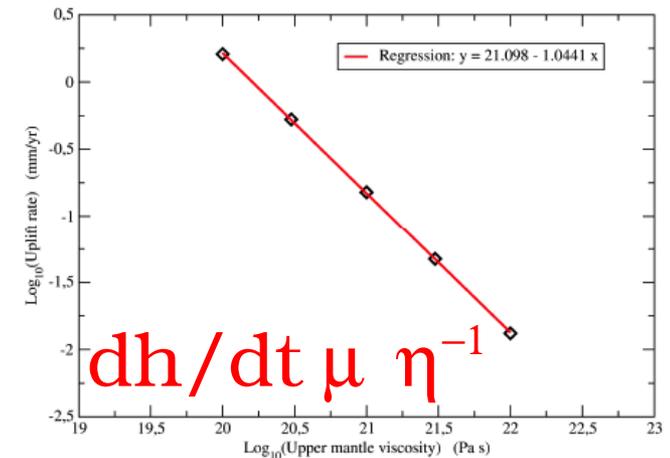
$$h \propto \mu \tau \mu \Delta\rho$$

$$dh/dt \propto \mu v h$$

$$\propto \mu \Delta\rho^2/\eta$$

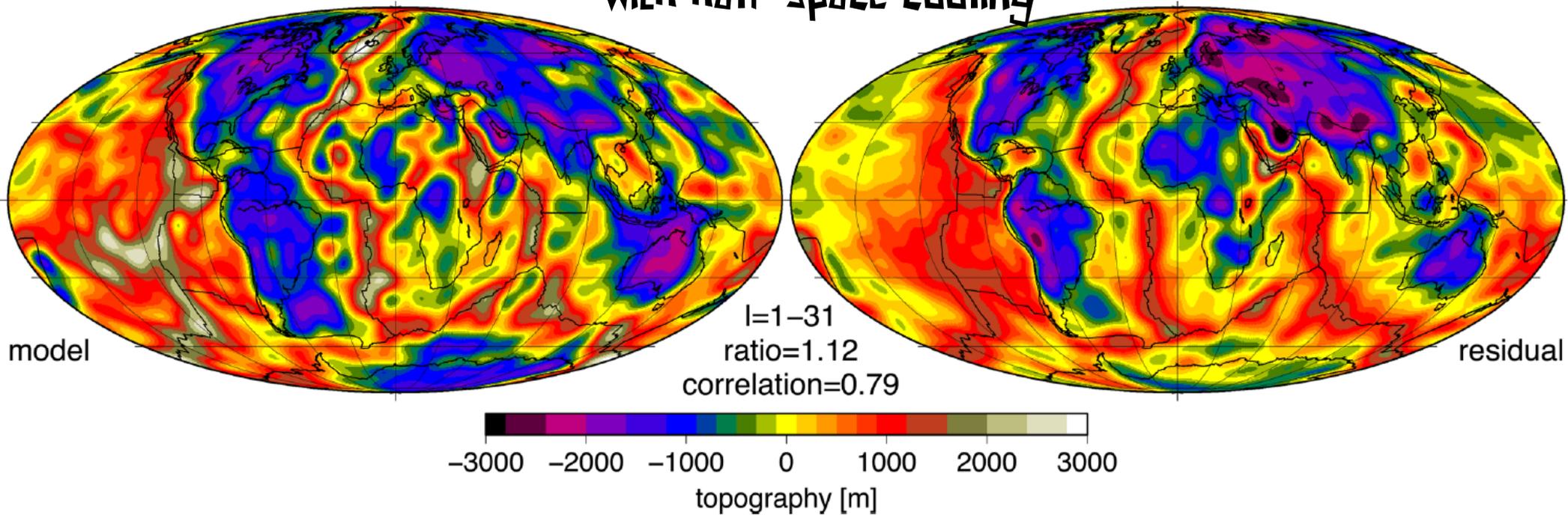
Note:

- Match topography and velocity, constrain both  $\Delta\rho$  and  $\eta$ ,
- match uplift, even better!

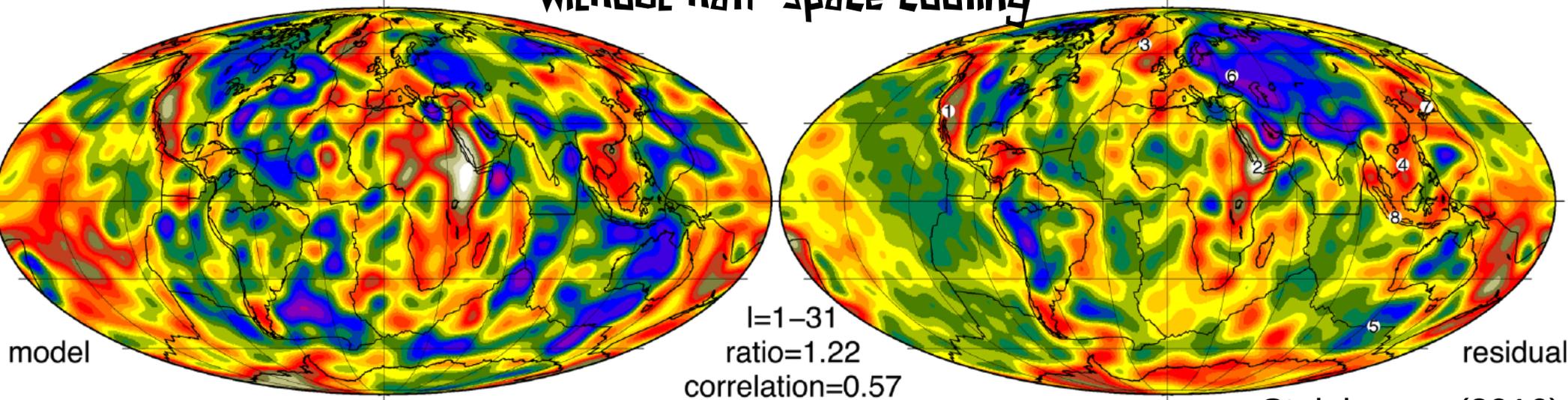


→ uplift rate,  $dh/dt \propto \mu$  inverse of viscosity,  $1/\eta$  (and density anomaly squared)

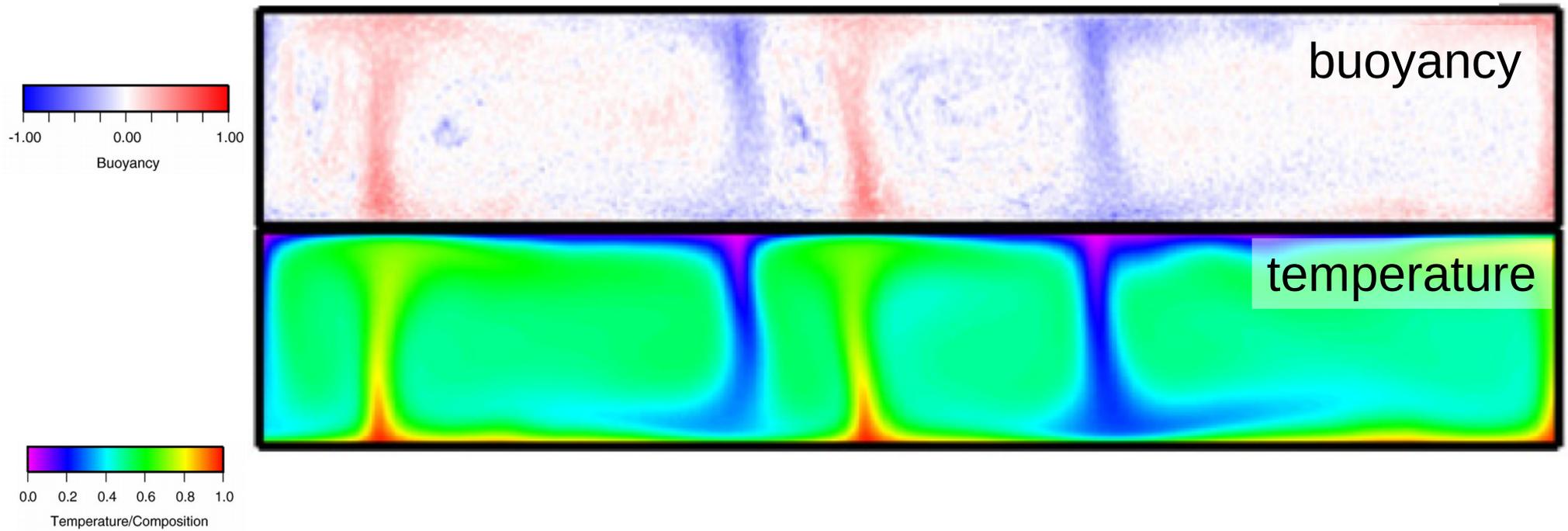
# Match of dynamic topography from mantle flow models to surface residual topography with half-space cooling



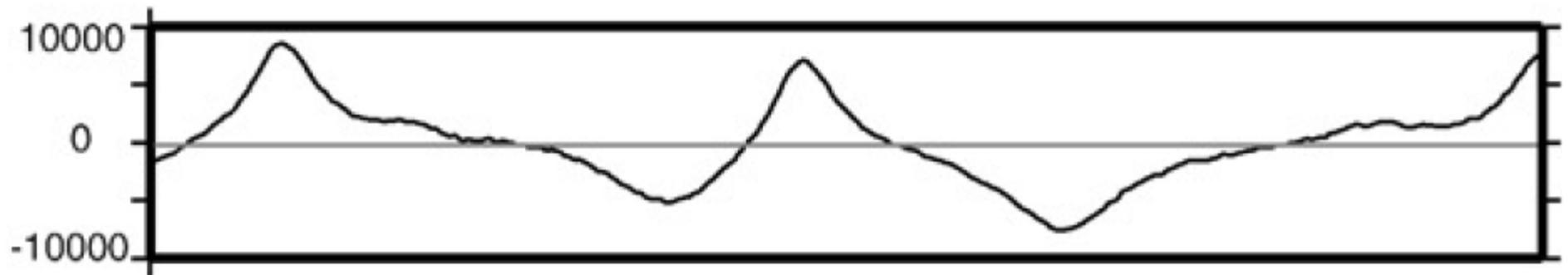
# without half-space cooling



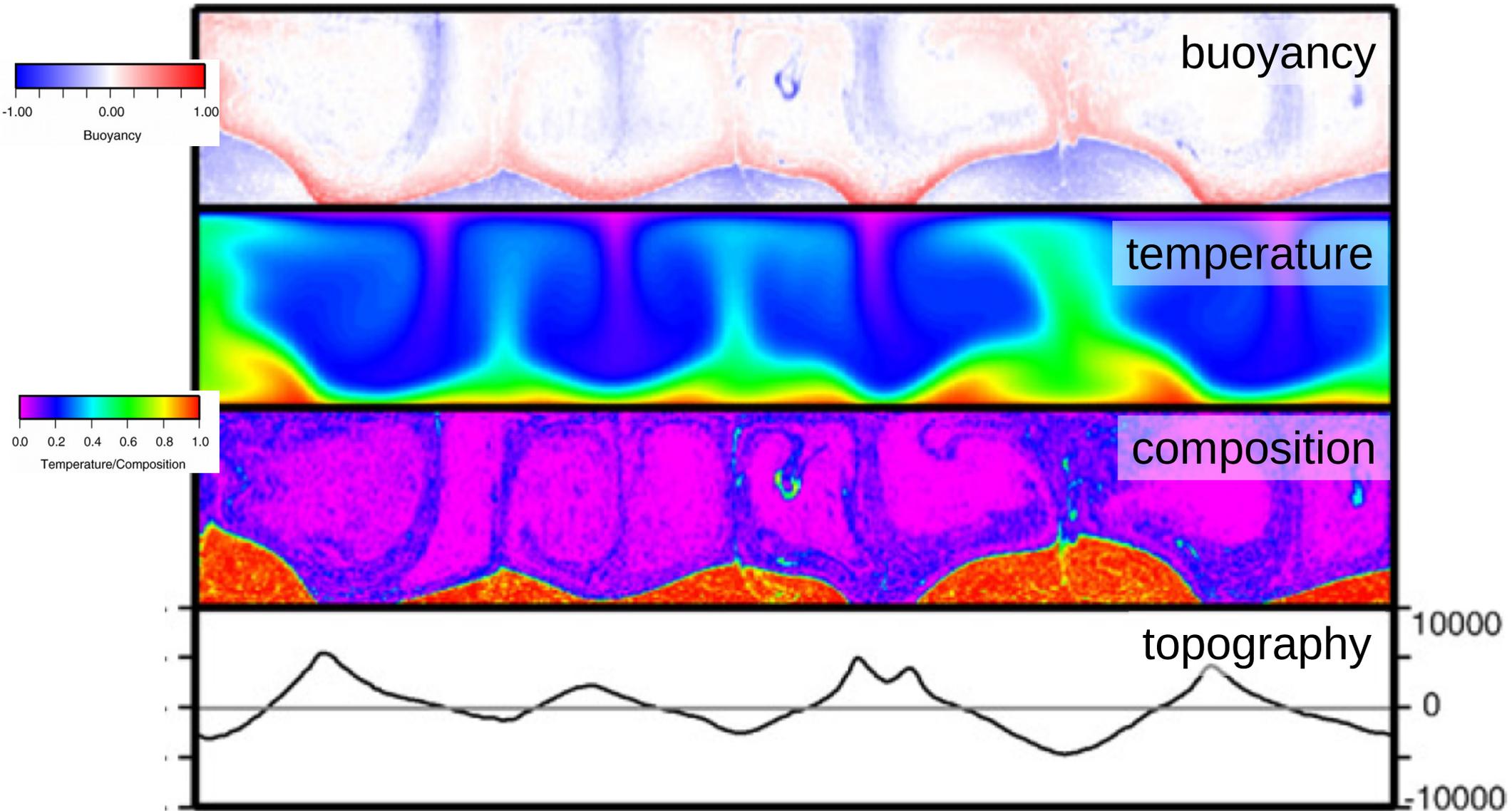
# Isochemical, $\Delta\rho = 1$



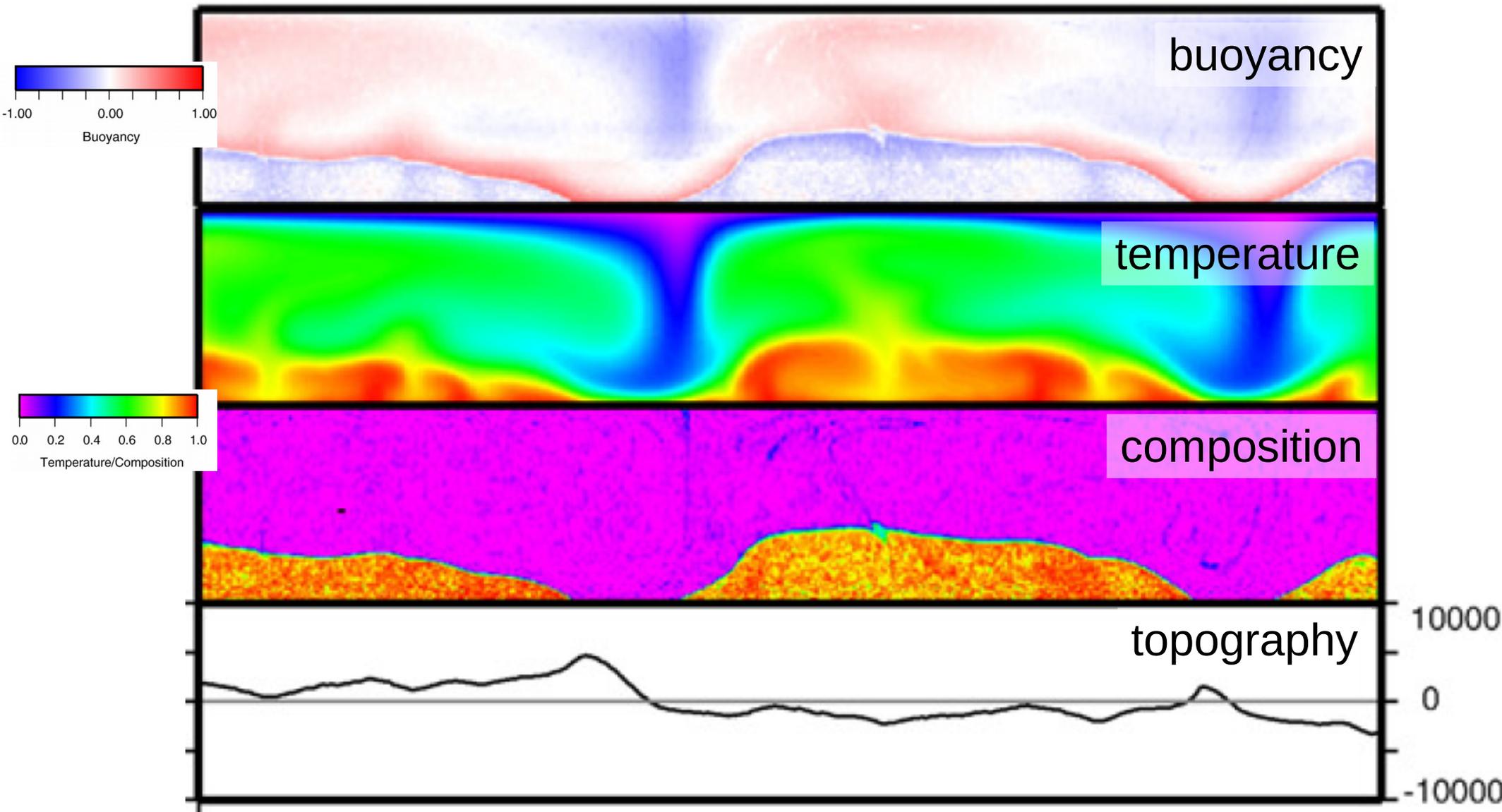
dynamic topography @ CMB



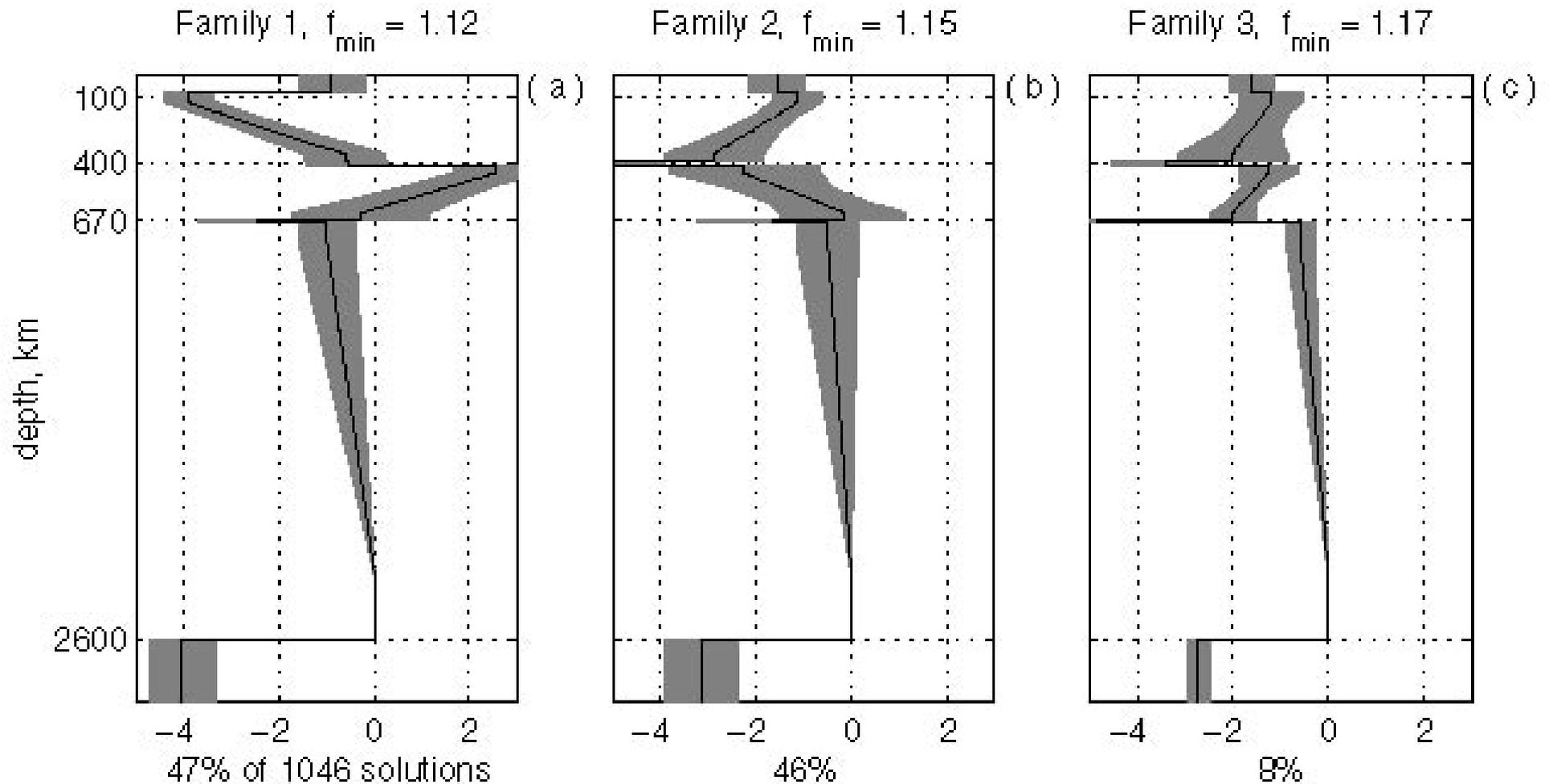
# Pilotania, $\Delta\rho = 1$



# Pilotania, $\Delta\rho = 1,000$



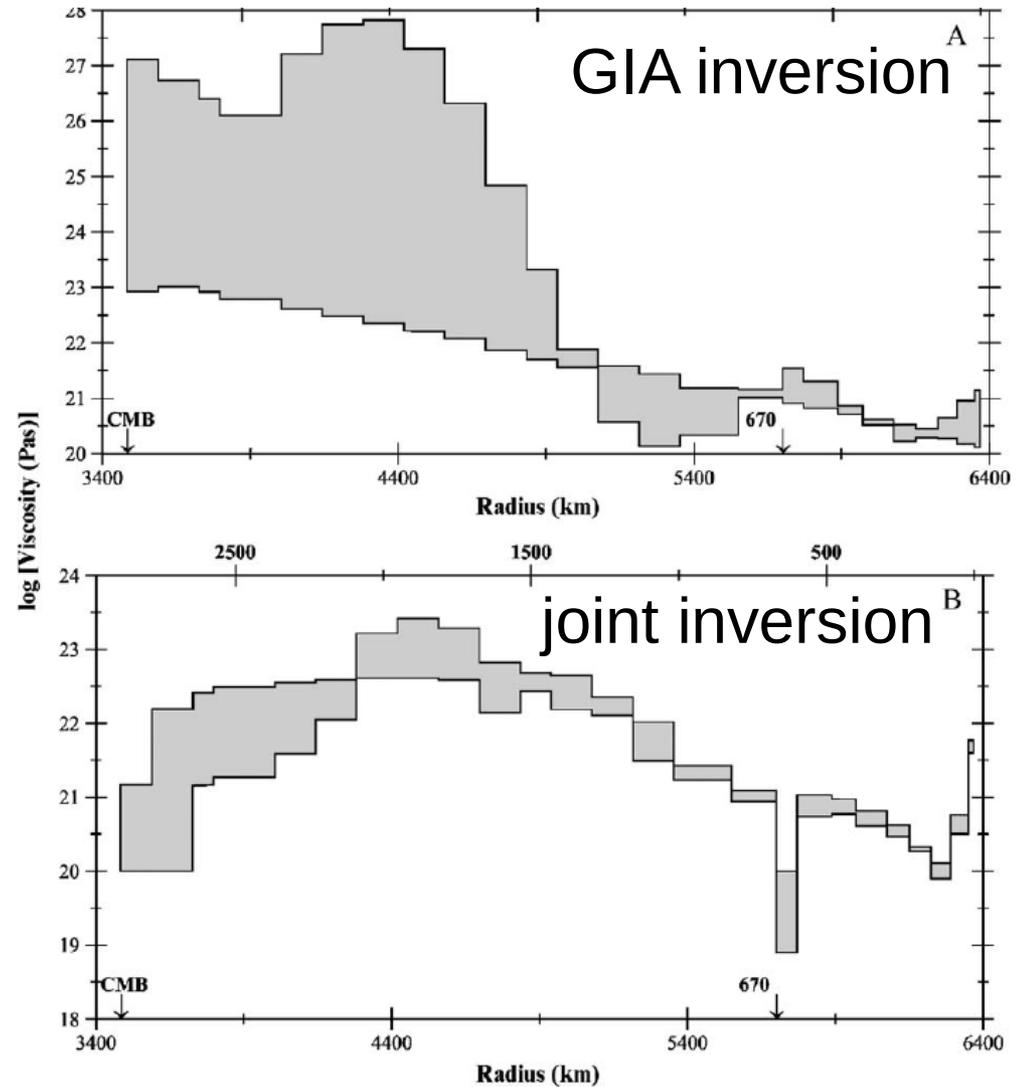
# Viscosity inversions are non-unique (Monte Carlo approach, based on geoid and surface dyn. topo)



# Combining post-glacial rebound (GIA) and geoid (still need to close loop with ice models)

## Notes:

- geoid gives relative  $\eta$  with depth
- GIA gives absolute value
- “Haskell constraint” is  $\sim 10^{21}$  Pas for *average* down to  $\sim 1200$  km (under cratons...)



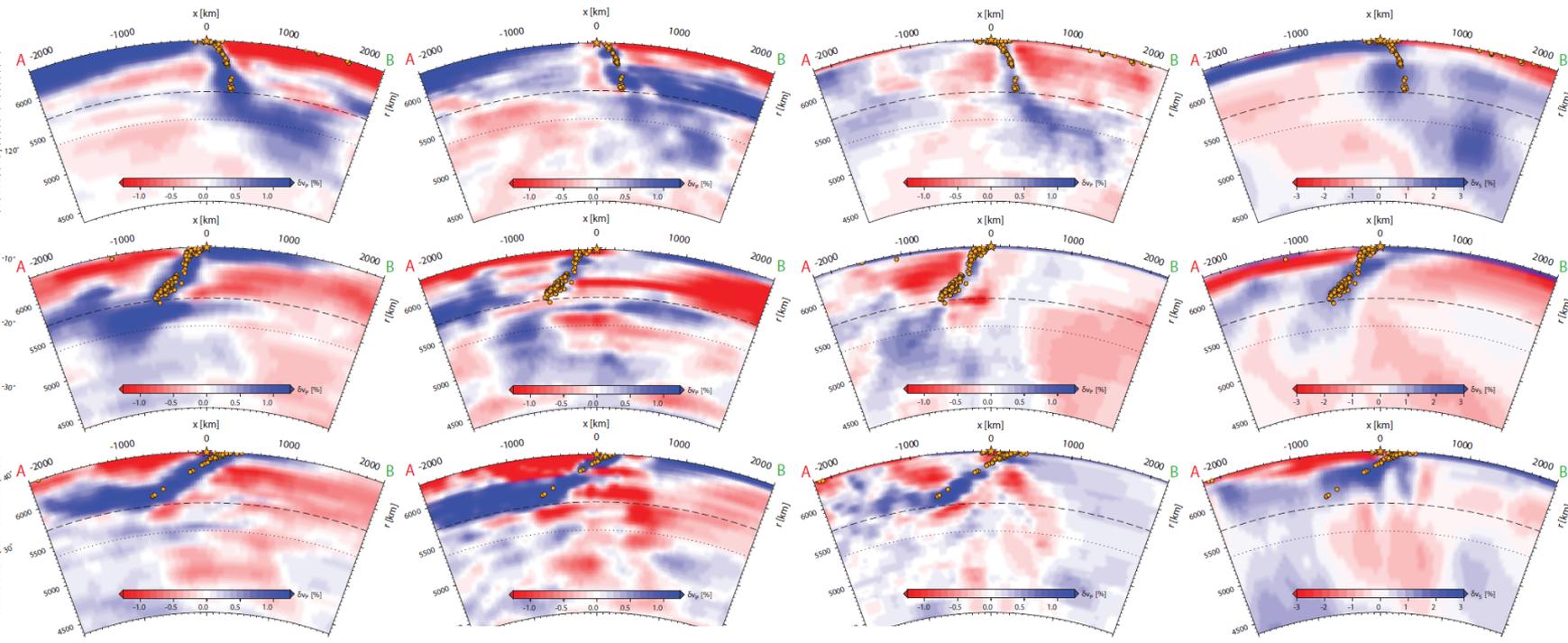
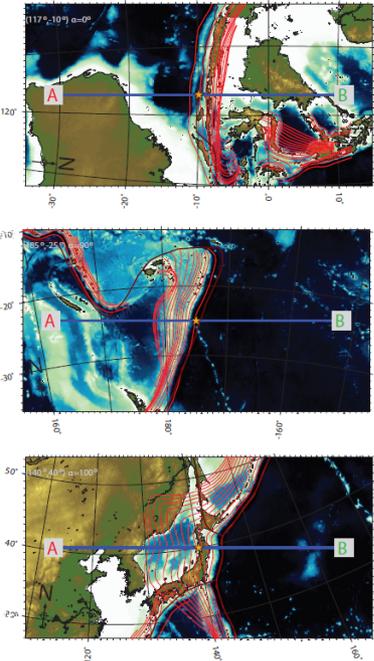
# Slab ponding diversity and viscosity stratification in transition zone

GAP-P4

LLNL-G3D

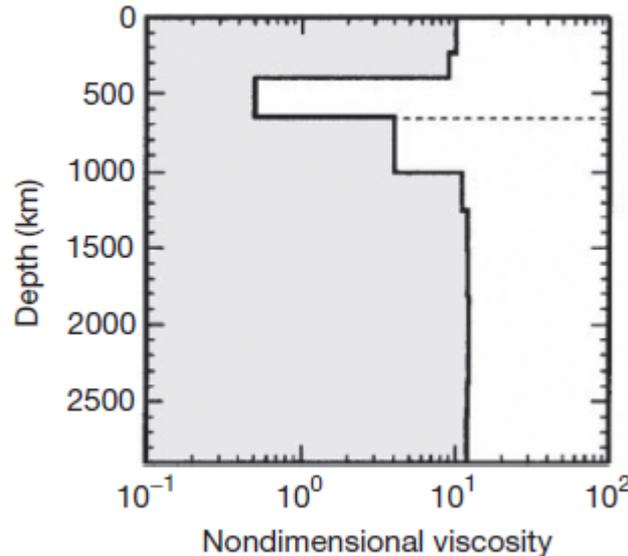
MIT-P08

SAVANI



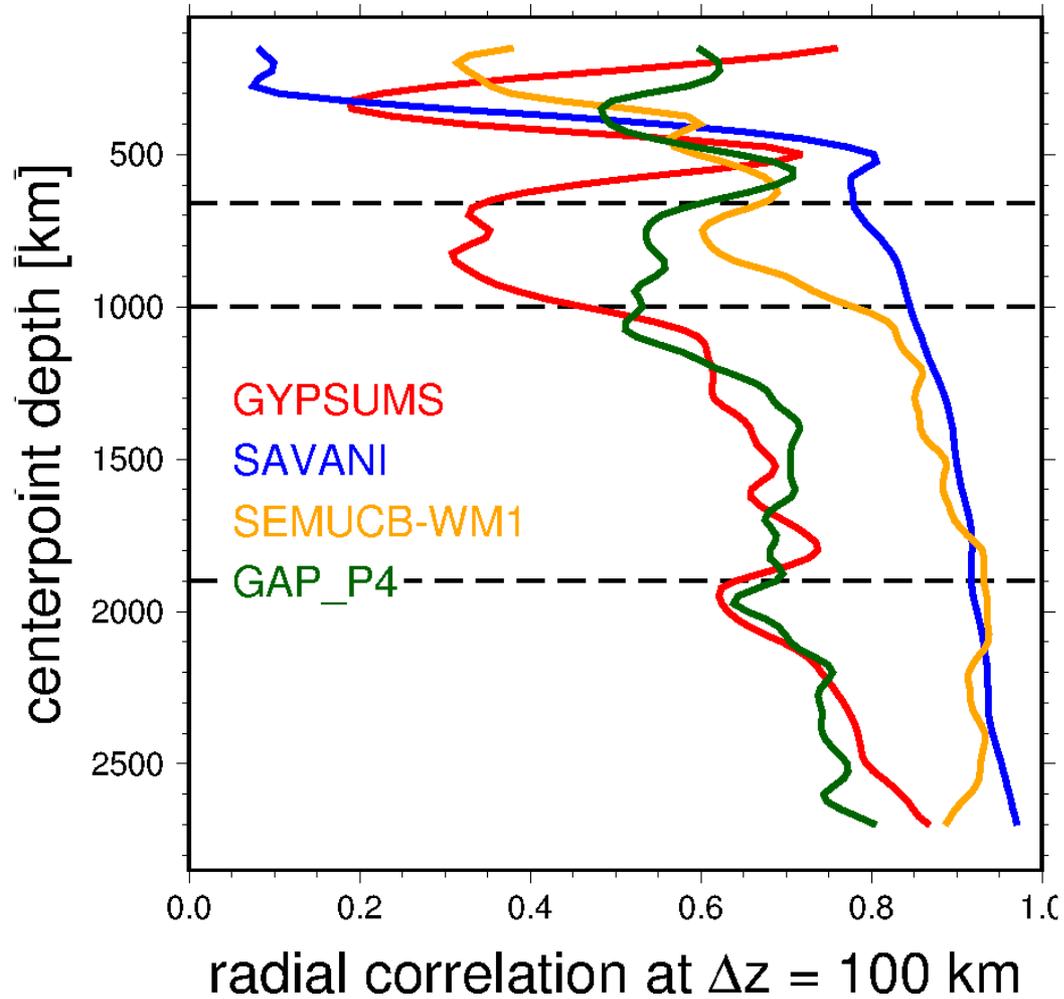
cf. Fukao and Obayashi (2013)

King and Masters (1992)

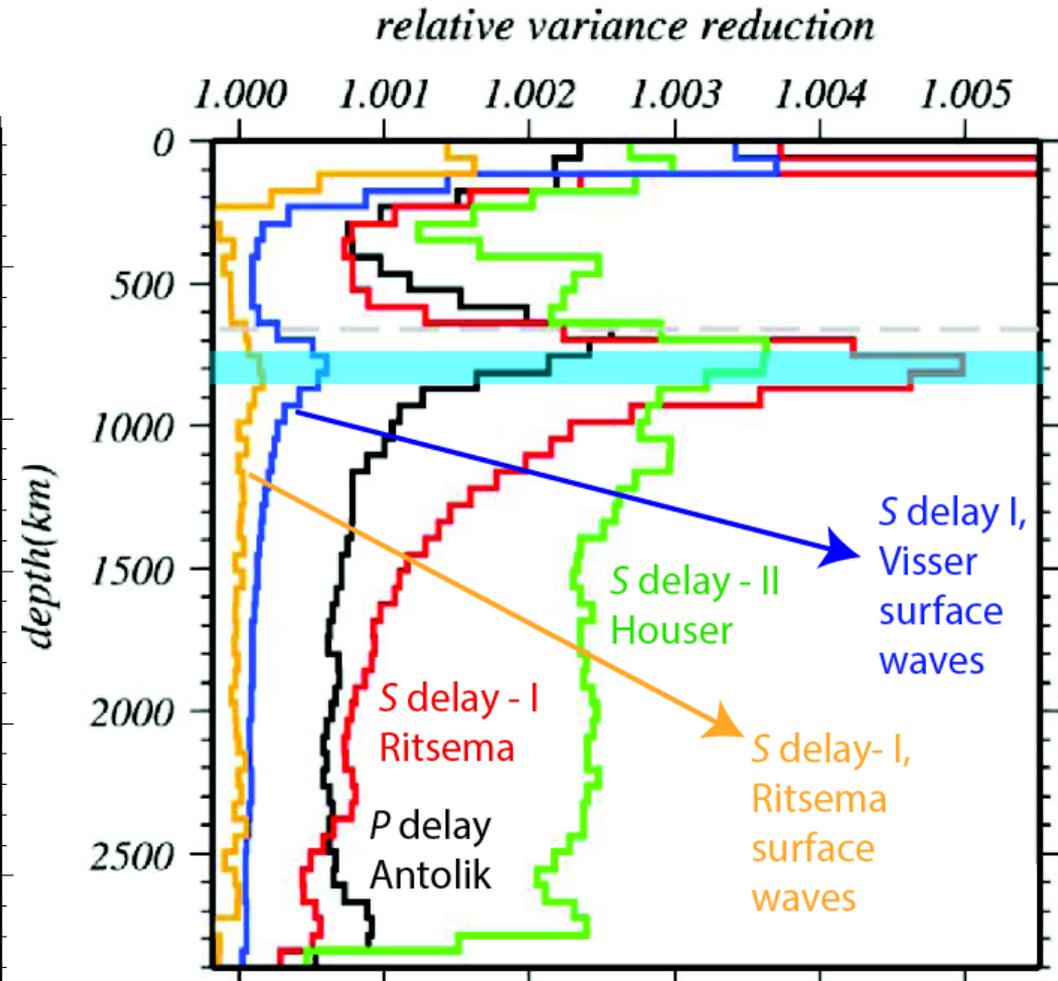
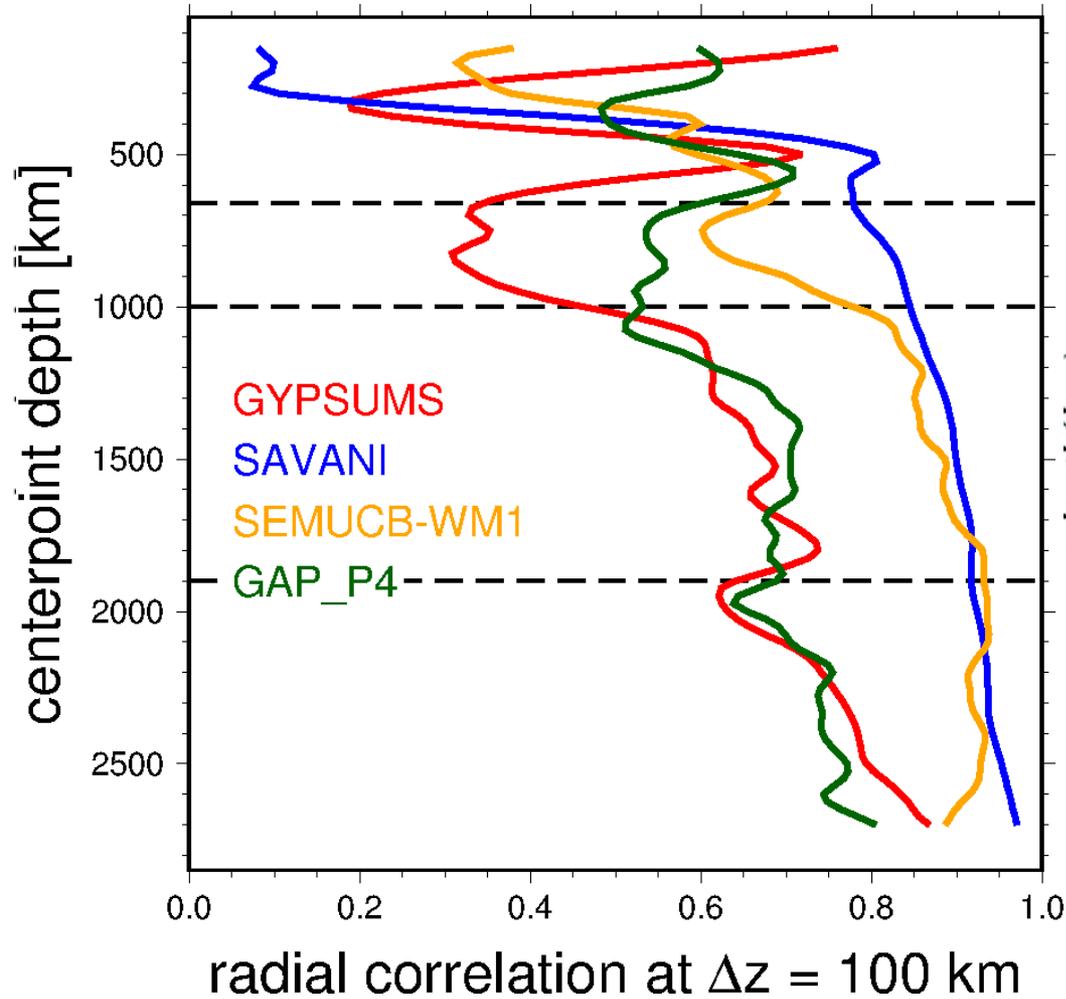


Genetic algorithm inversion based on geoid

# Global, stochastic view: Radial correlation functions



# Query data for discontinuity depth



Becker and Boschi (2011)

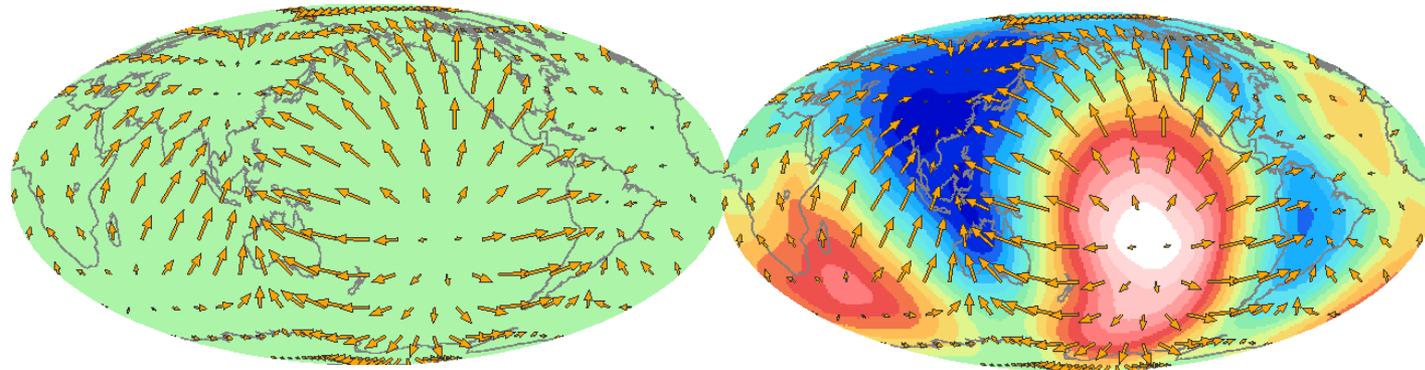
cf. Puster and Jordan (1997),  
Tackley (1998, 2002)

see Max Rudolph talk on Rudolph et al. (2015)

# Adding a constraint: Predicting plate motions

## Flow model with only radial viscosity variations

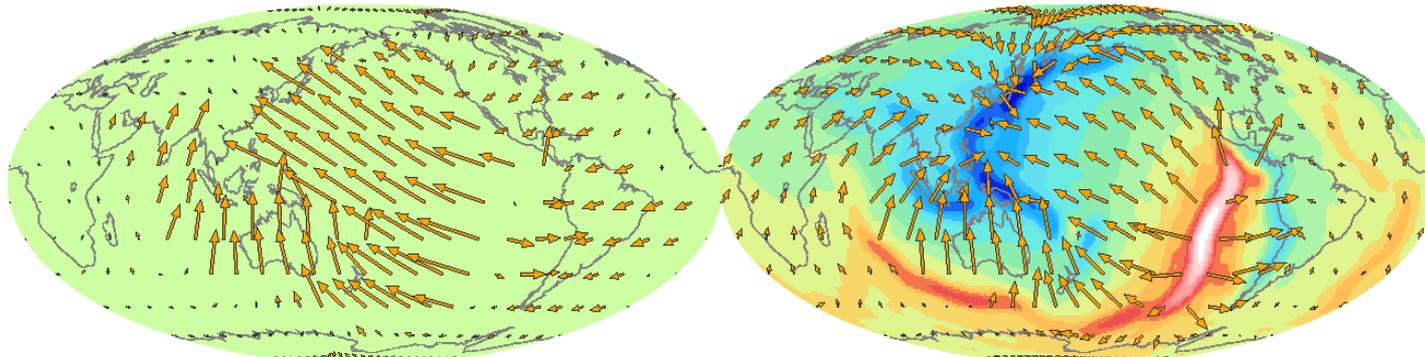
### Poloidal component



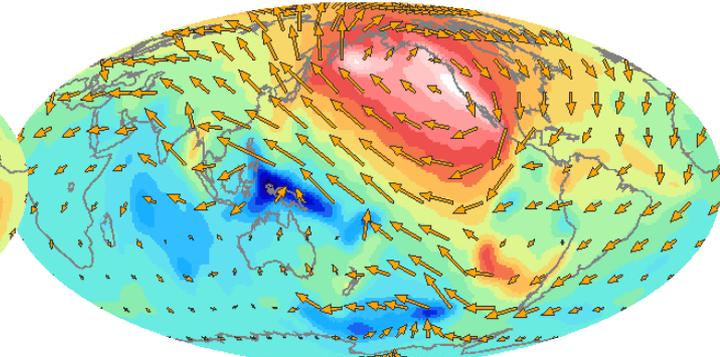
- No toroidal flow without lateral viscosity variations (no PT coupling)
- Strain-rates not very plate-like

## Observed plate velocities in hot spot reference frame

### Poloidal component



### Toroidal component



Sources and sinks

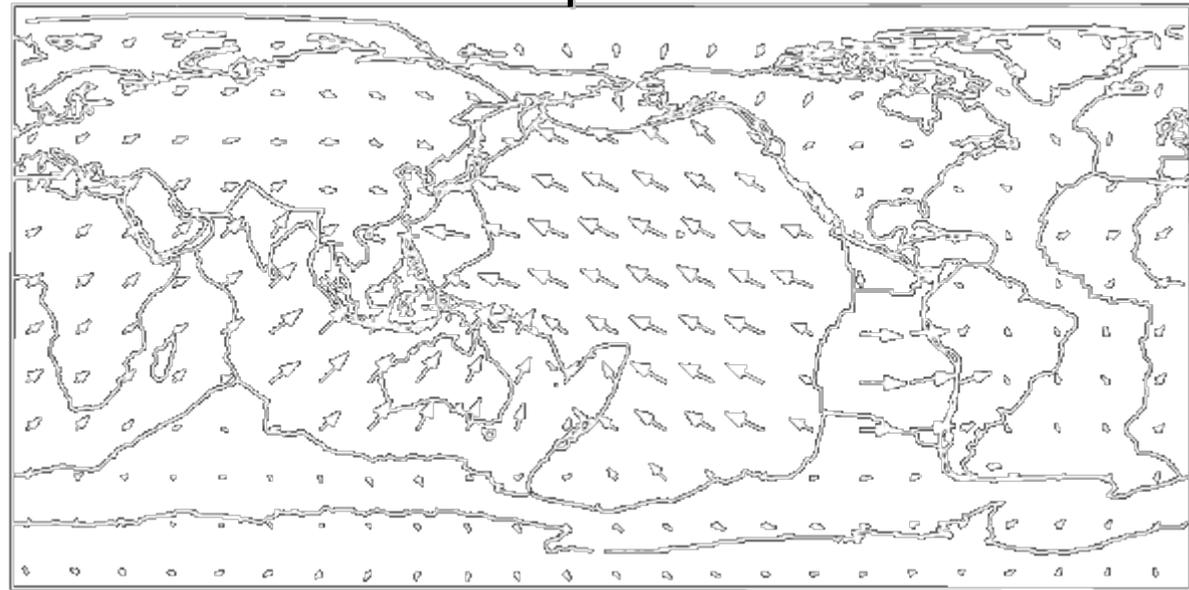
Strike slip motion, spin



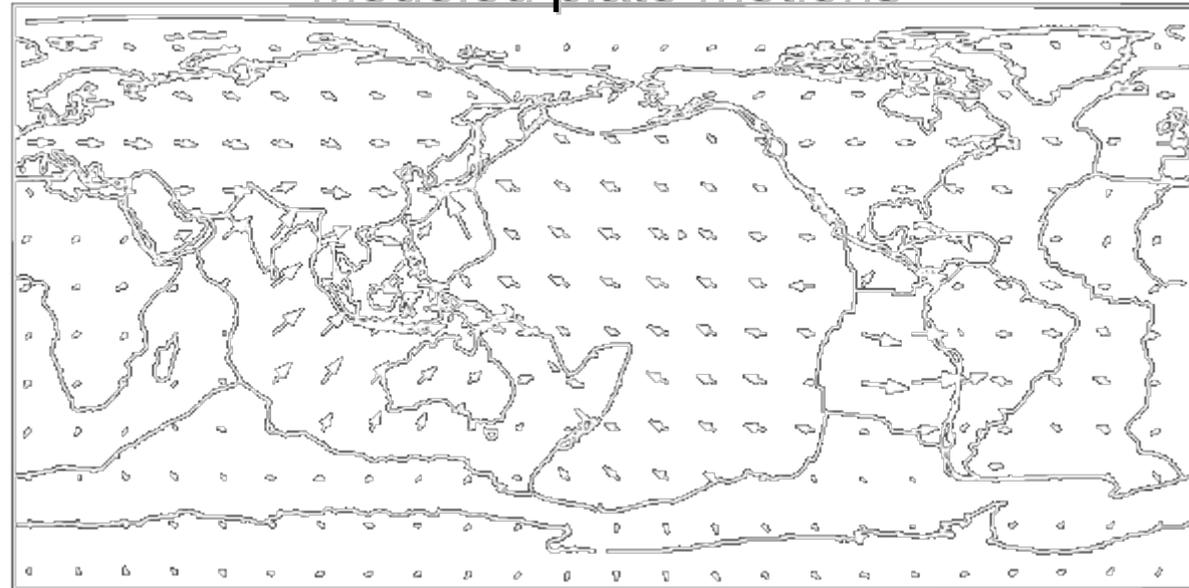
# Match to observed plate motions

- Velocity model
  - Prescribe weak plate boundaries
  - Compute plate drag coupling and driving torques
  - Solve for Euler vectors for rigid plates
- Correlations good, but oceanic plates move as fast as continental ones

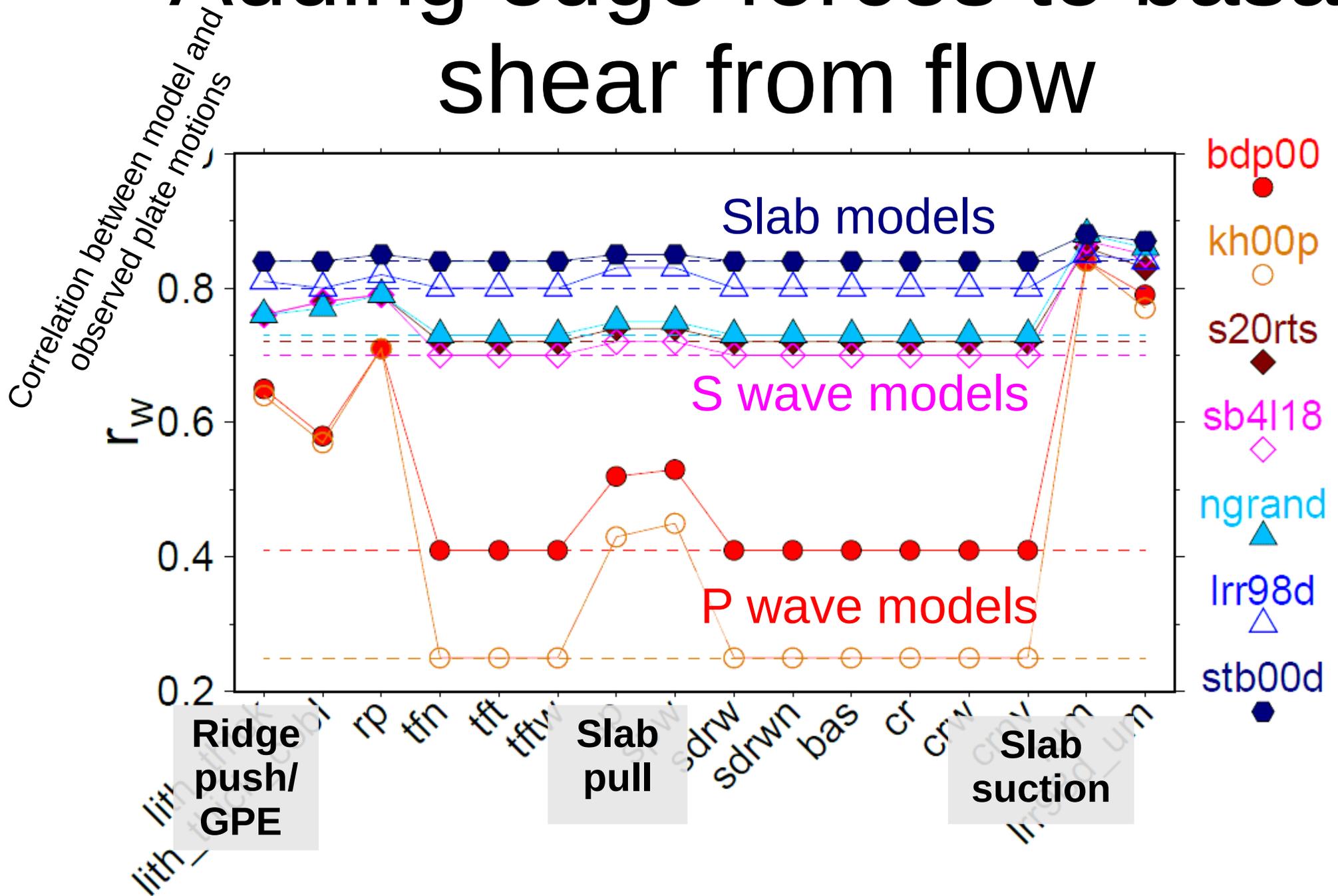
Observed plate motions



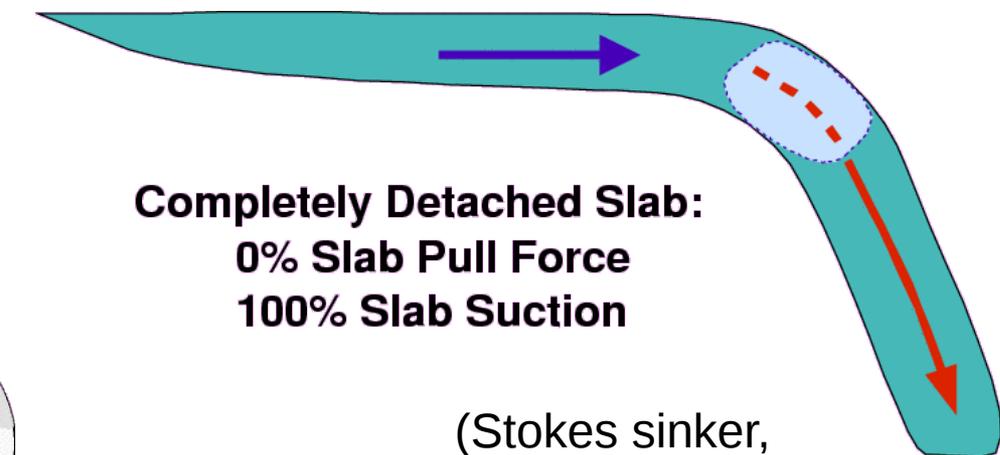
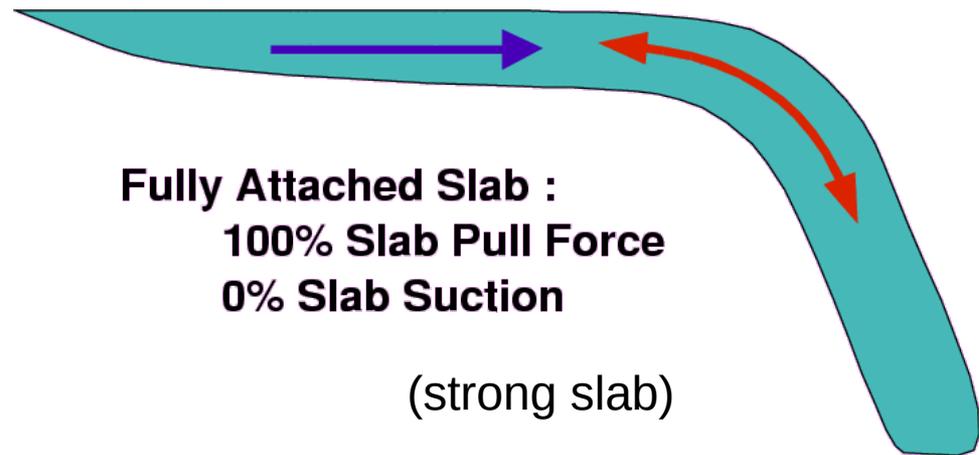
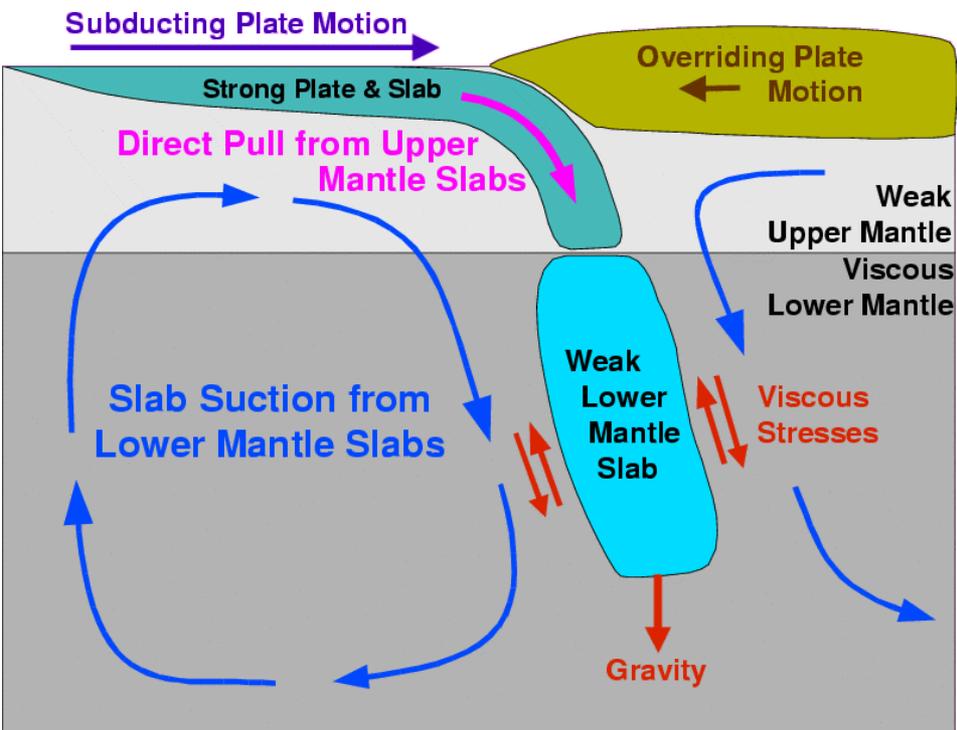
Modeled plate motions



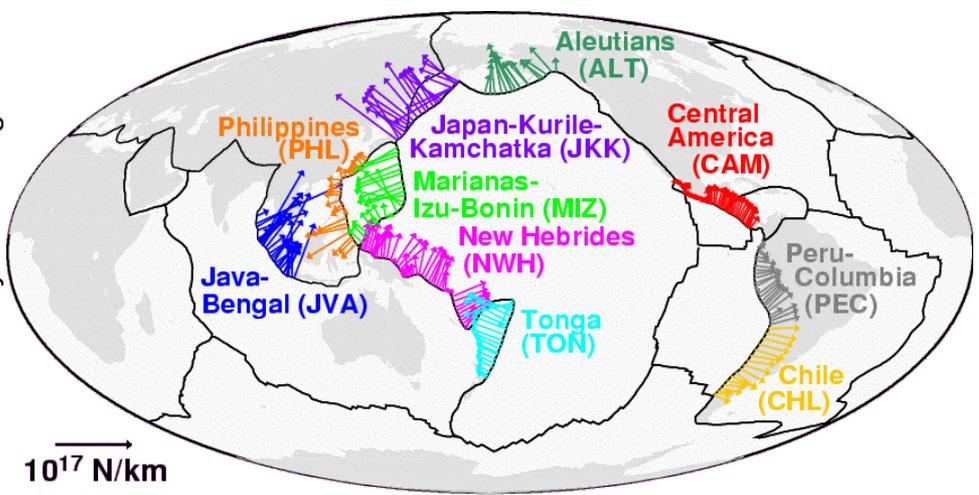
# Adding edge forces to basal shear from flow



Becker & O'Connell (2001)



Which slabs must be detached to produce the best fit to plate motions?



**Completely Detached Slab:**  
 0% Slab Pull Force  
 100% Slab Suction

(Stokes sinker,  
 weak slab)

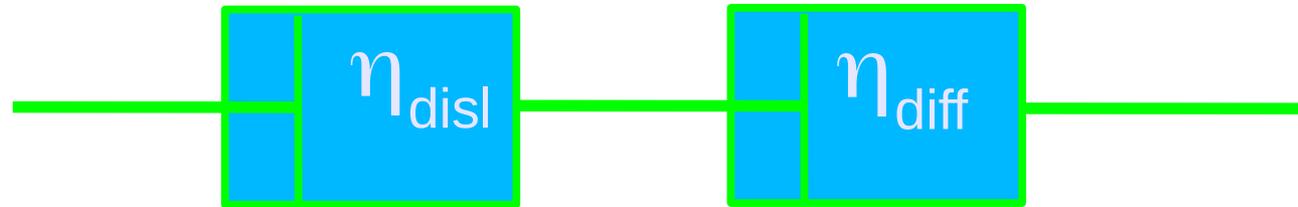
# Mantle rheology

$\eta$ : viscosity  
 $\dot{\epsilon}$ : strain-rate

→ use generic temperature and/or stress dependence of viscosity on top of radial variations

→ use effective, olivine creep law with diffusion & dislocation creep

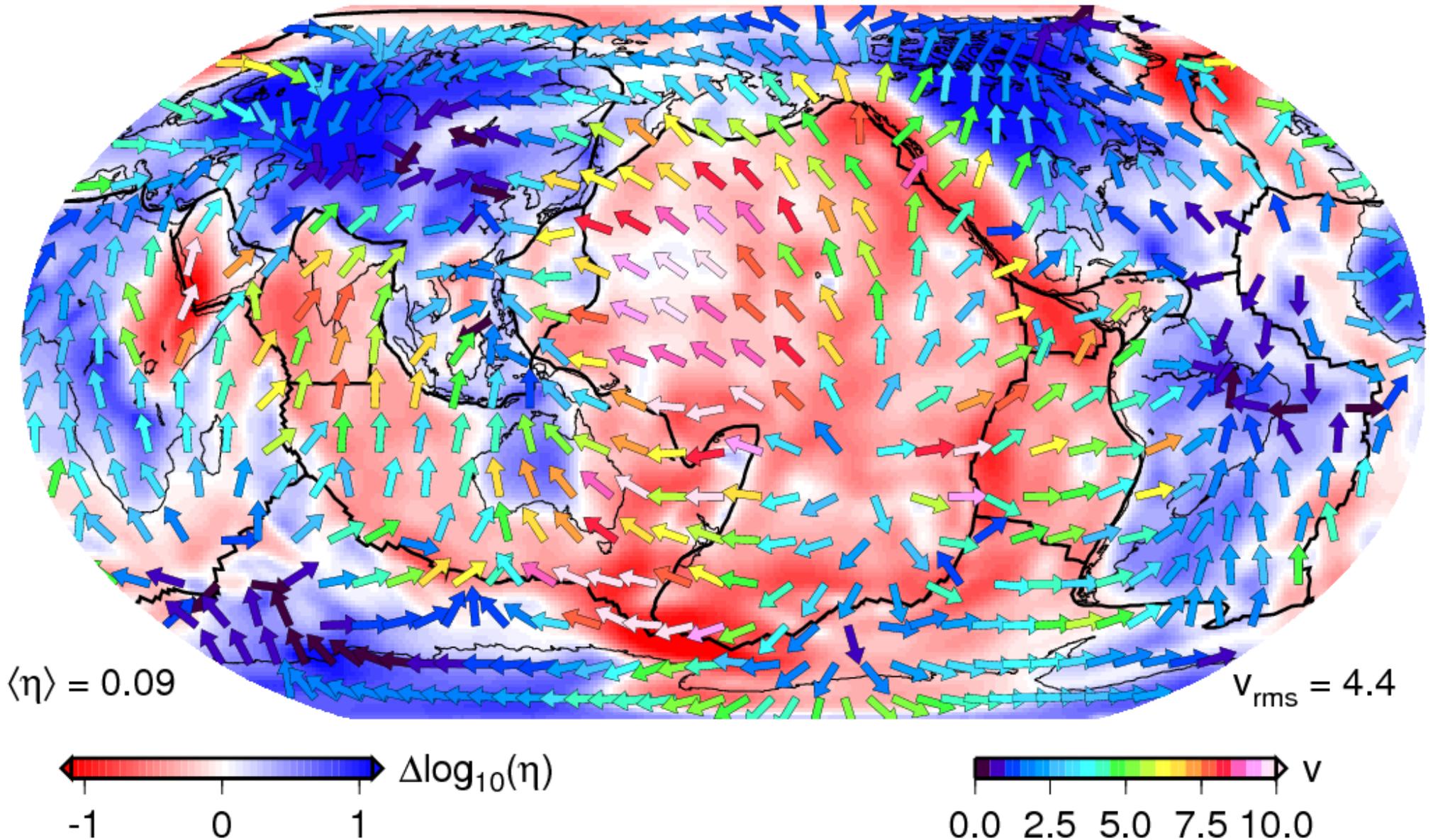
$$\dot{\epsilon}_0 = \dot{\epsilon}_{\text{disl}} + \dot{\epsilon}_{\text{diff}}$$



$$\eta_{\text{eff}} = \frac{\eta_{\text{disl}} \eta_{\text{diff}}}{\eta_{\text{disl}} + \eta_{\text{diff}}}$$

$$\eta = \left( \frac{d^m}{A'} \right)^{\frac{1}{n}} \dot{\epsilon}_{II}^{\frac{1-n}{n}} \exp \left( \frac{E^* + pV^*}{nRT_r(T_c + T)} \right)$$

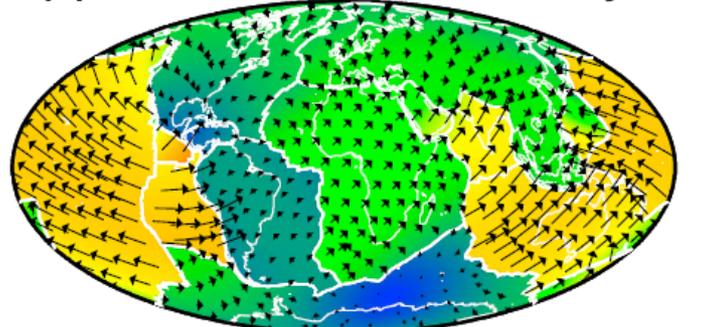
# Sub-oceanic vs. continental speeds (LVV GCM @ 250 km)



Hager & O'Connell (1981); Ricard & Vigny (1989); Zhang & Christensen (1993); Cadek & Fleitout (2003); Becker (2006);

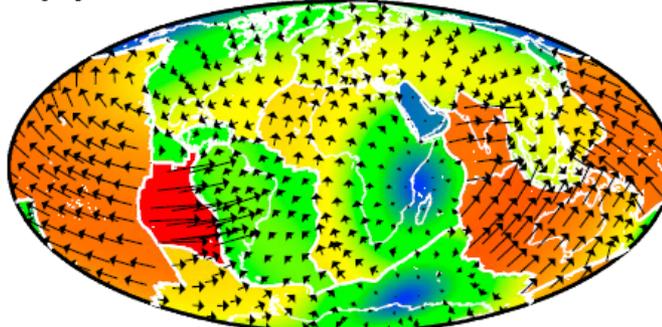
Stadler et al. (2010); Alisic et al. (2014)

(a) NUVEL-1A Plate Velocity



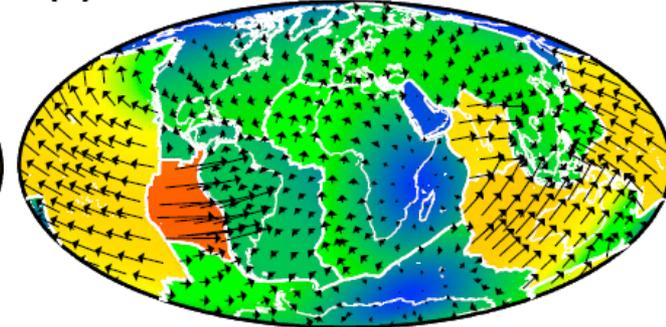
$V_{\text{subd.}}/V_{\text{non-subd.}} = 3.4$   
 $V_{\text{aver}} = 3.7 \text{ cm/yr}$   
 Plate Velocity Magnitude [cm/yr]

(b) No Asthenosphere



$V_{\text{subd.}}/V_{\text{non-subd.}} = 3.2$  Misfit = 0.23  
 $V_{\text{aver}} = 7.1 \text{ cm/yr}$   $f_{\text{sp}} = 100\%$

(c) Shallow Continental Roots



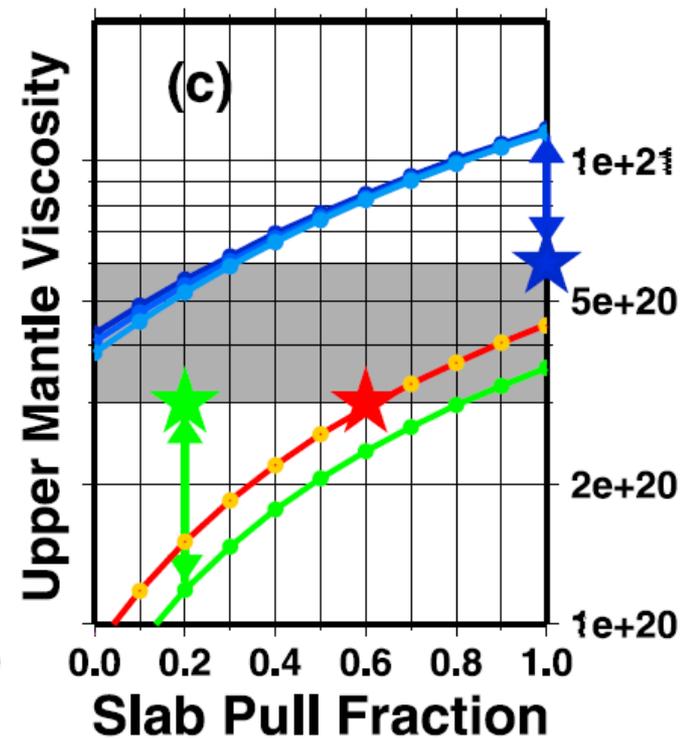
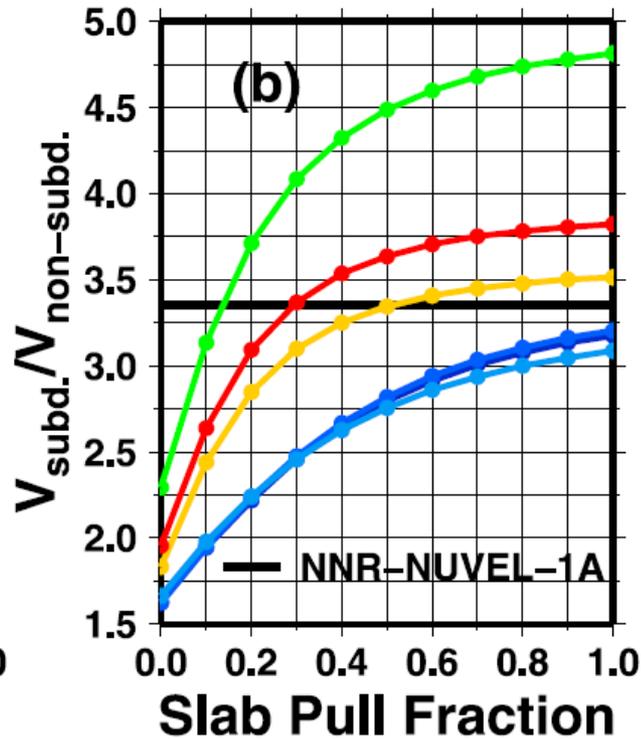
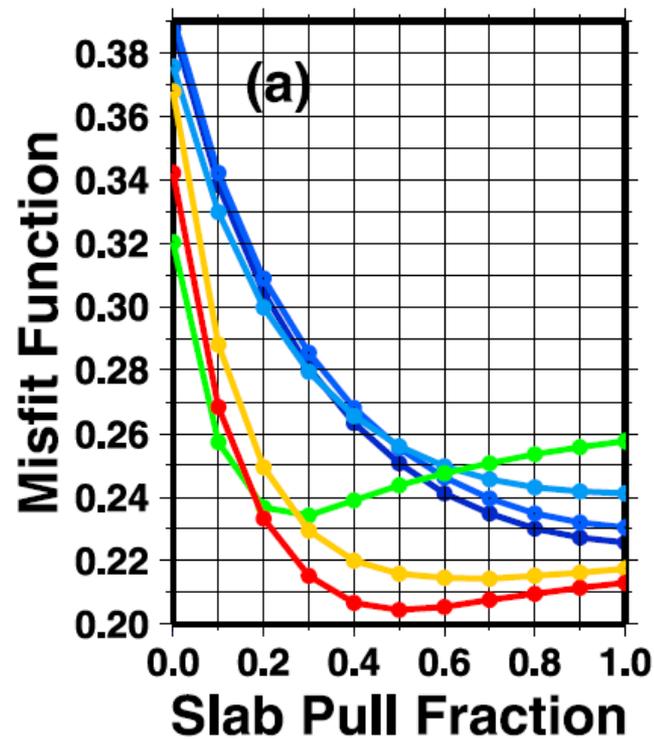
$V_{\text{subd.}}/V_{\text{non-subd.}} = 3.7$  Misfit = 0.21  
 $V_{\text{aver}} = 3.6 \text{ cm/yr}$   $f_{\text{sp}} = 60\%$

With Asthenosphere:

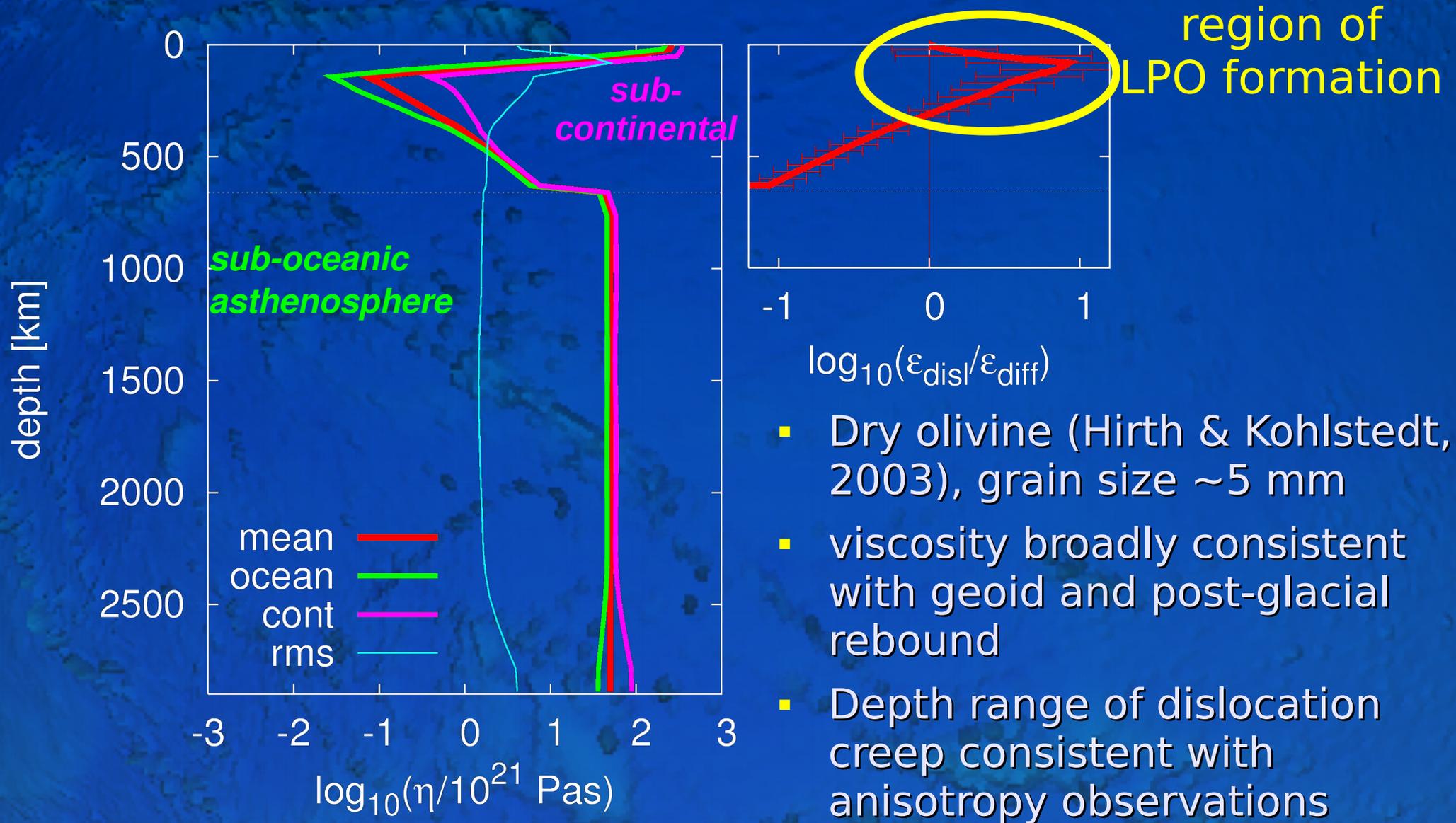
- Uniform Thickness Plates
- Shallow Roots
- Deep Roots

No Asthenosphere:

- Uniform Thickness Plates
- Shallow Roots
- Deep Roots



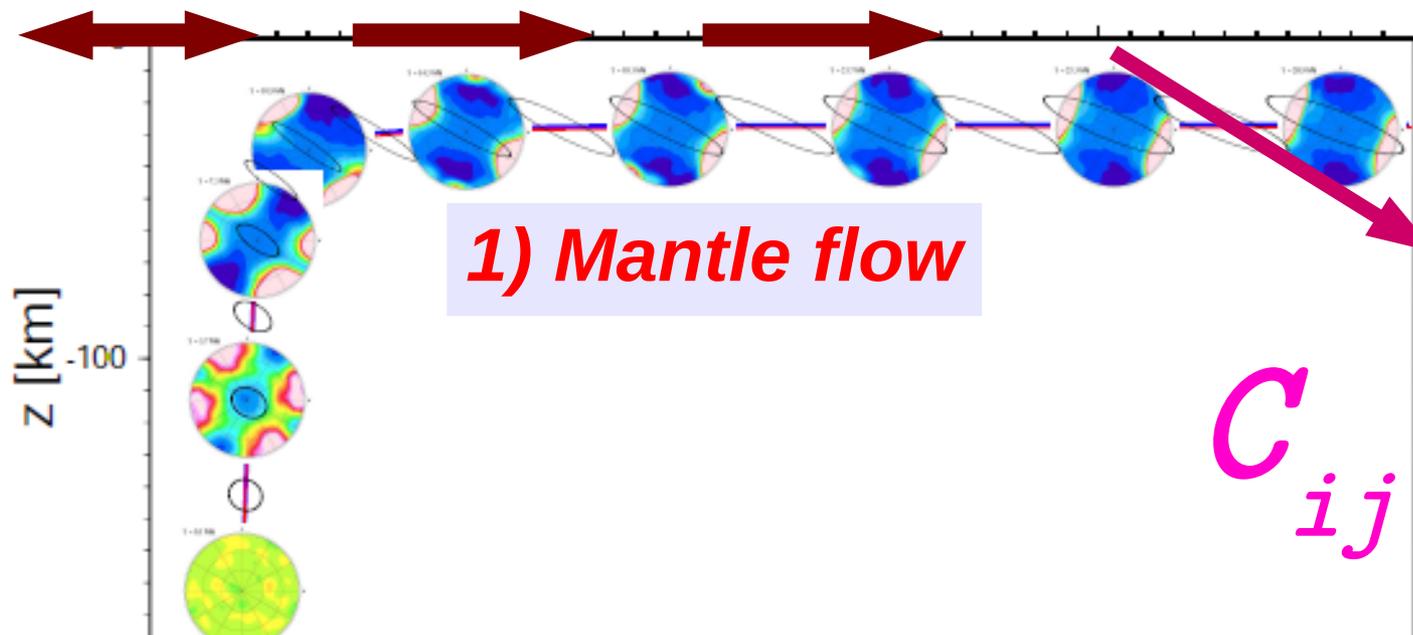
# Average mantle viscosities for olivine



Cadek & Fleitout (2003); McNamara et al. (2003);

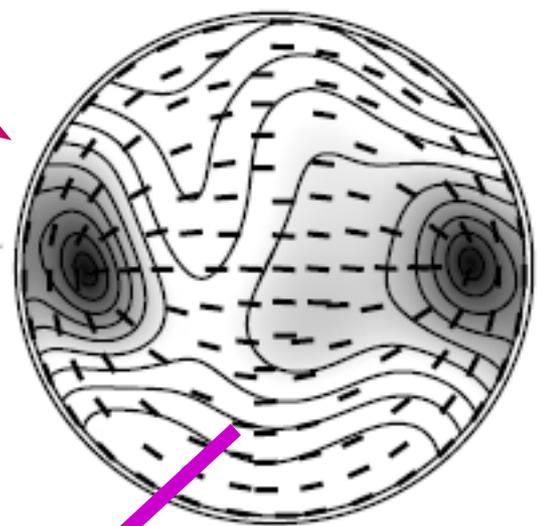
Podolevsky et al. (2005); Becker (2006)

# Seismic anisotropy from flow



**1) Mantle flow**

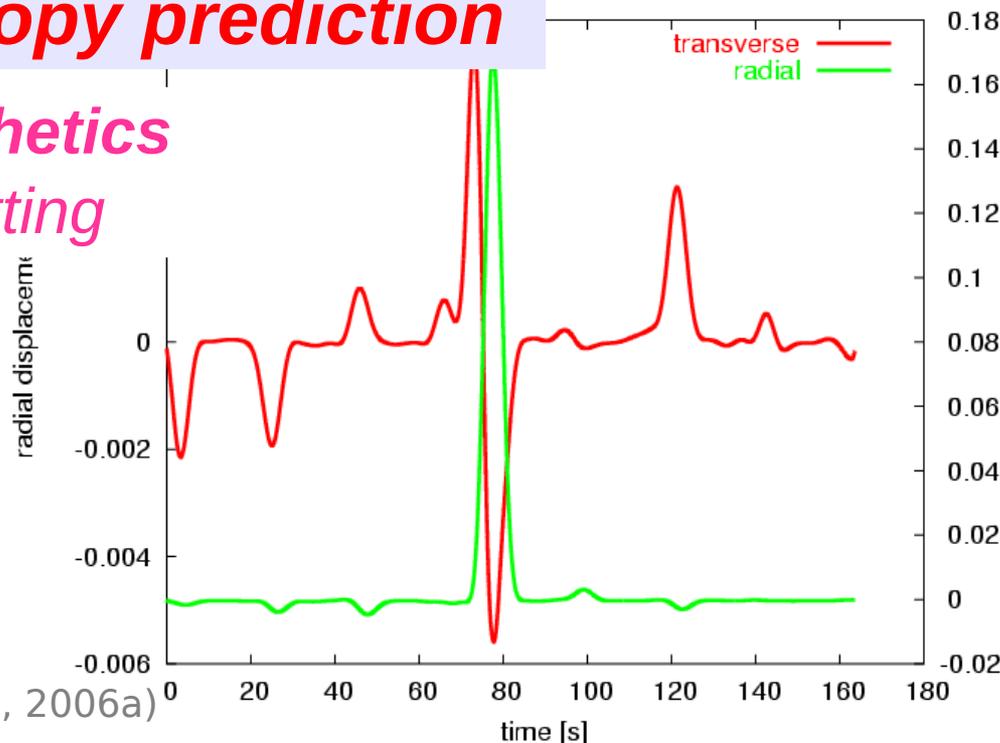
**2) LPO prediction**



$C_{ij}$

**3) Anisotropy prediction**

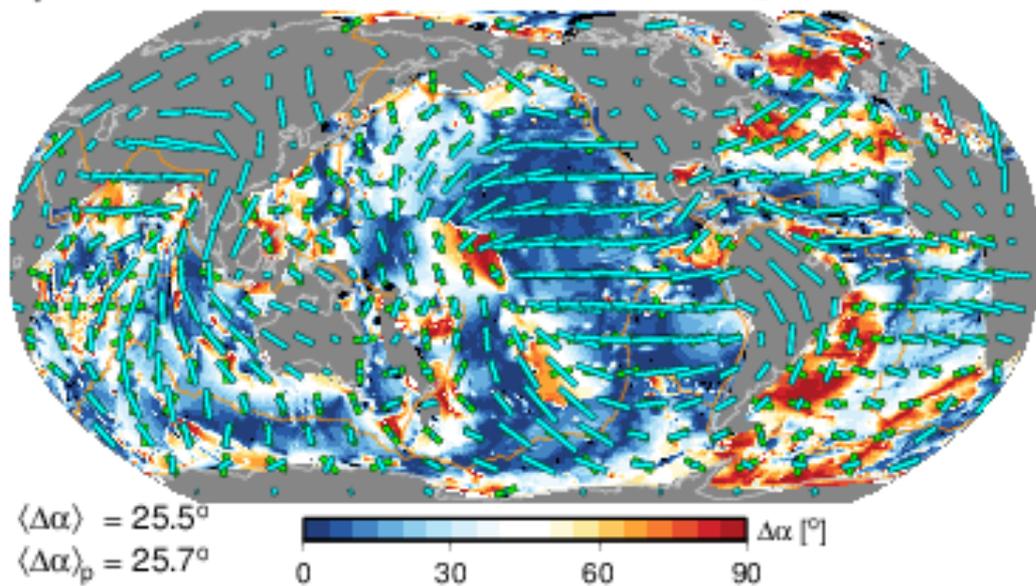
*model synthetics*  
*S wave splitting*



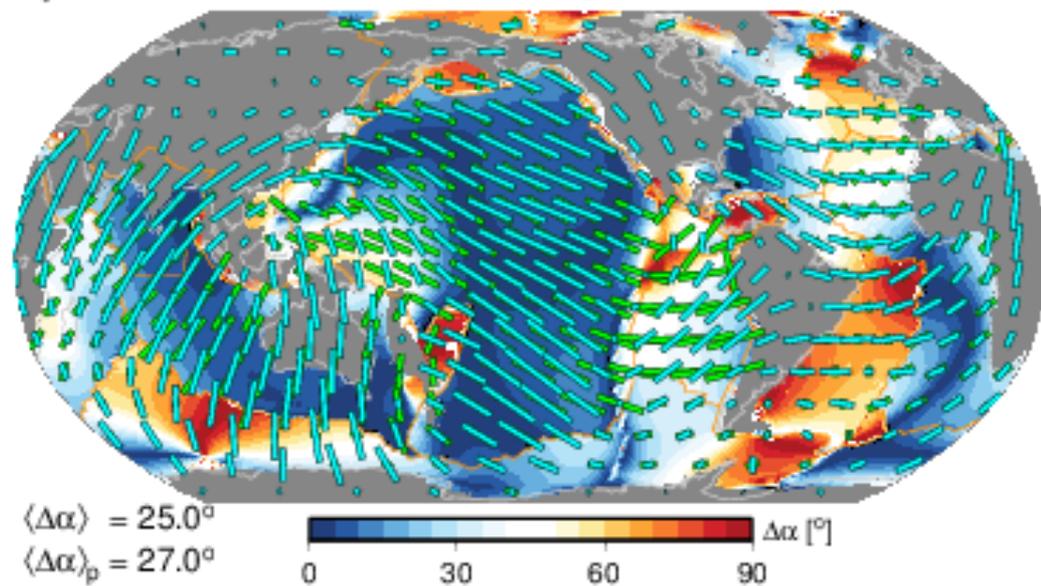
(80% hex, 12% orth.  
@ 75% SC saturation)

generate  
synthetic  
waveforms  
from 3D  
anisotropic  
model

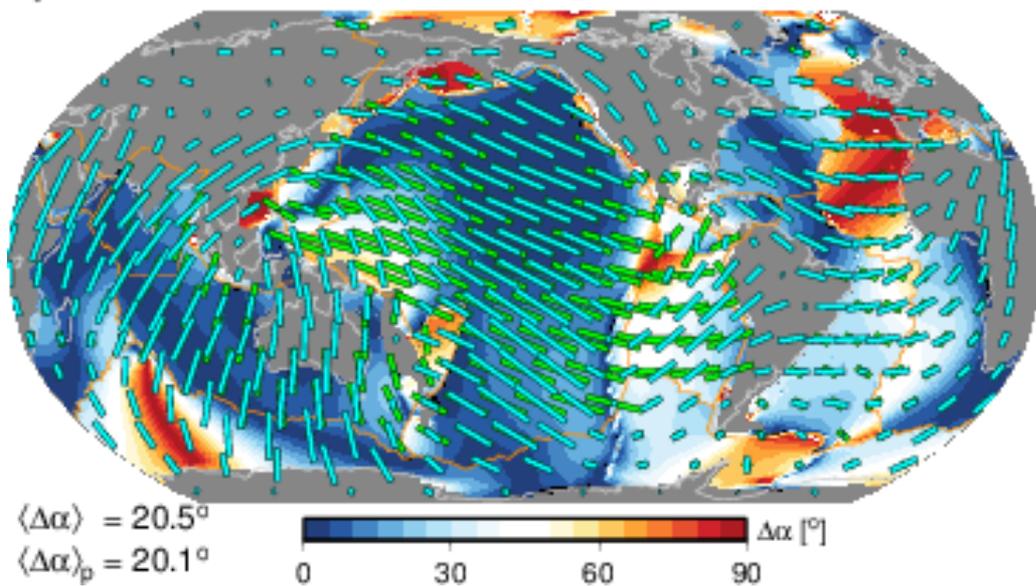
a) **SL2013SVA vs. SPREADING @ 50 km**



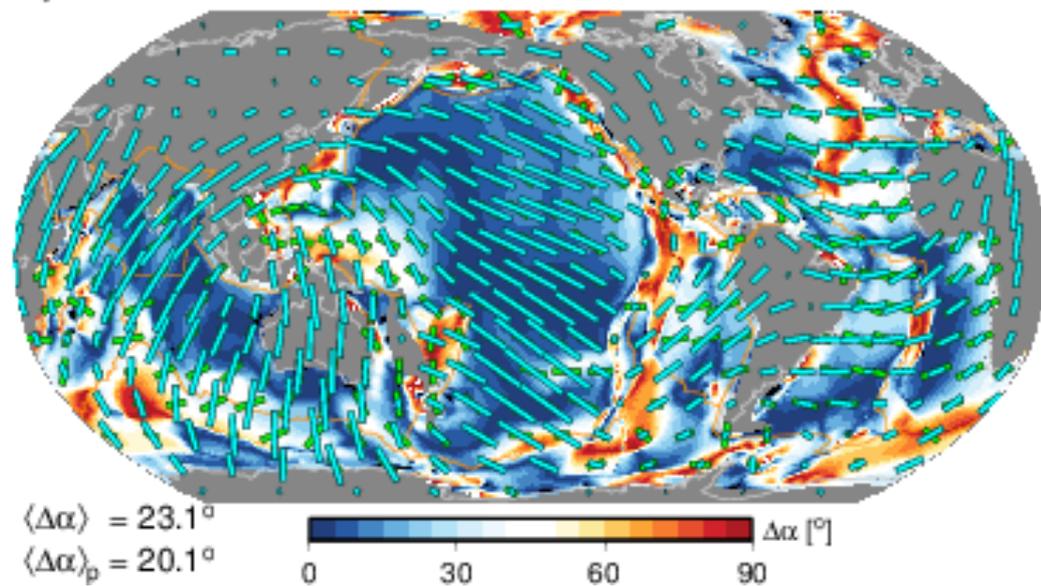
b) **SL2013SVA vs. NNR @ 200 km**



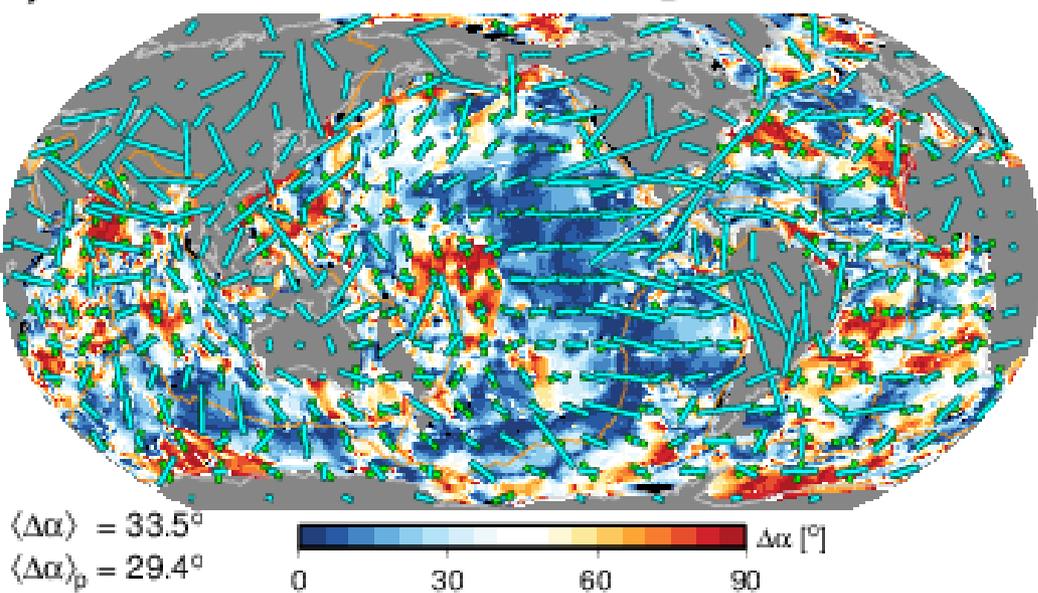
c) **SL2013SVA vs. RNR @ 200 km**



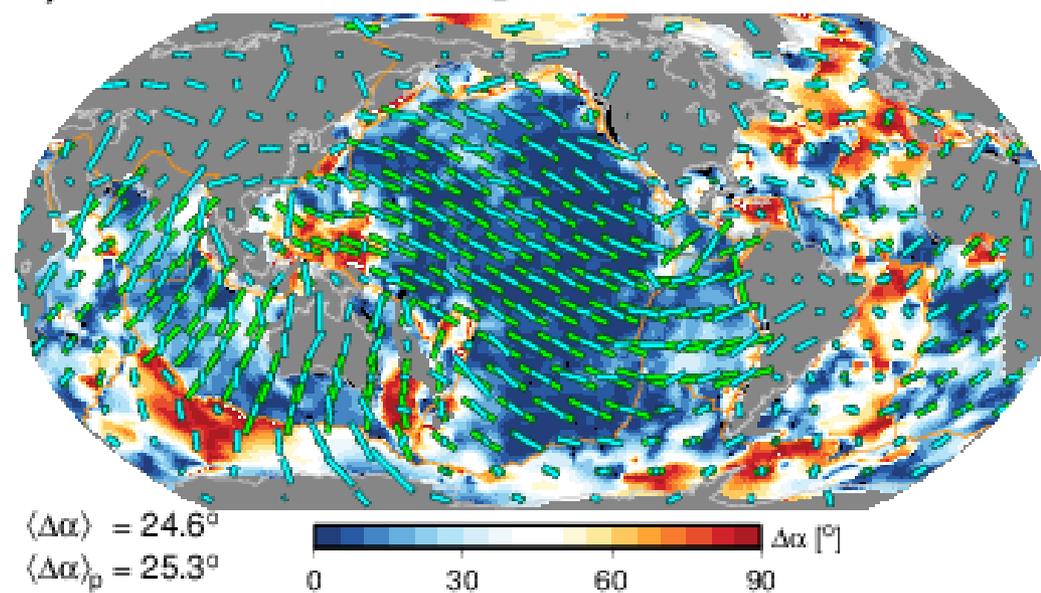
d) **SL2013SVA vs. LPO @ 200 km**



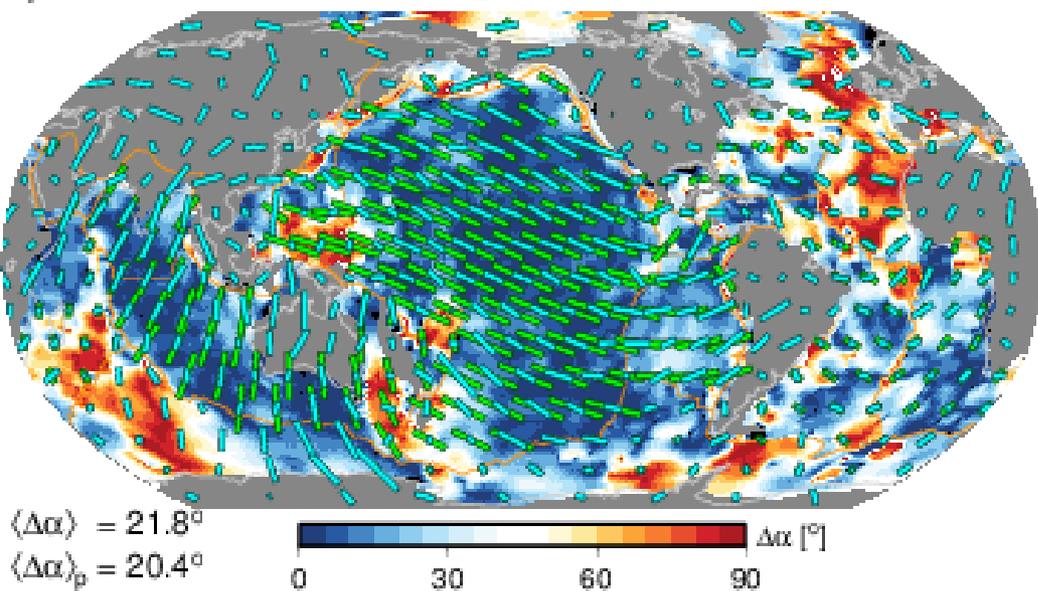
a) **DR2015 vs. SPREADING @ 50 km**



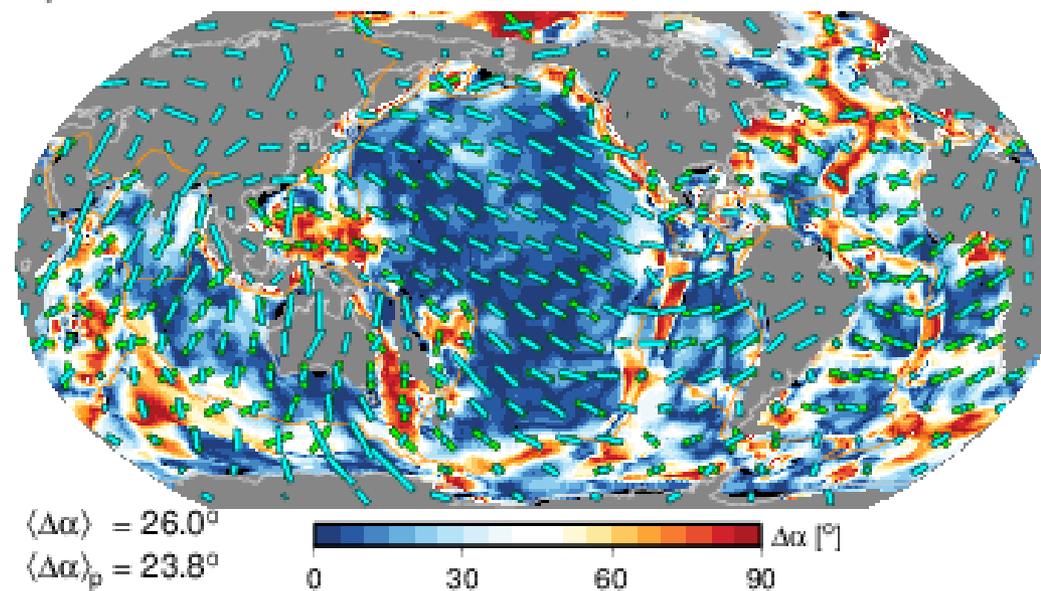
b) **DR2015 vs. NNR @ 200 km**



c) **DR2015 vs. RNR @ 200 km**

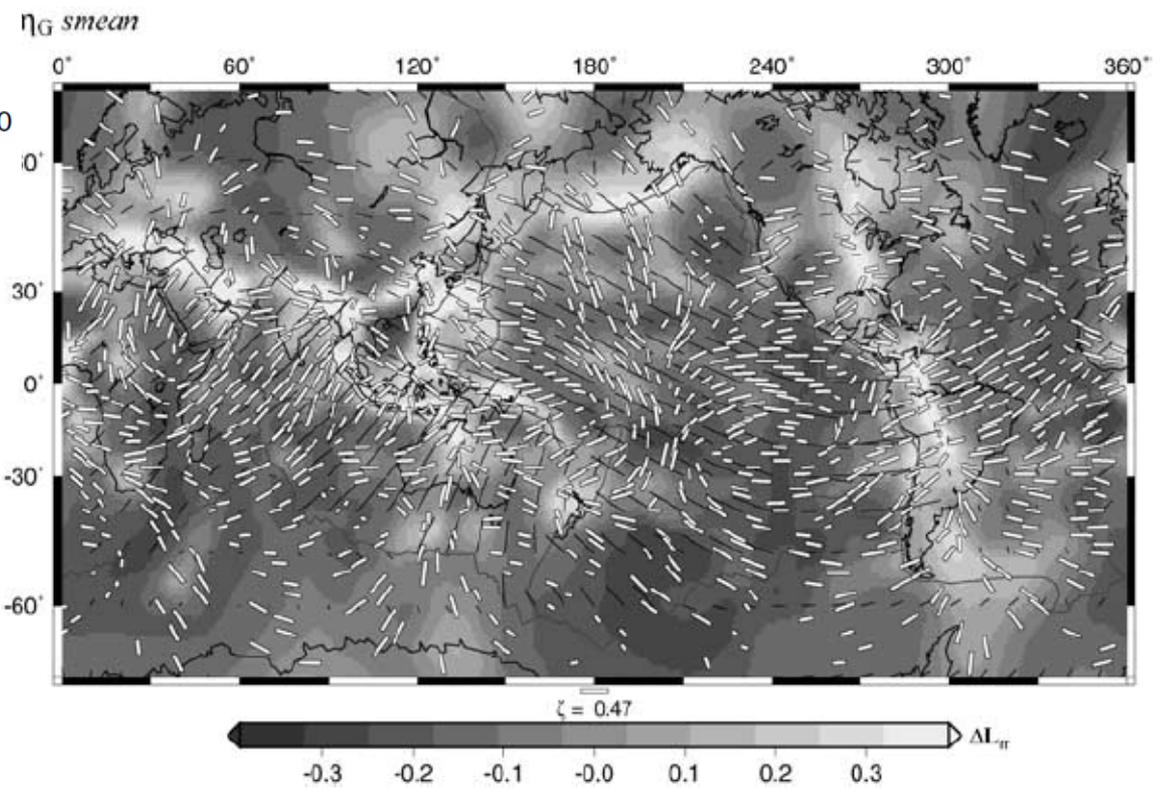
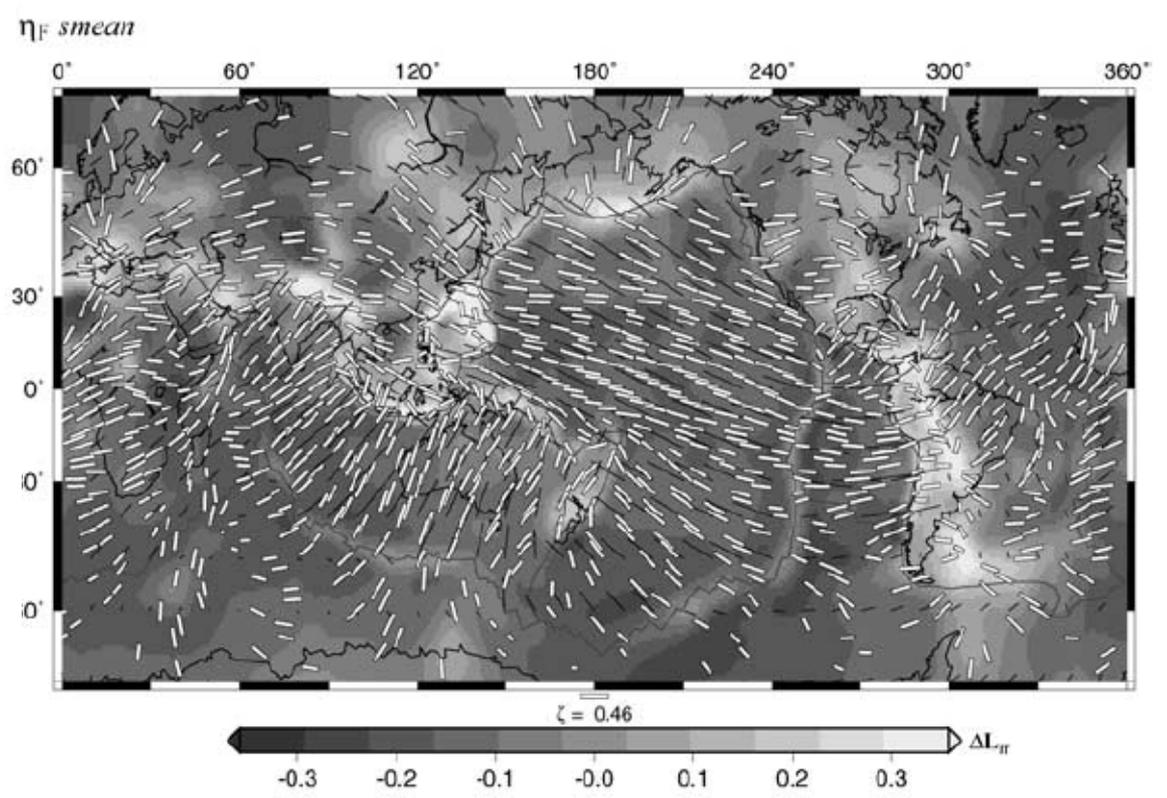
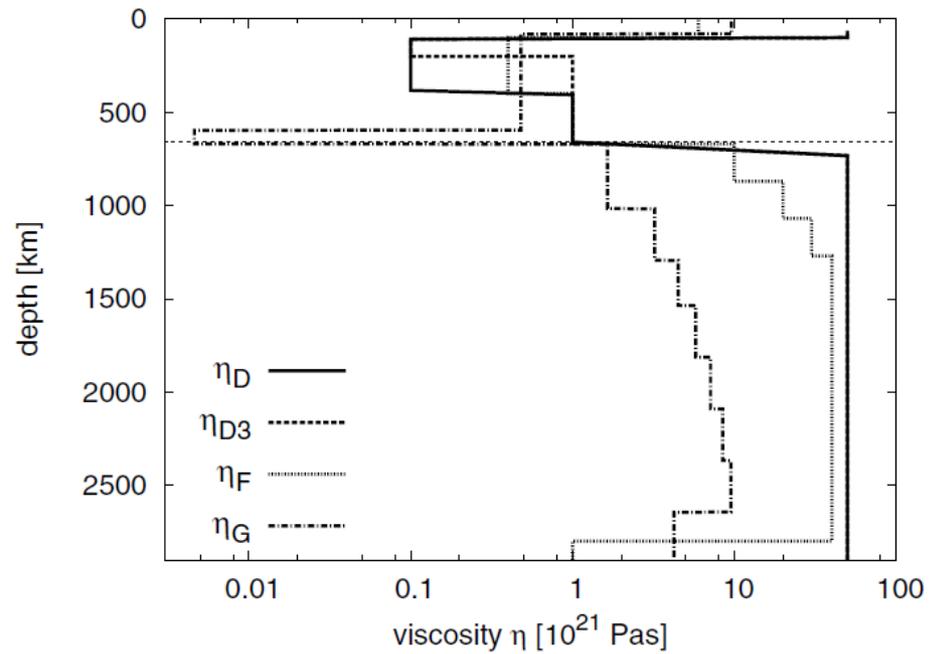


d) **DR2015 vs. LPO @ 200 km**

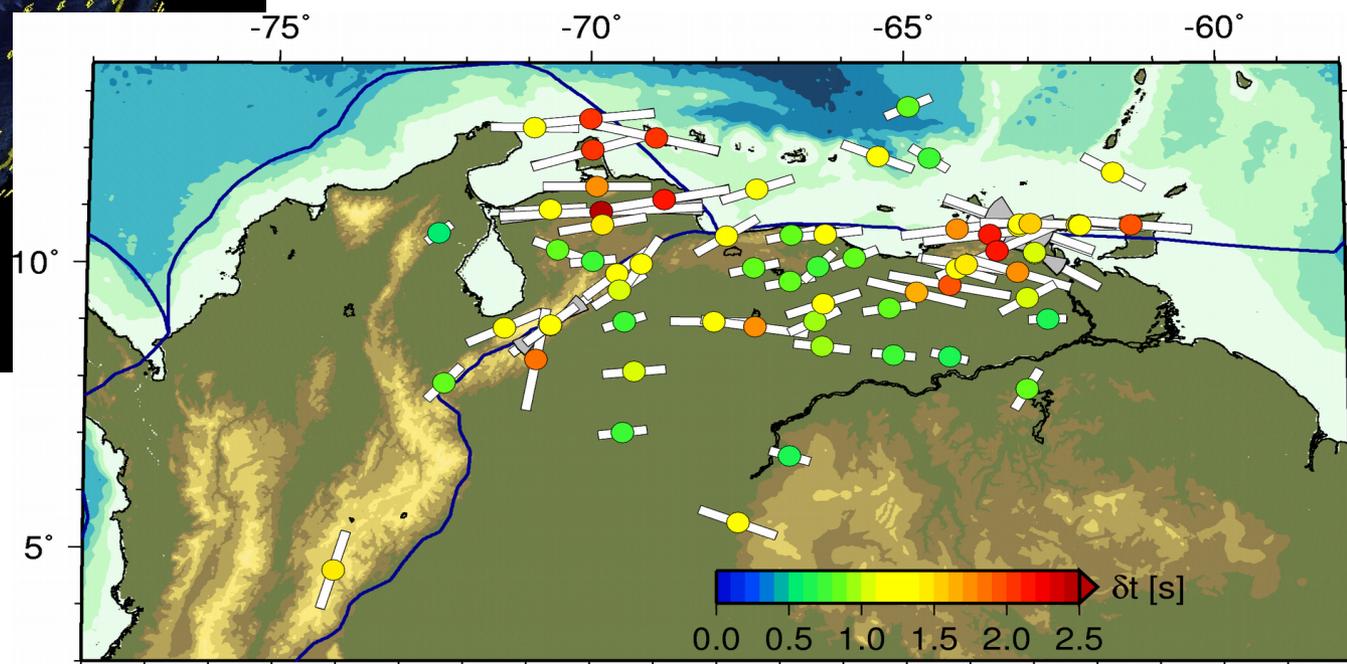
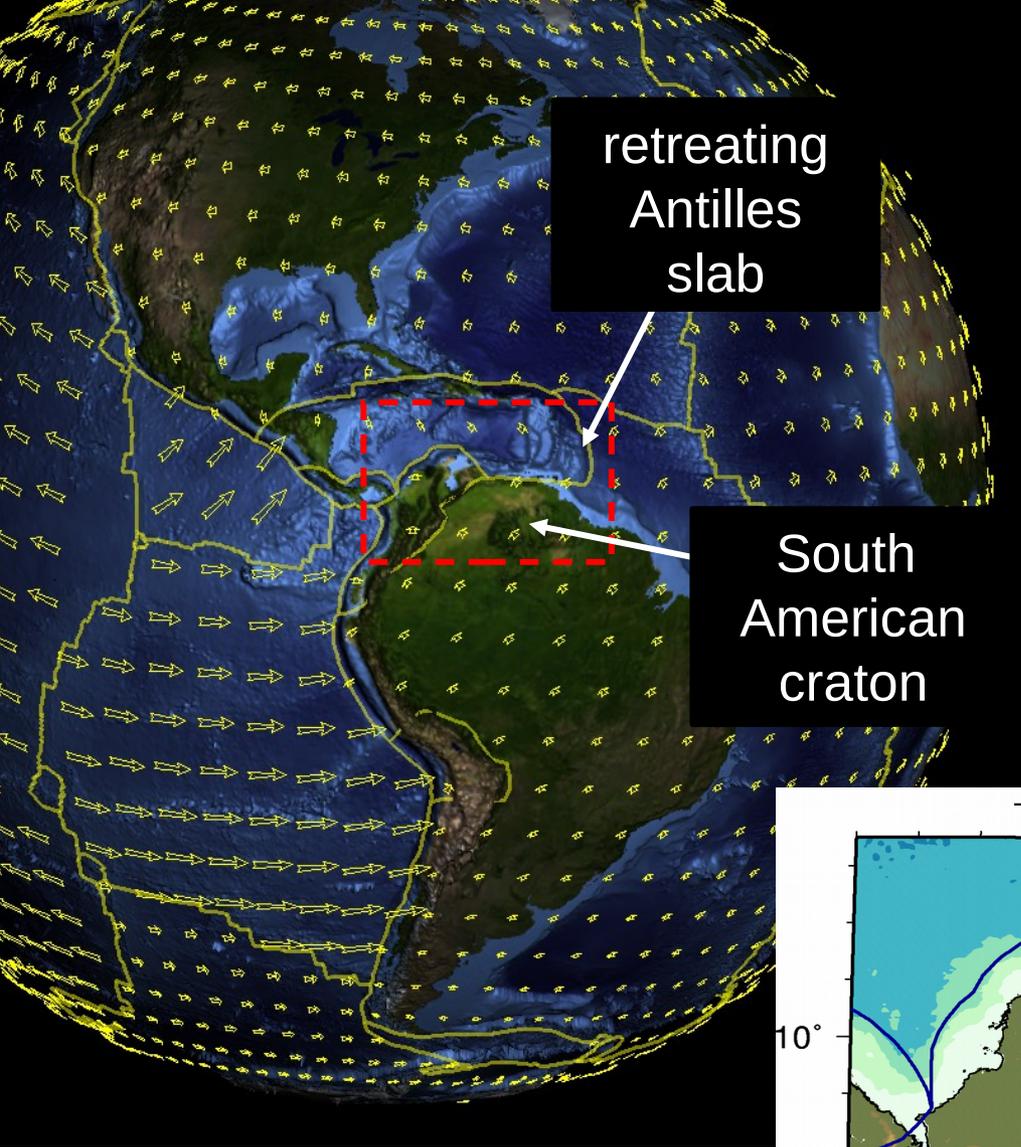


using Debayle et al.'s (2015) model

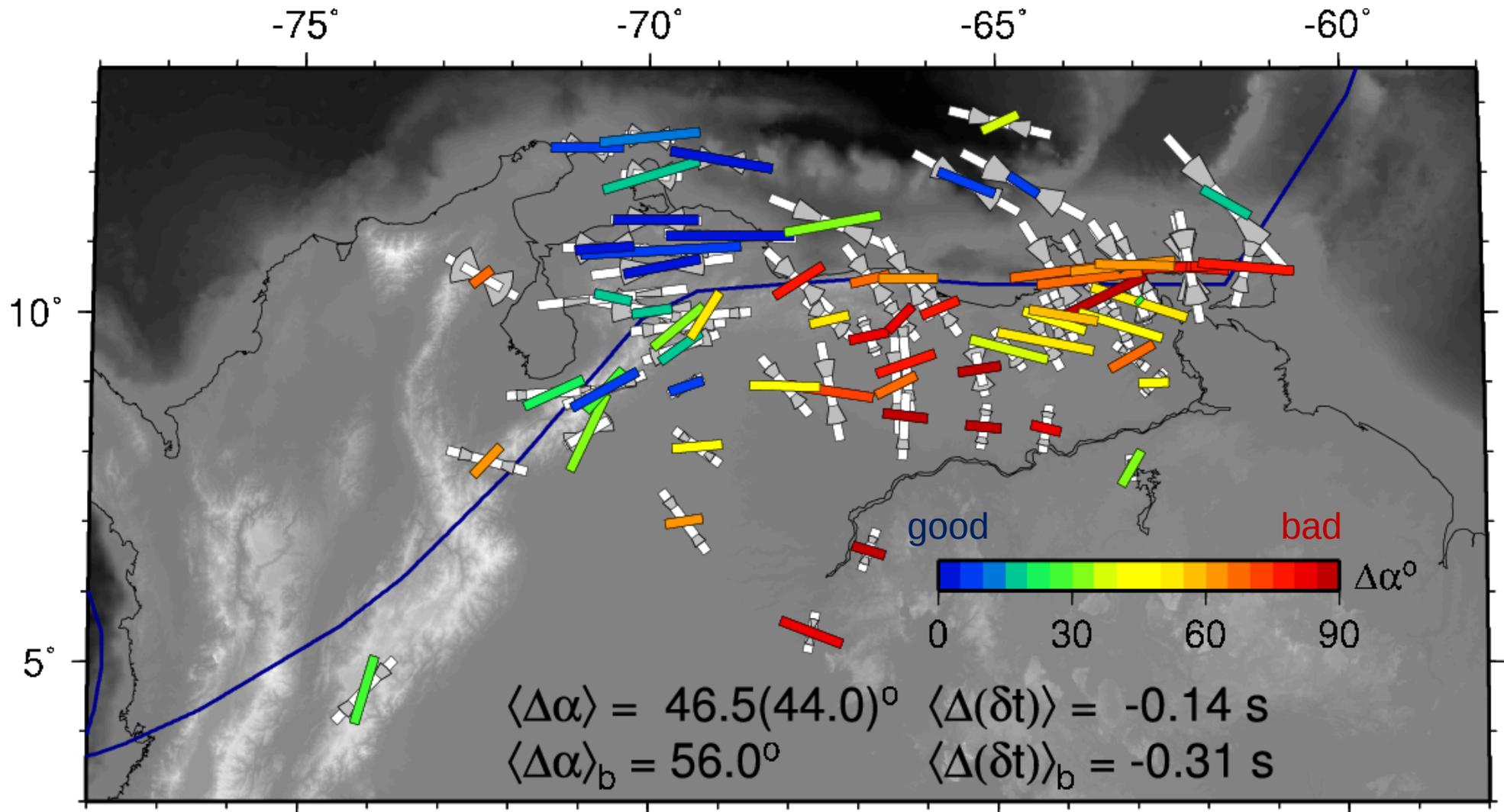
# Test for decoupling layer



# Regional *SKS* splitting

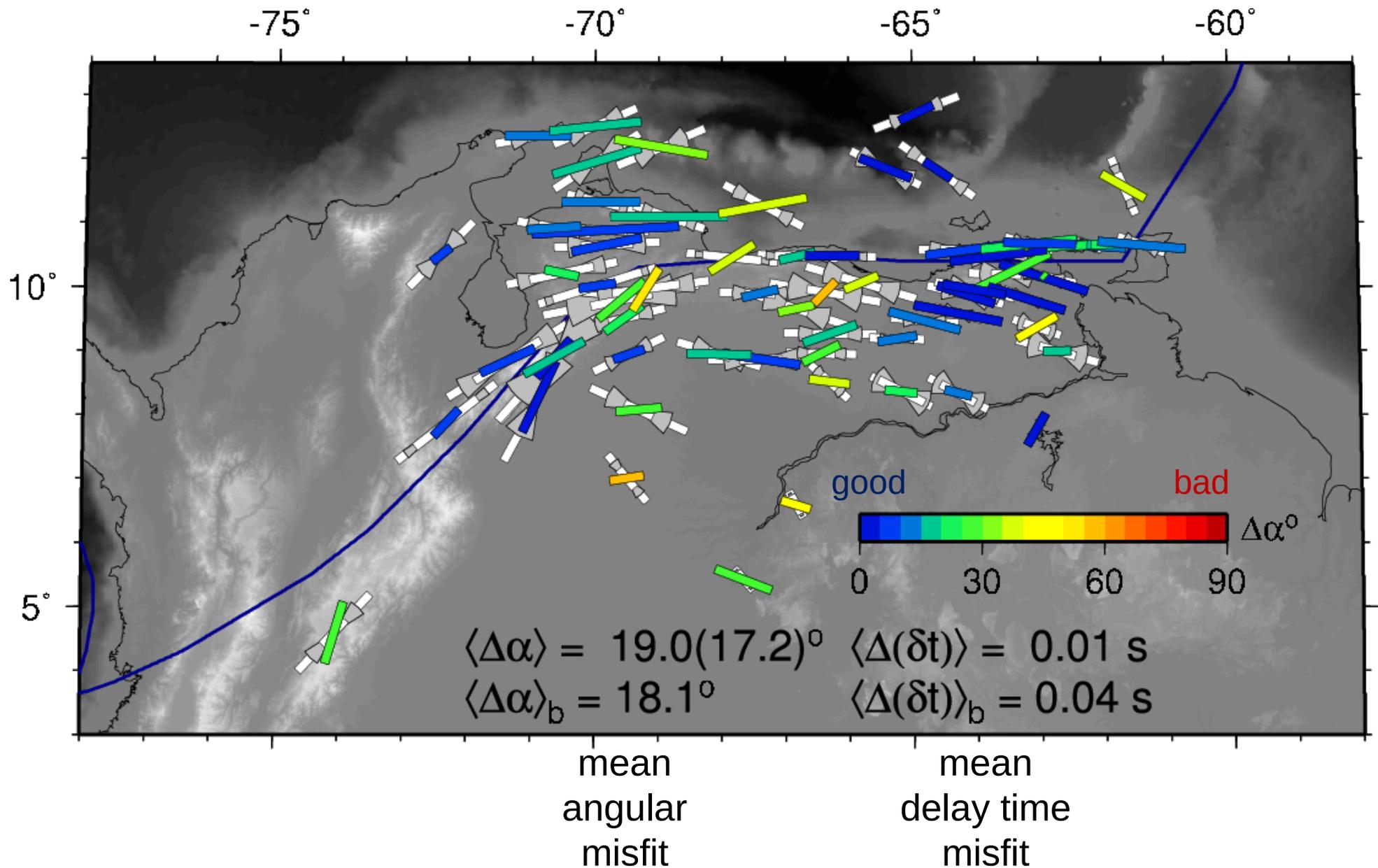


# SKS splitting from reference model (tomography driven)



Colored bars: measurements colored by misfit  
White bars: predictions from full waveform modeling,  
wedges indicating back-azimuthal variation of  $\delta t$ ,  $\alpha$

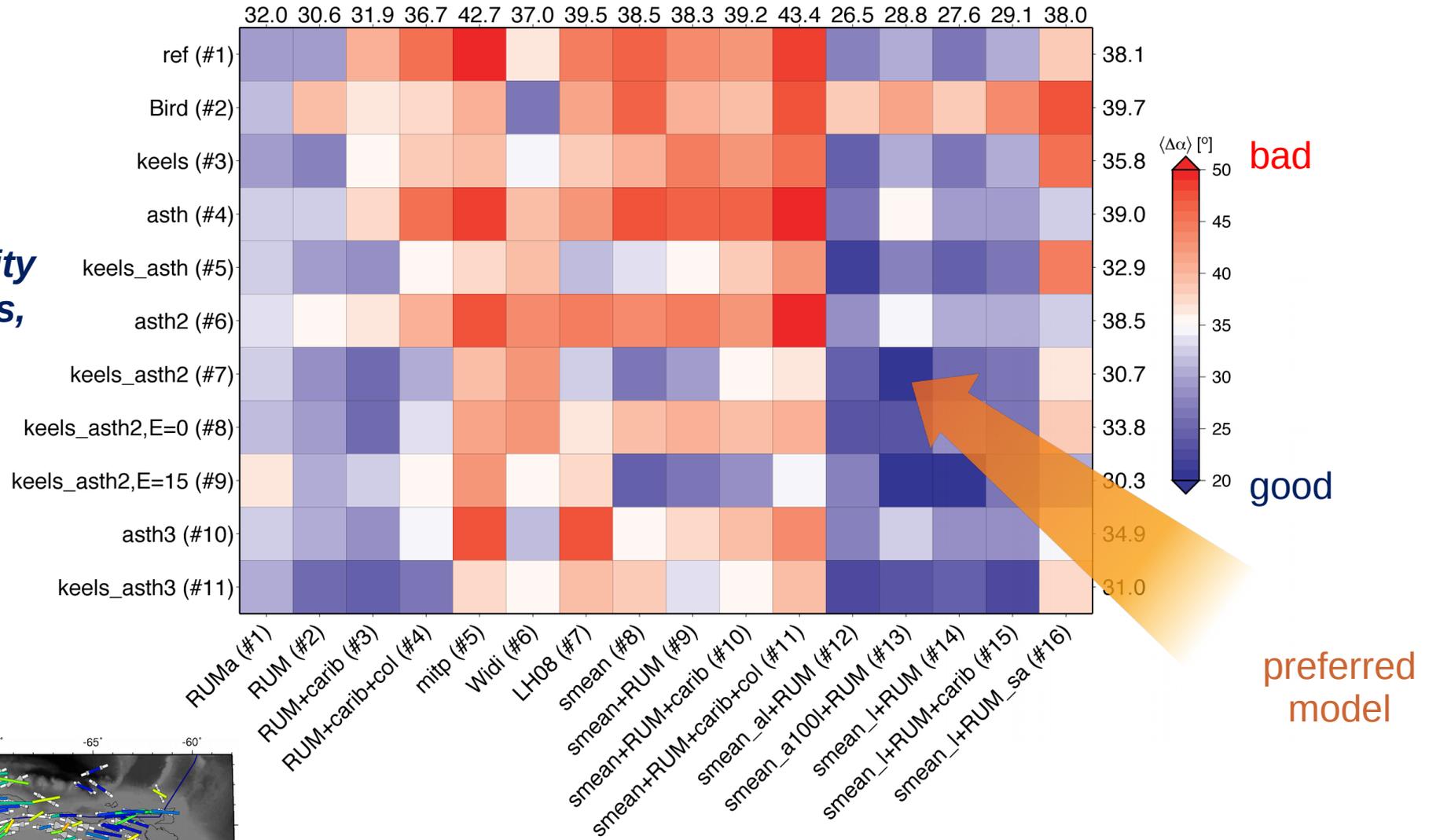
# Splitting from slab model with a weak asthenosphere and keel



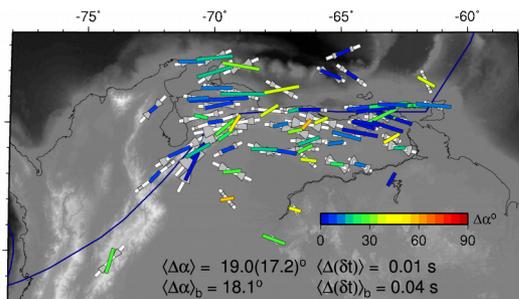
# Inverse geodynamics: Regional splitting azimuth misfit

$$u \propto \frac{\Delta \rho}{\eta}$$

*viscosity  
models,  
 $\eta$*

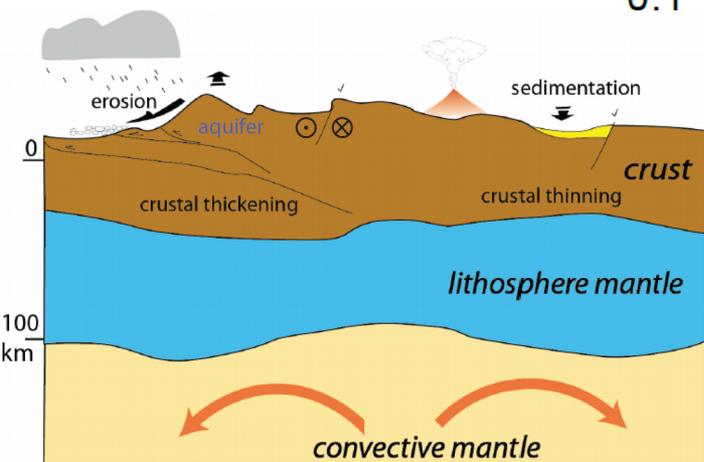
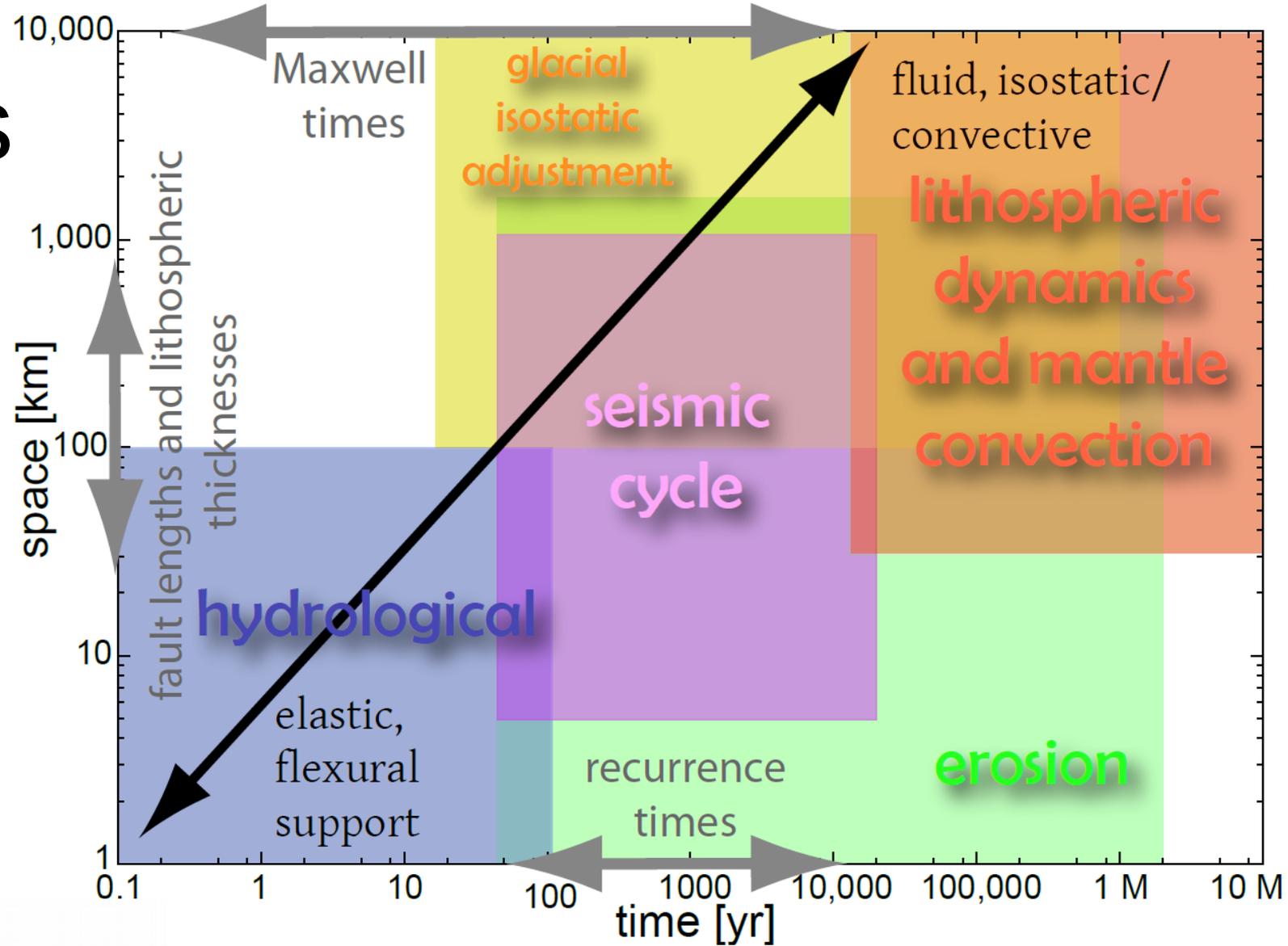


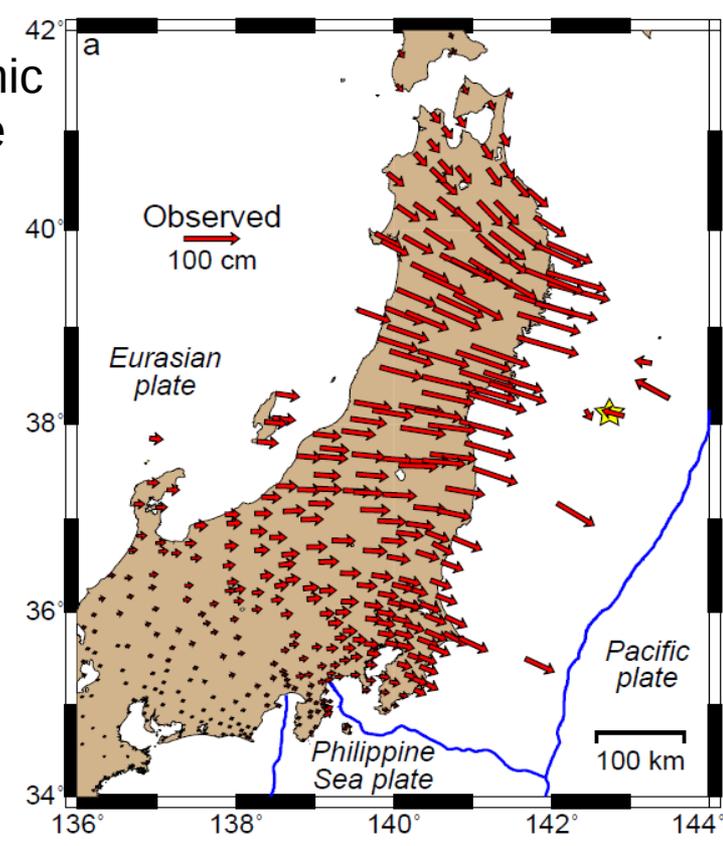
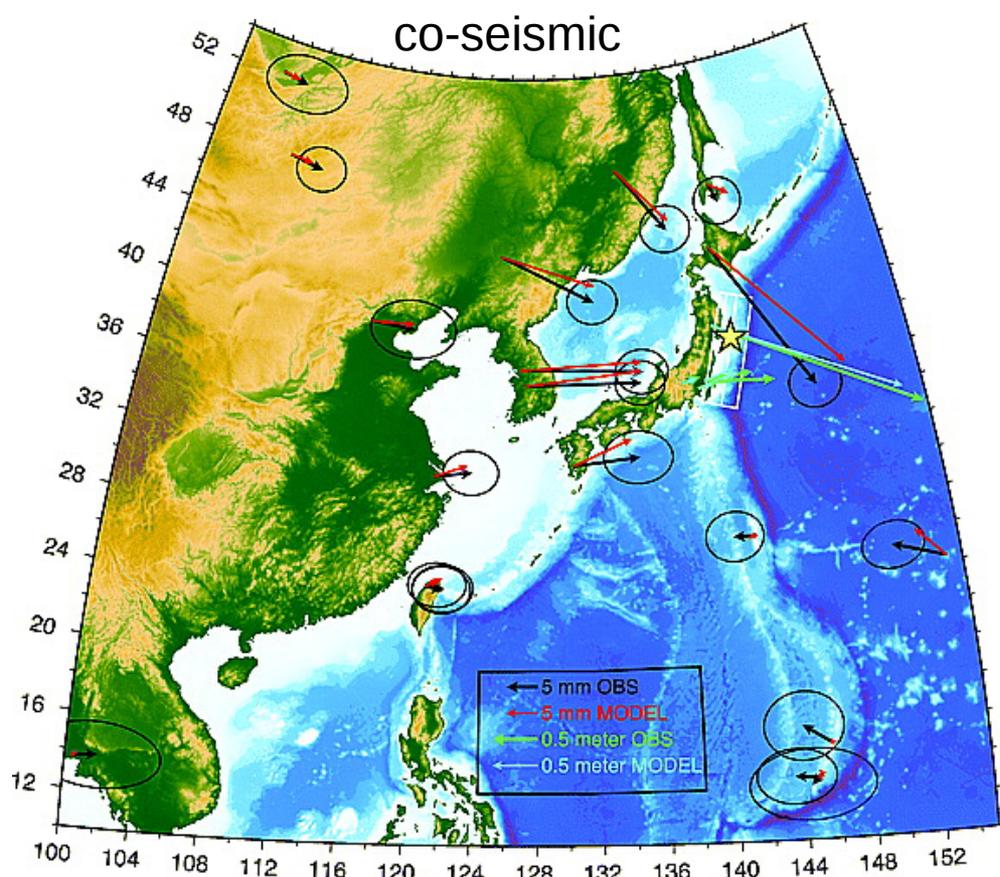
*density  
models,  $\Delta \rho$*



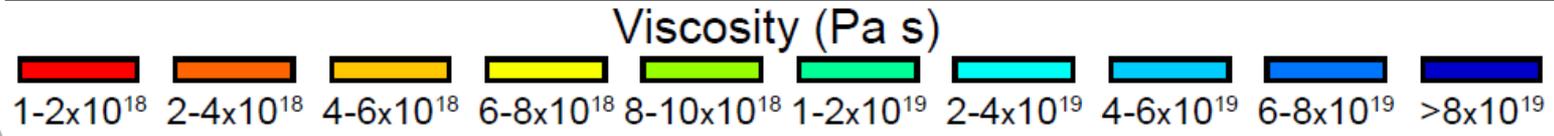
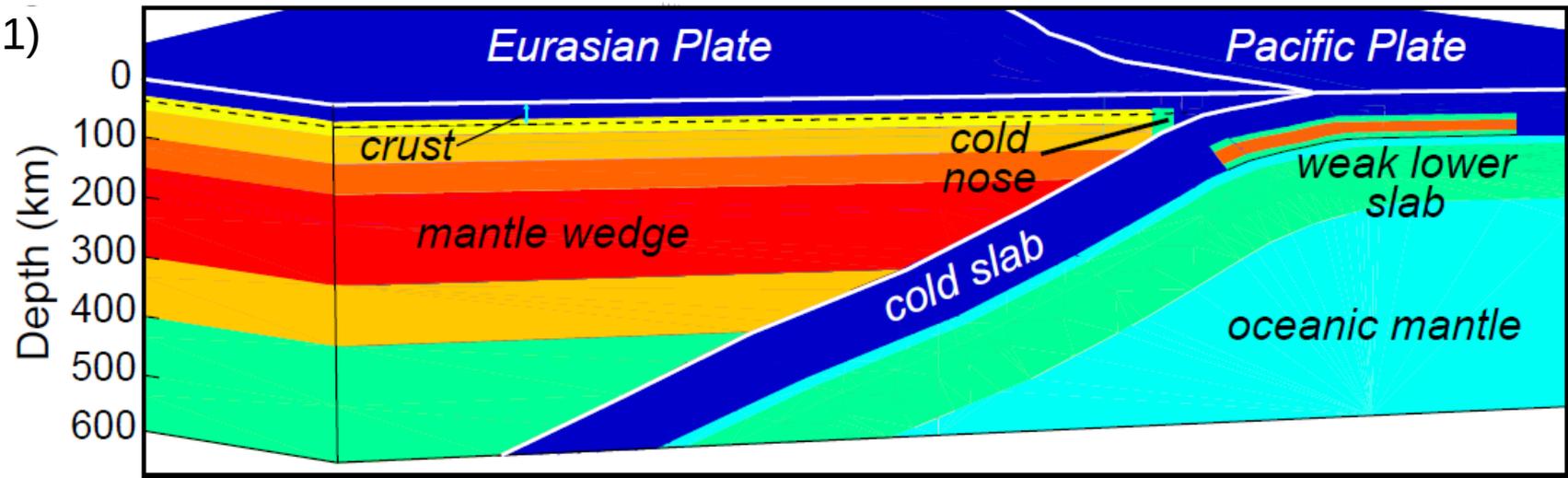
How to resolve some of the  
non-uniqueness and  
uncertainties?

# Verticals



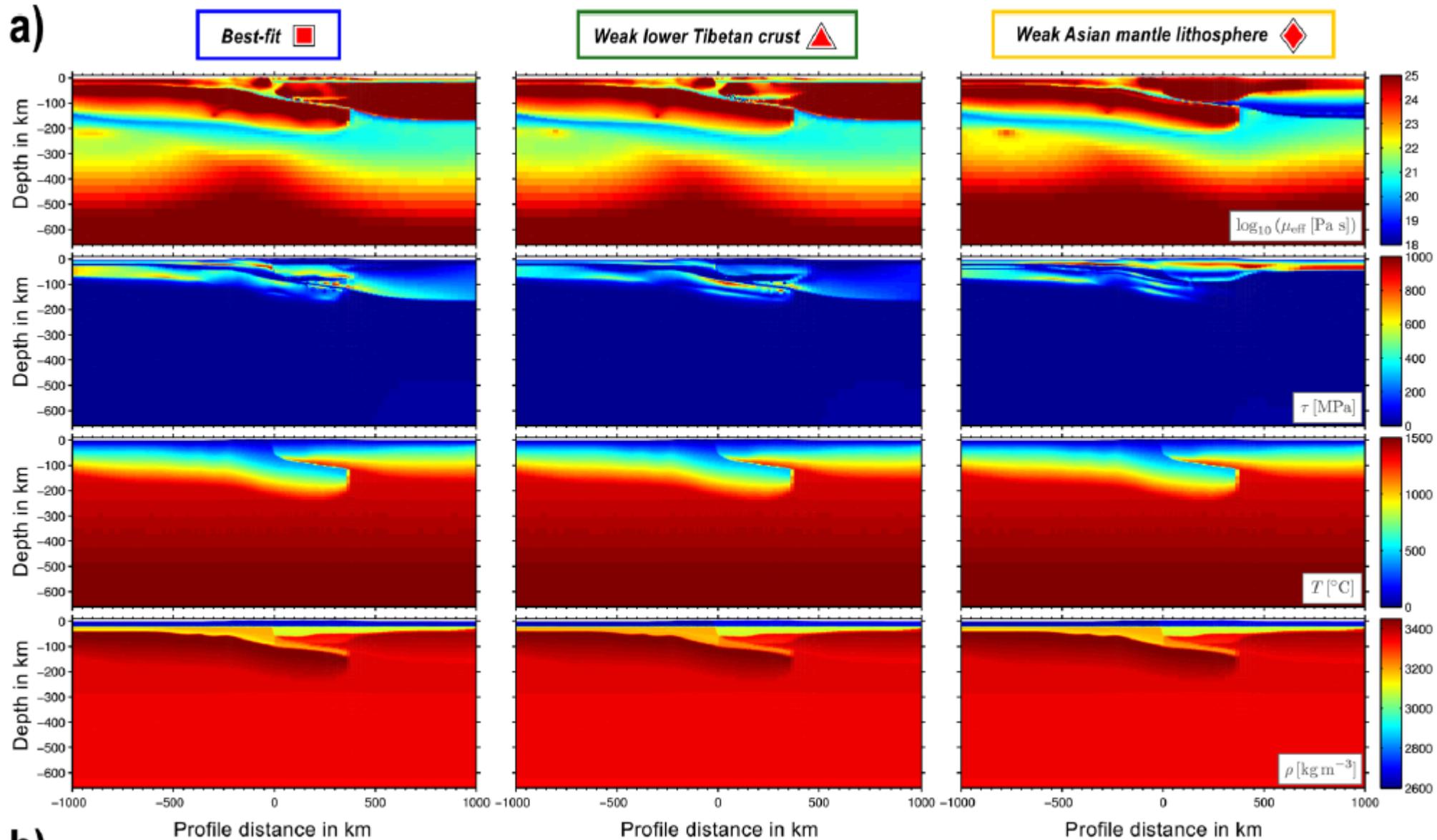


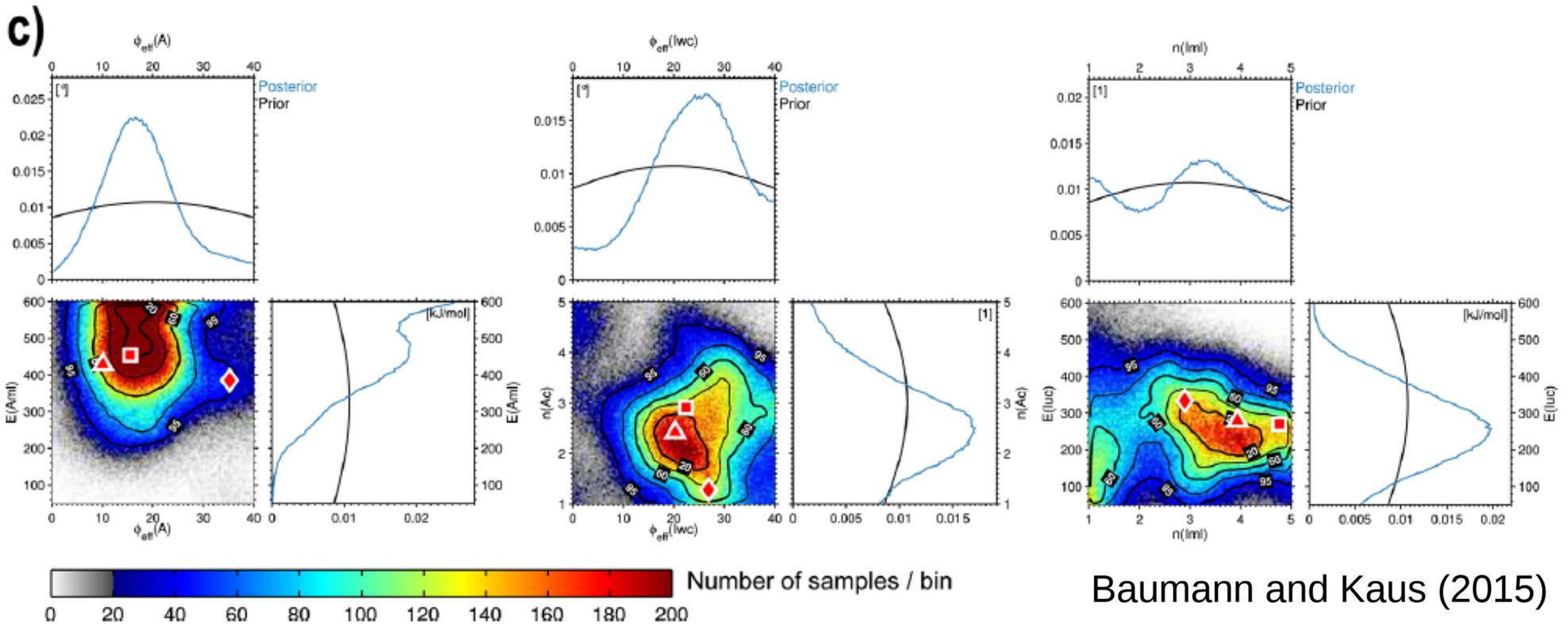
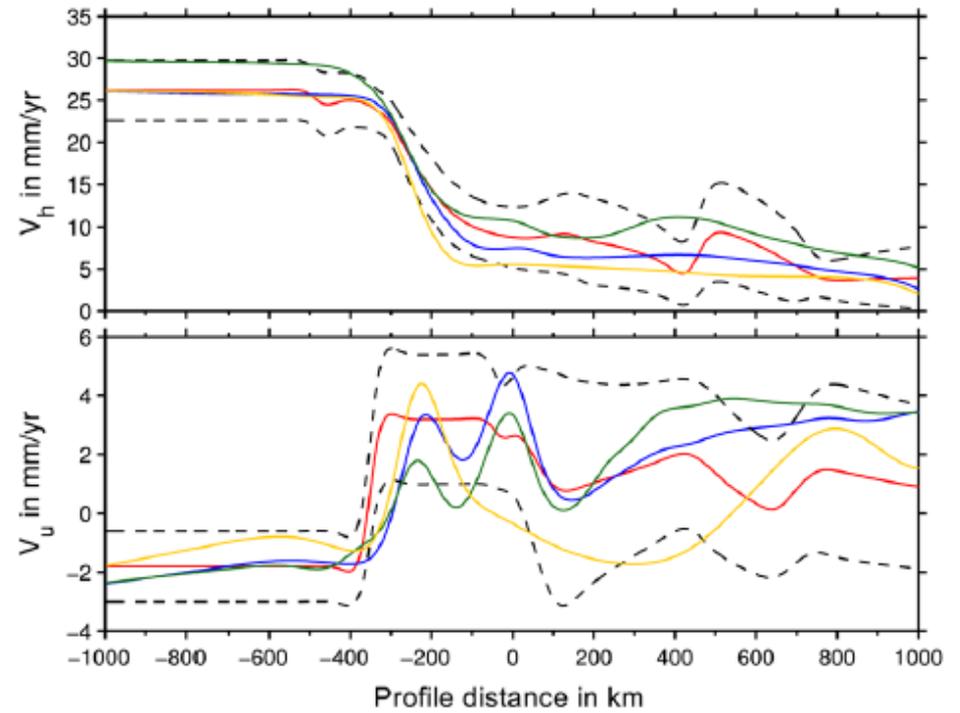
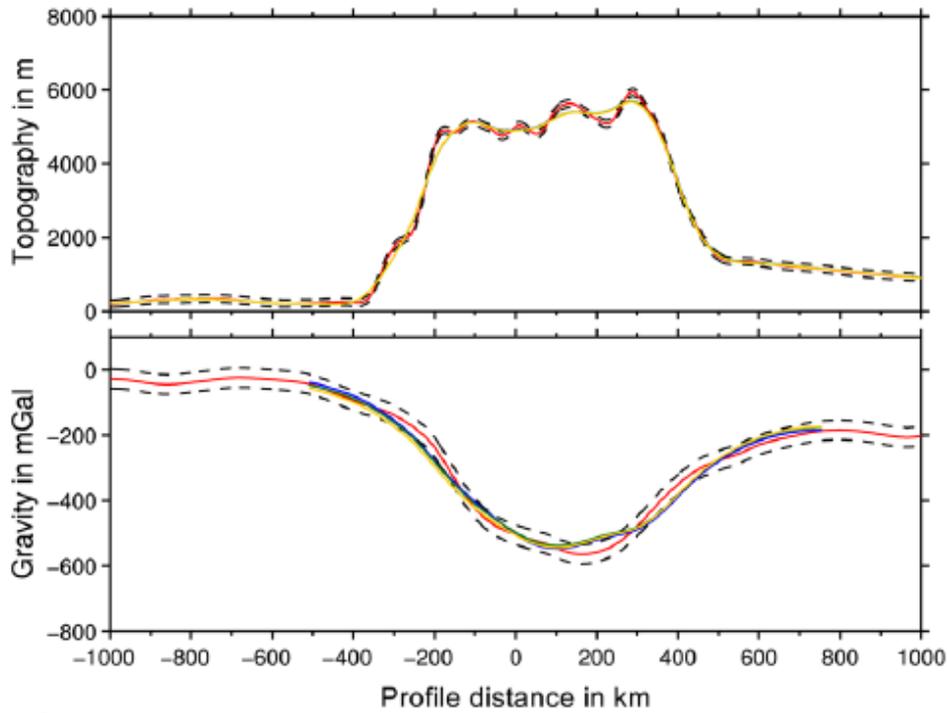
Pollitz et al. (2011)



Freed et al. (submitted)

# Geodynamic inversions





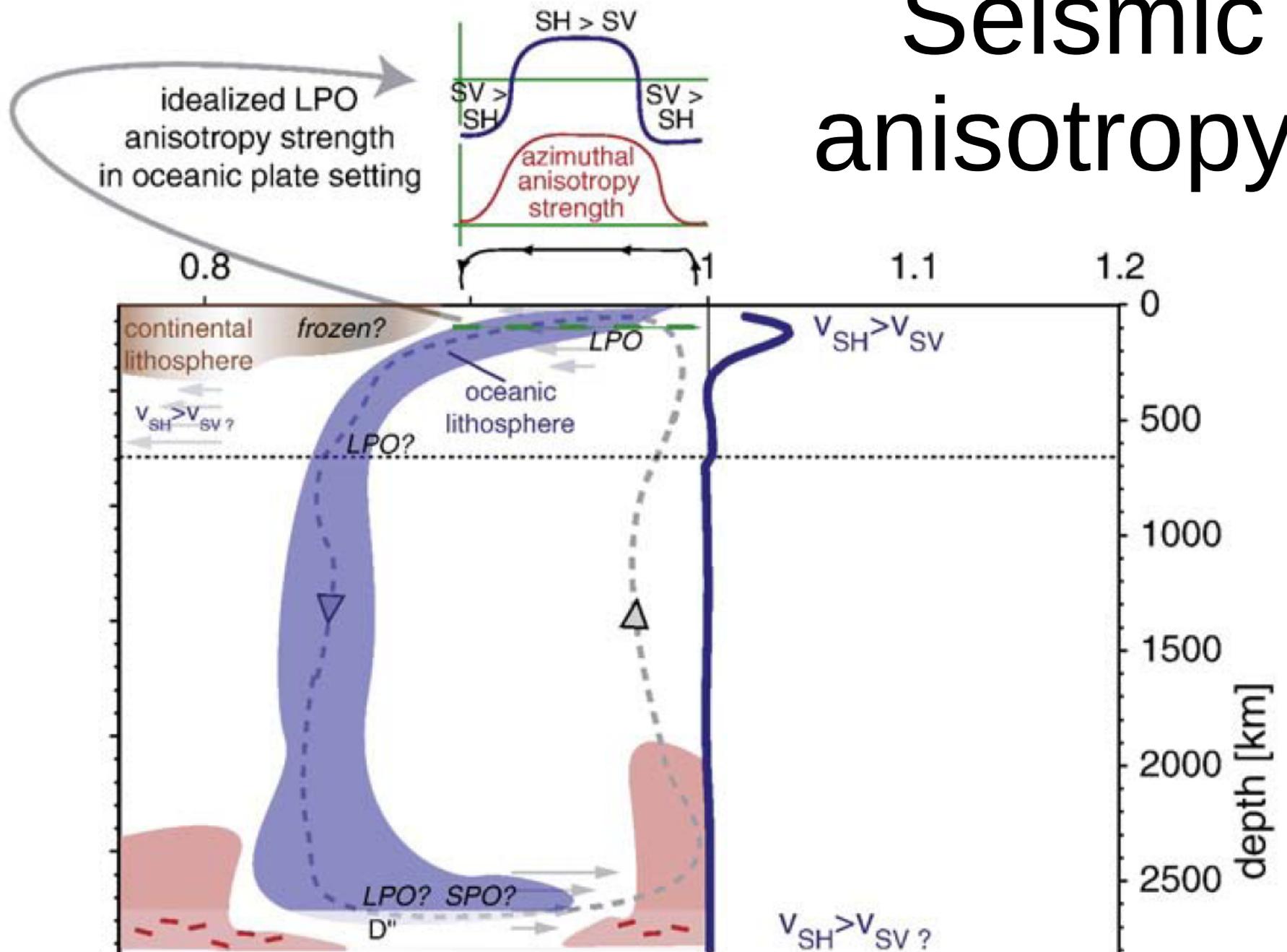
Baumann and Kaus (2015)

# State of affairs

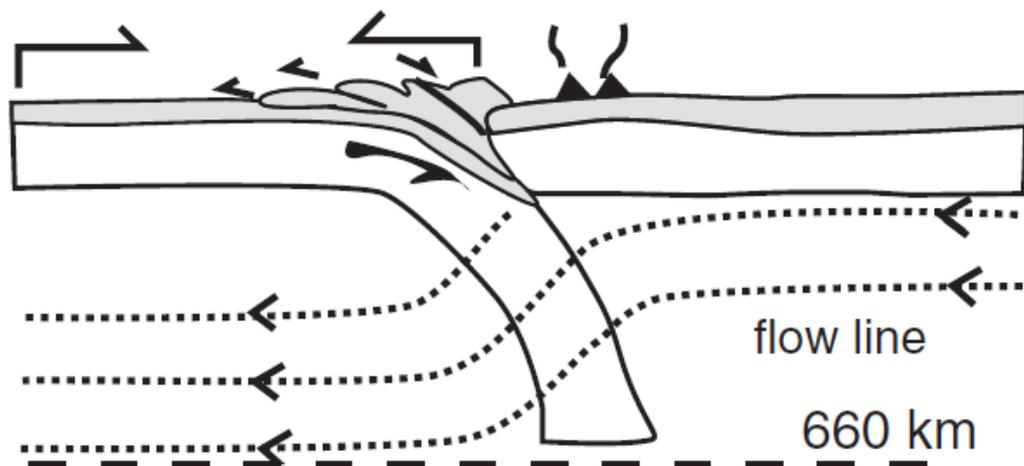
- Can explain plate velocities, geoid, dynamic topography and ~seismic anisotropy with global mantle circulation models
- Provides constraints on rheology and effective density distribution, needed to understand terrestrial planet evolution
- Frontiers: Time evolution and continental dynamics, e.g.
  - predict intraplate deformation
  - experimental design/hypothesis testing

**Additional slides**

# Seismic anisotropy

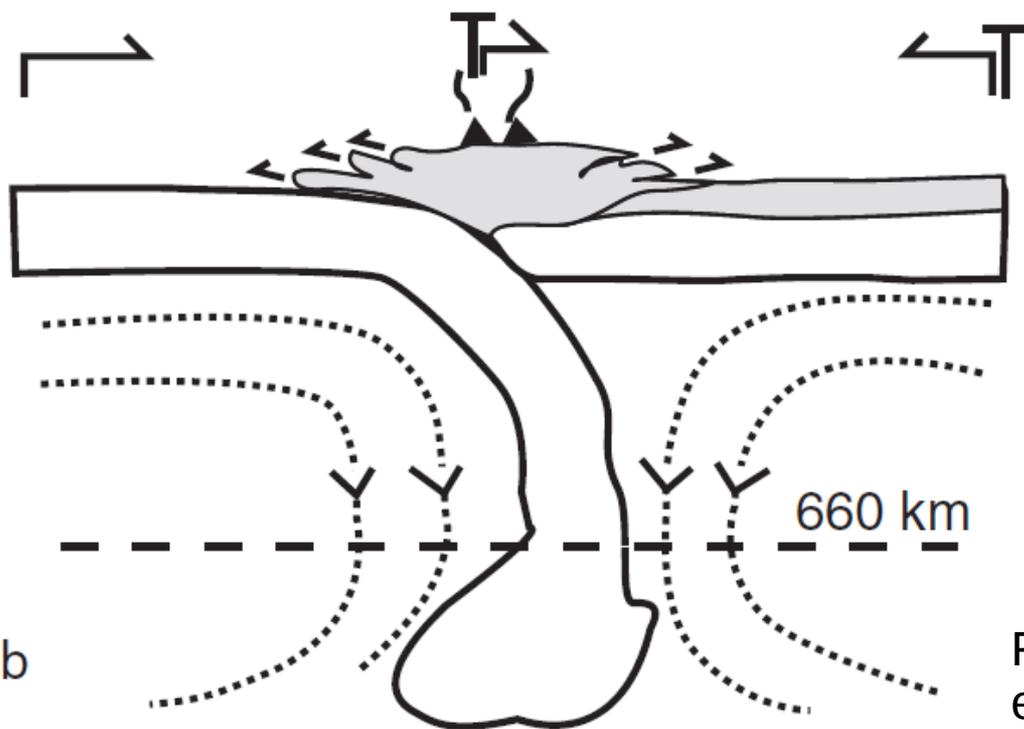


# slab pull - orogeny



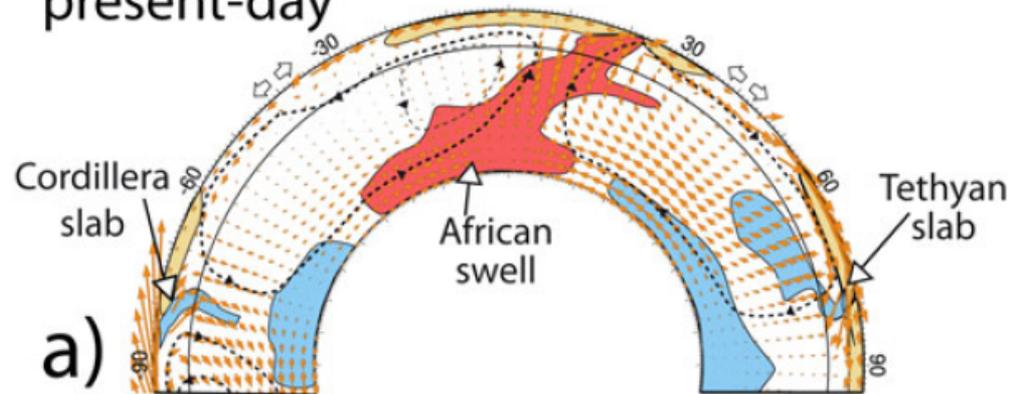
a

# slab suction - orogeny



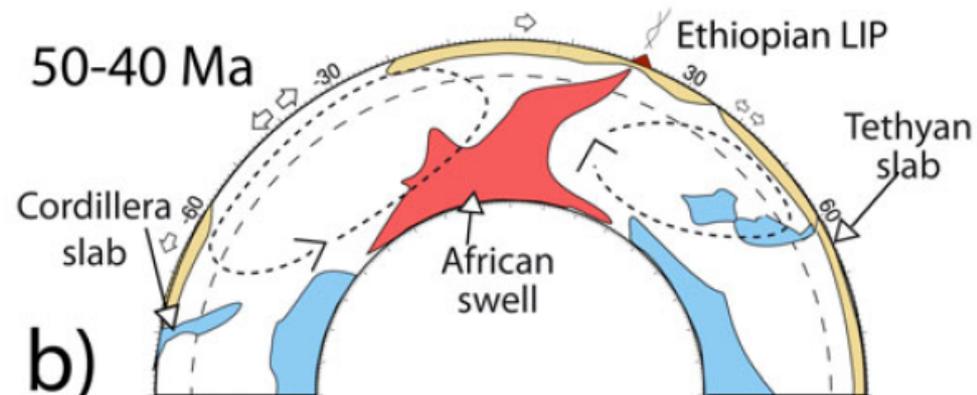
b

# present-day



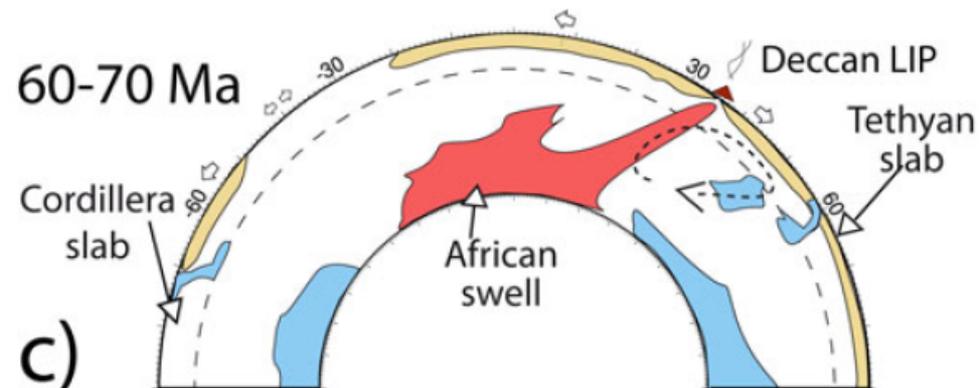
a)

# 50-40 Ma



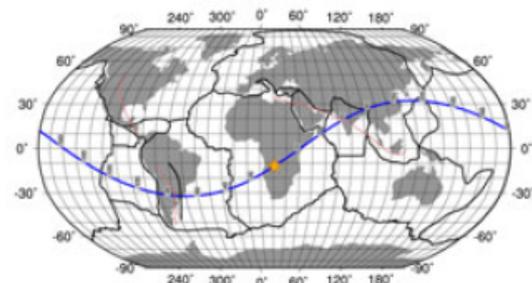
b)

# 60-70 Ma

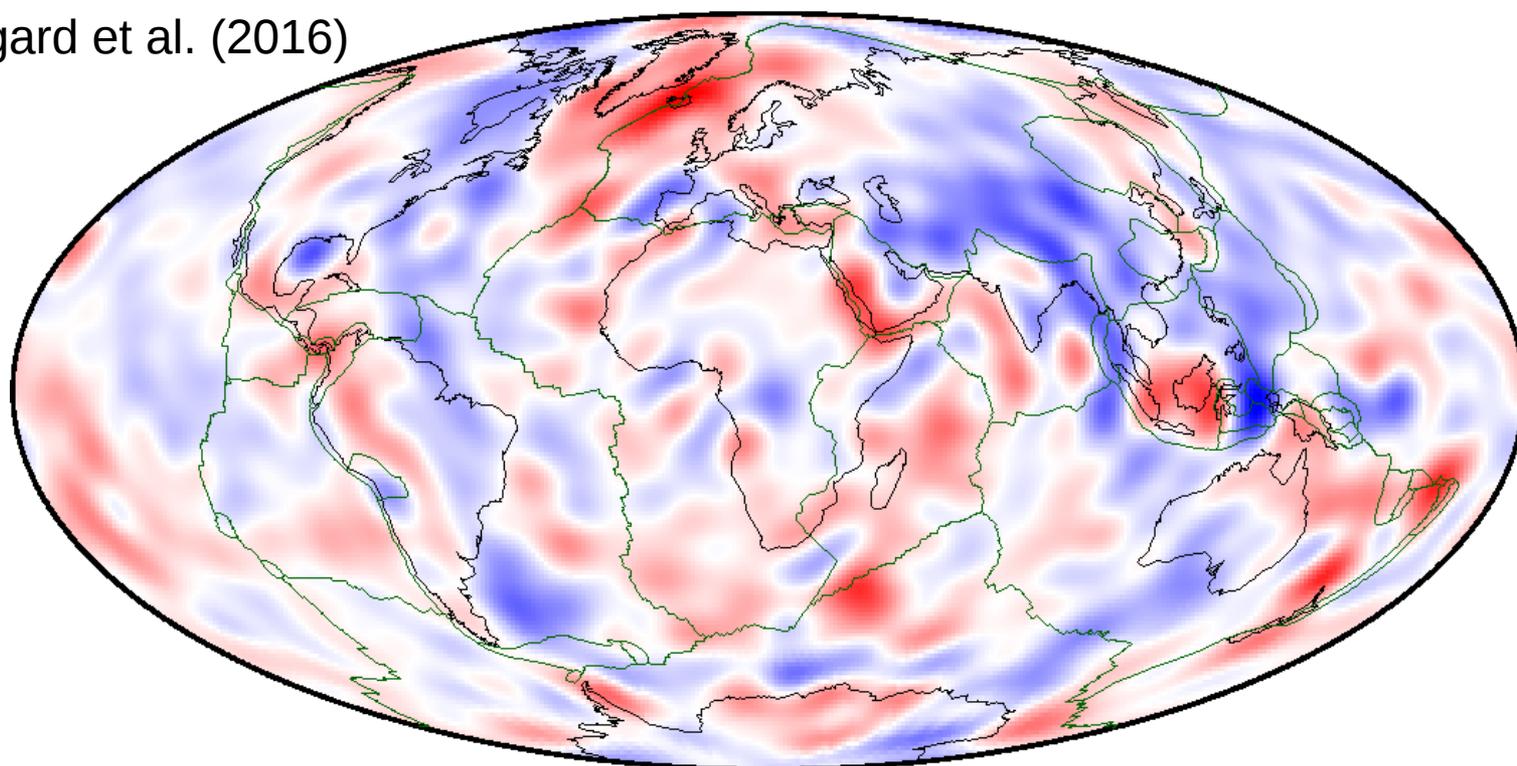


c)

Faccenna  
et al. (2013)



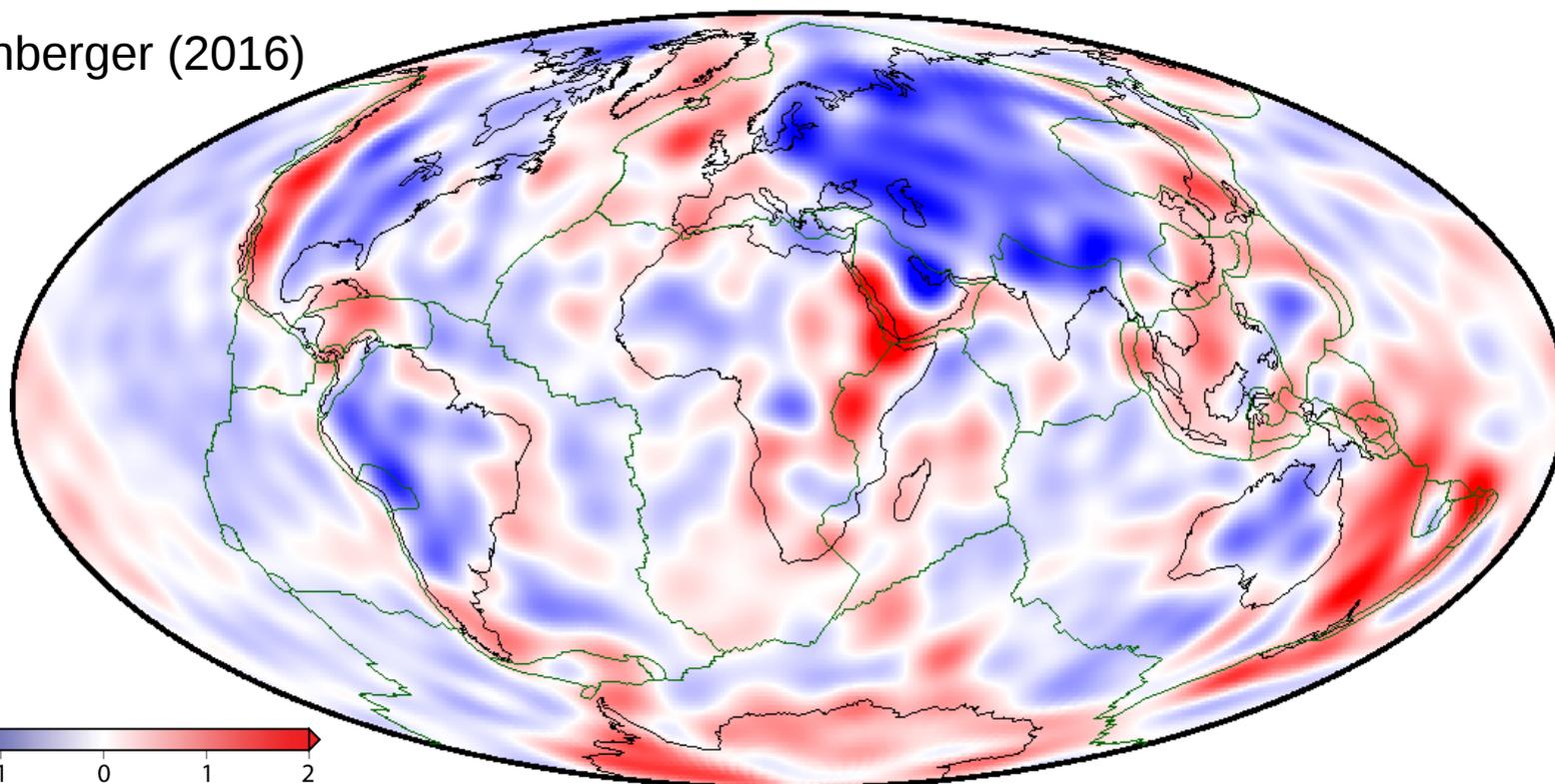
Hoggard et al. (2016)



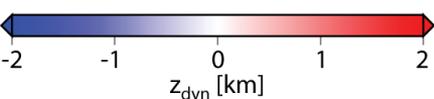
**Residual  
topography  
models**

crust for sparse,  
oceanic sites from  
active source,  
corrected to plate  
model, *continental  
areas from free-air*

Steinberger (2016)

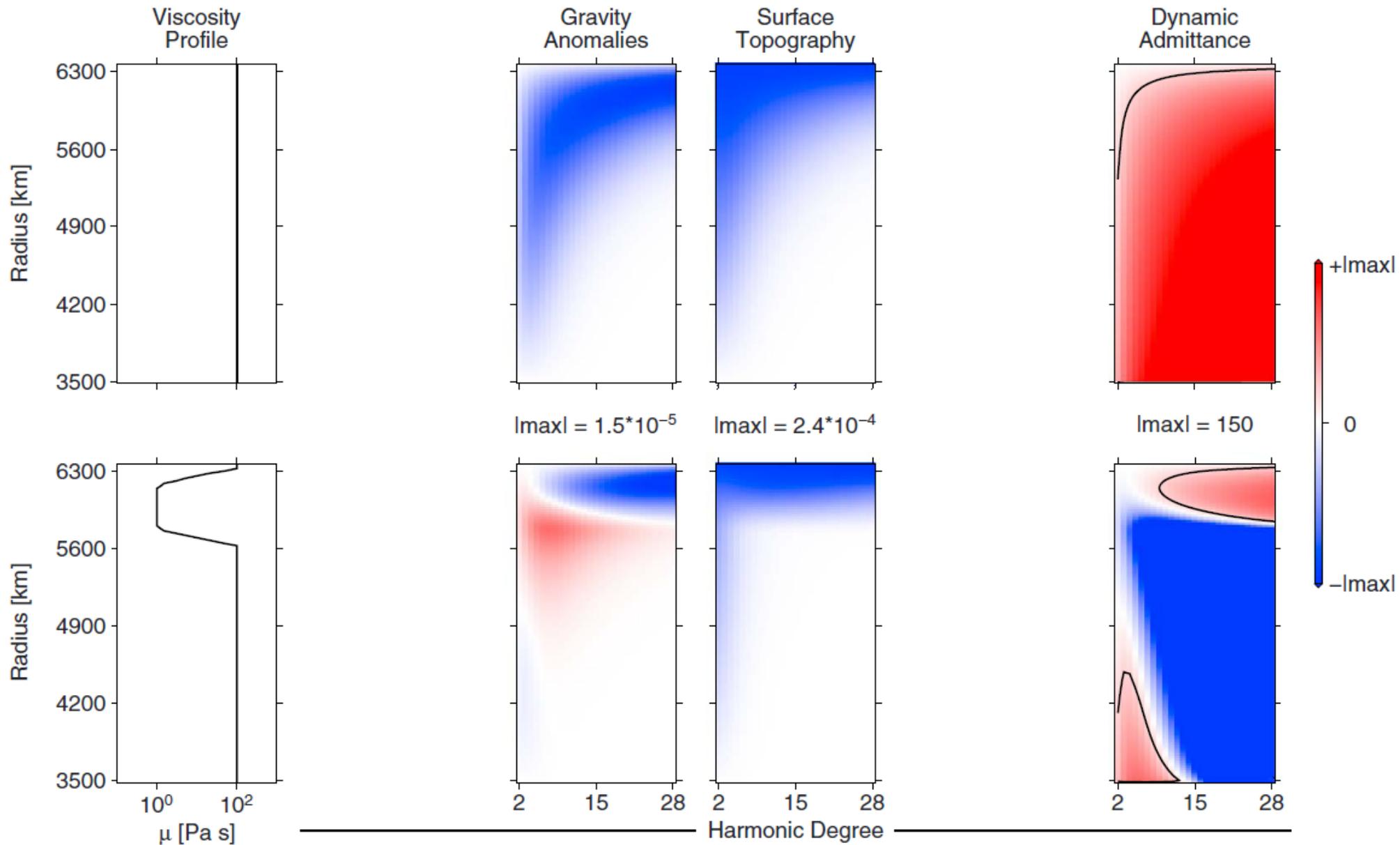


crust from  
CRUST1,  
lithospheric  
Model,  
corrected for  
half-space  
cooling



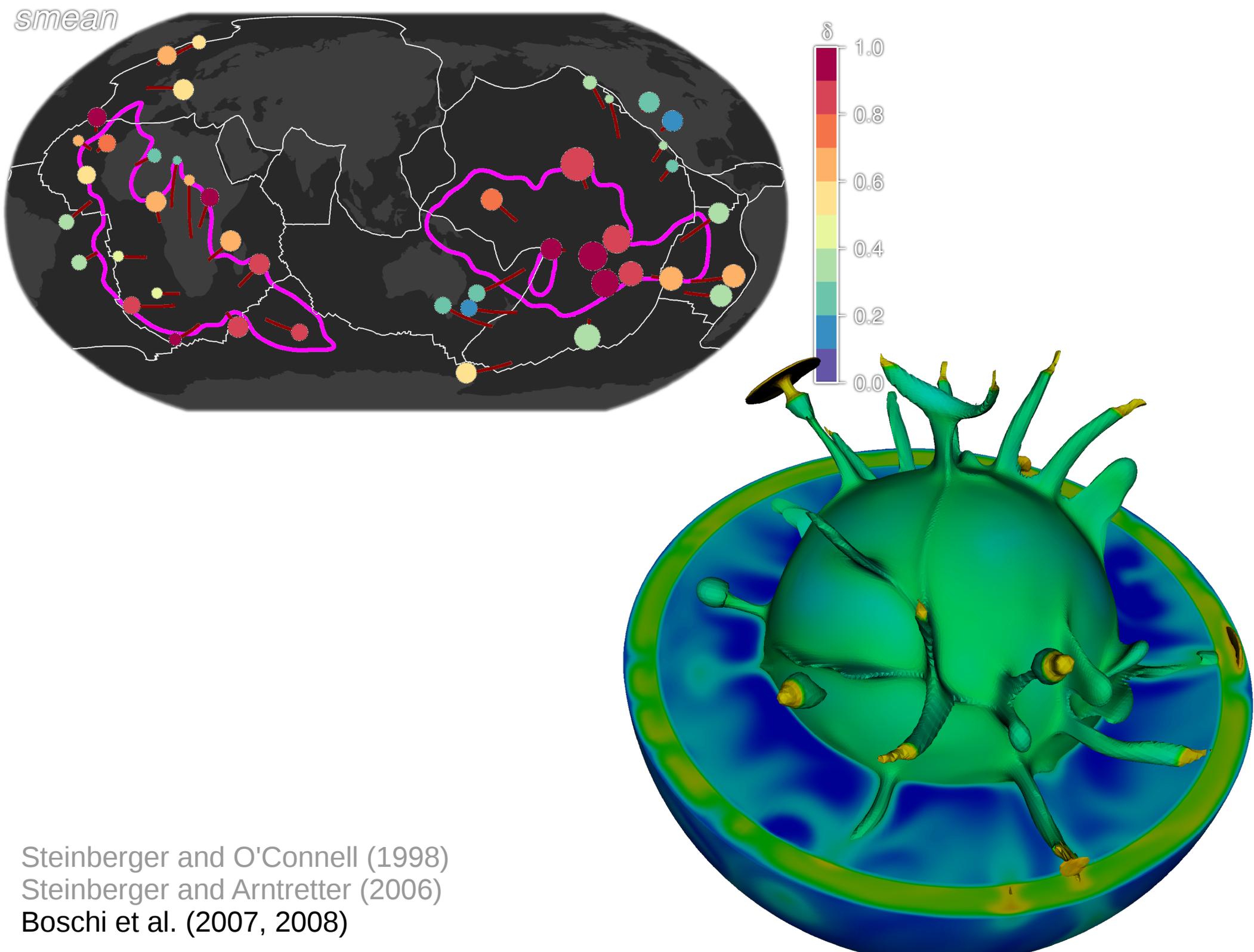
# Admittance $\neq$ constant

$$\frac{\text{gravity}}{\text{dynamic surface topography}} \neq \text{constant}$$



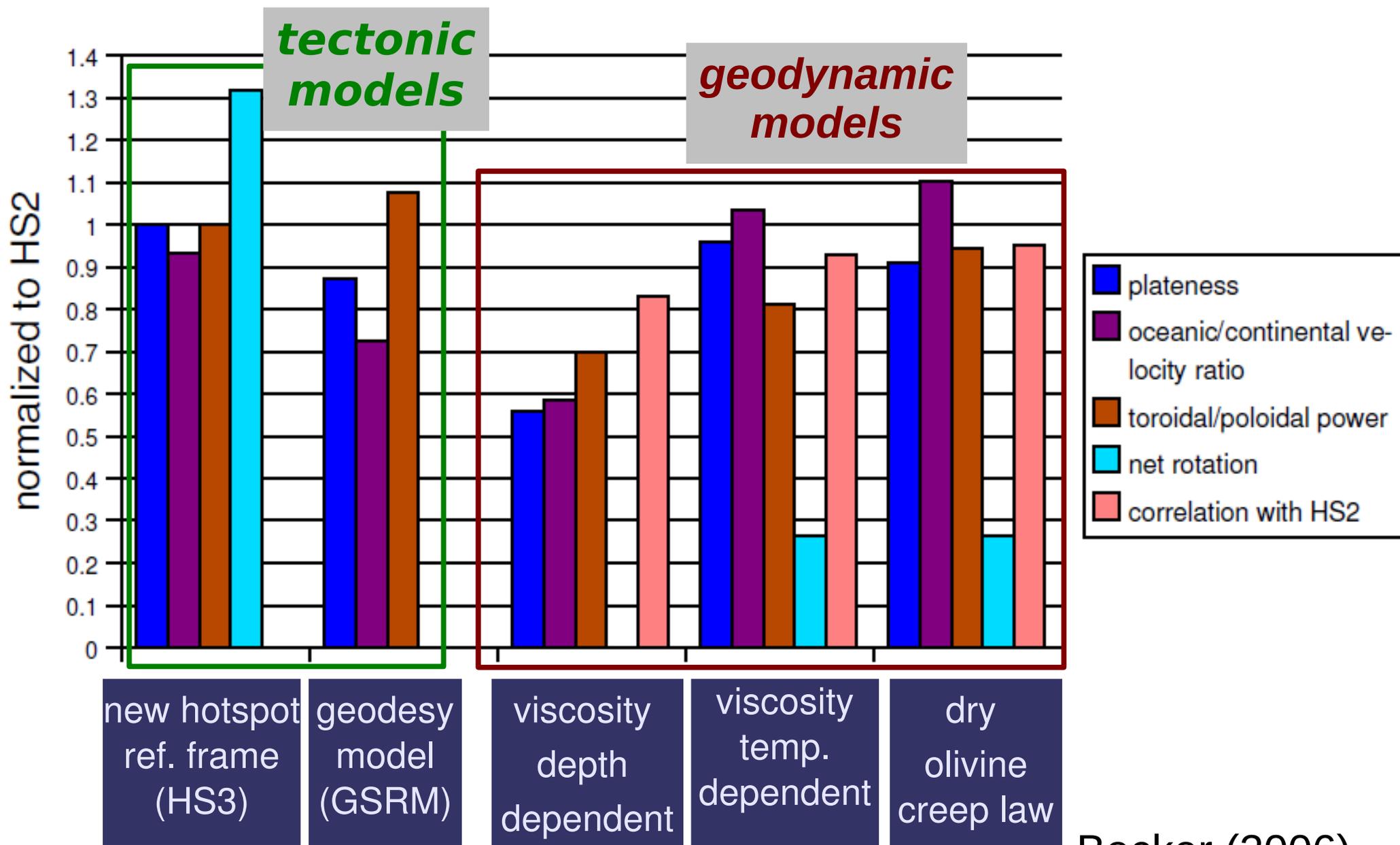
(CIDER supported...)

*smean*

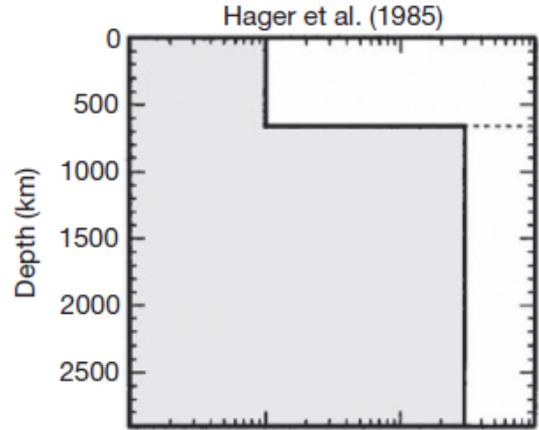


Steinberger and O'Connell (1998)  
Steinberger and Arnttetter (2006)  
Boschi et al. (2007, 2008)

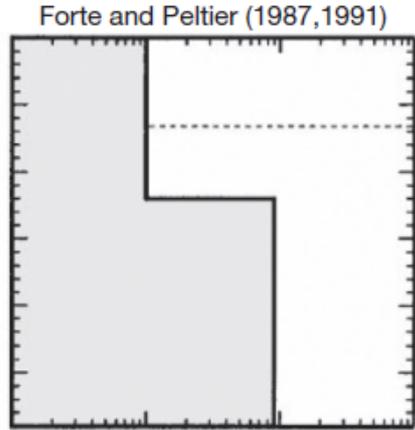
# Prediction of plate scores with LVVs



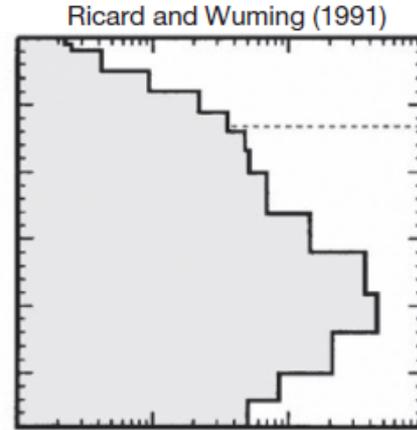
Becker (2006)



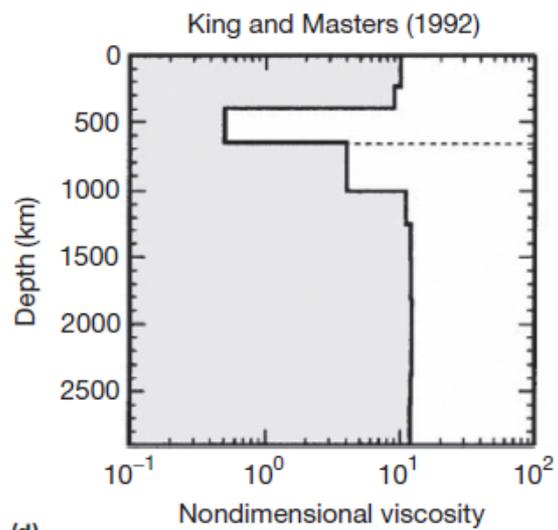
(a)



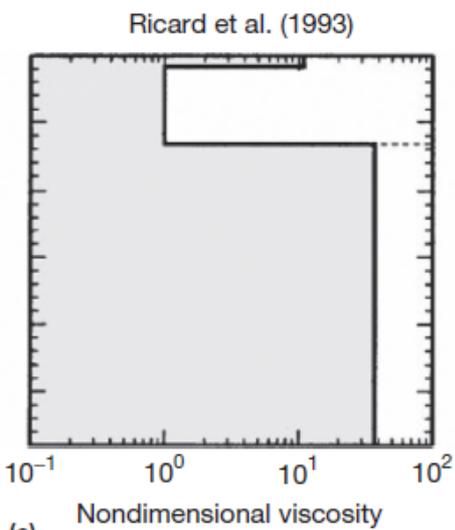
(b)



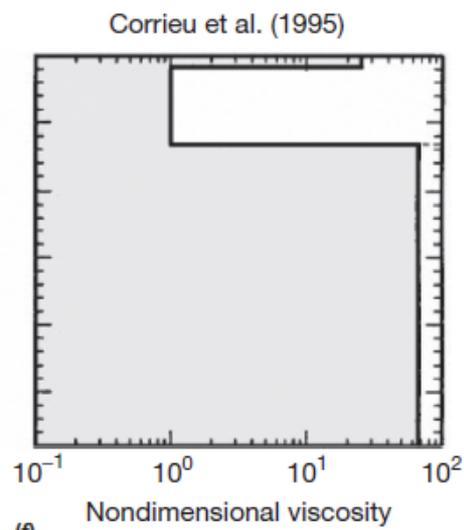
(c)



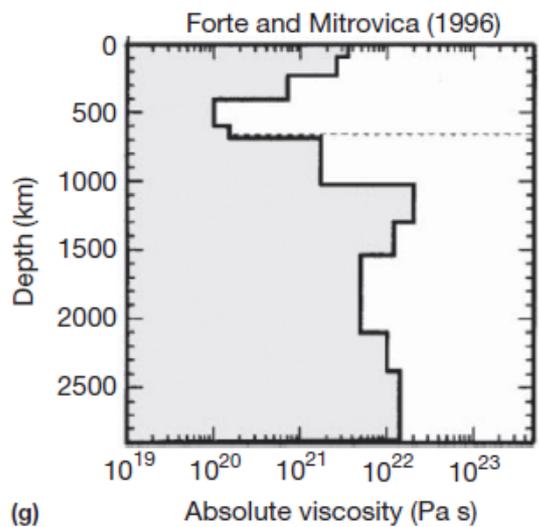
(d)



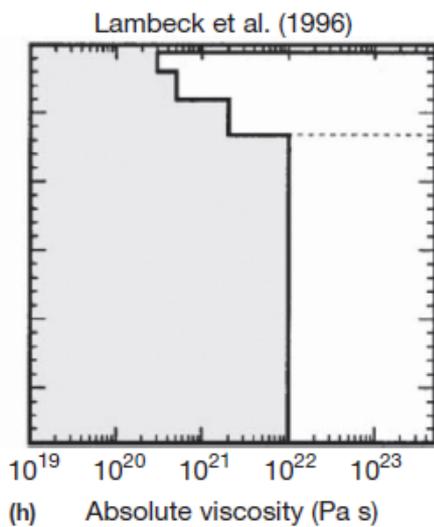
(e)



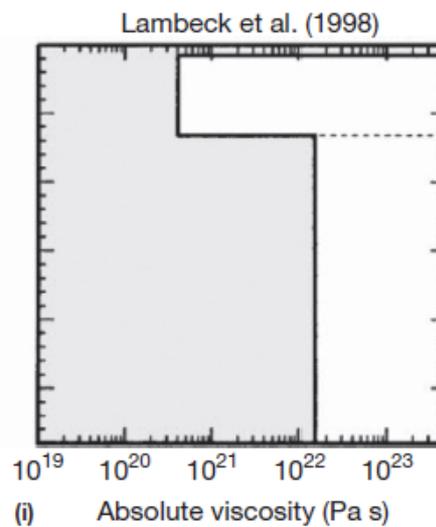
(f)



(g)



(h)



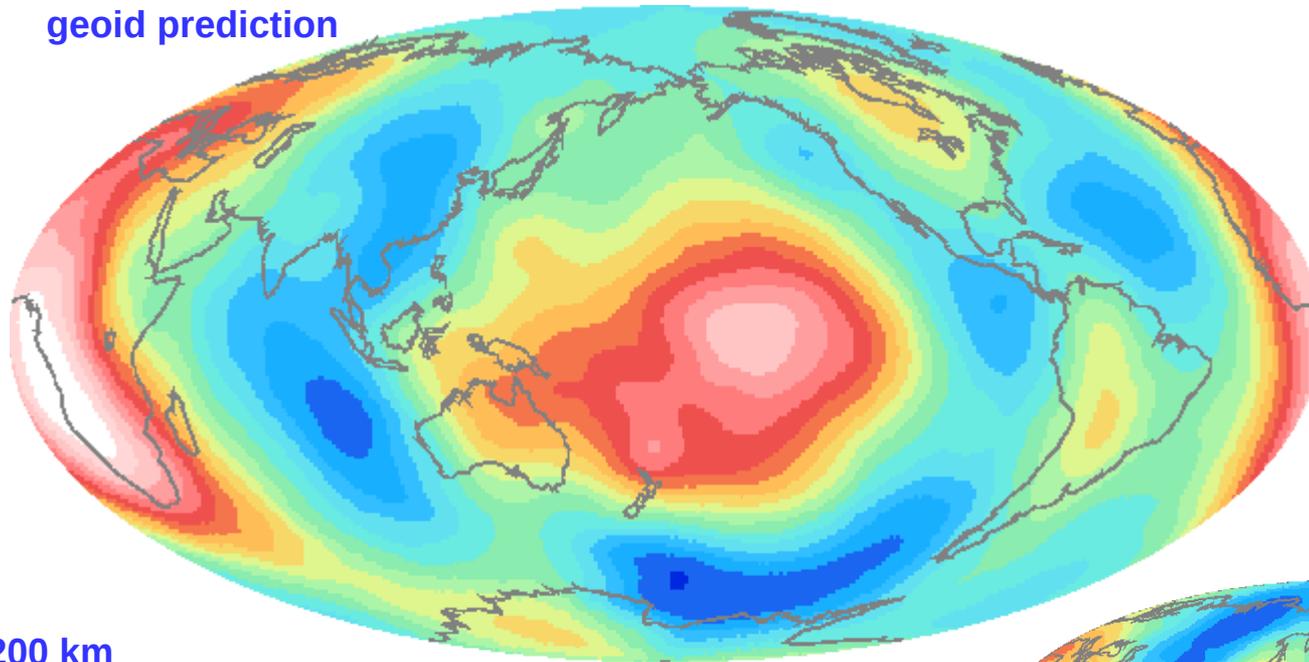
(i)

Forte (2015)

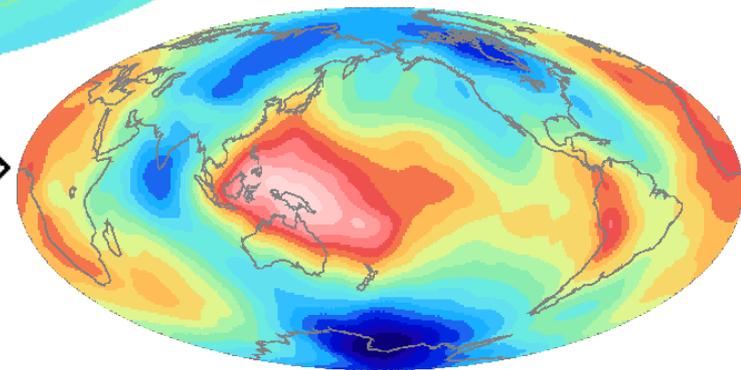
# Geoid for tomography driven flow

- Lower mantle stiffer

geoid prediction

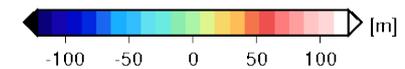
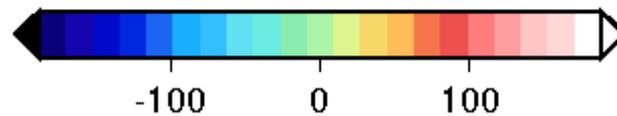
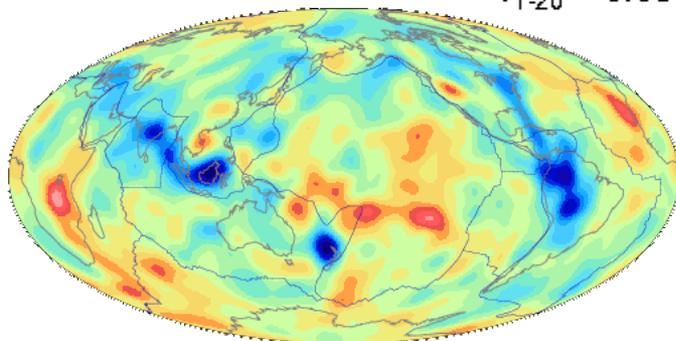


observed geoid

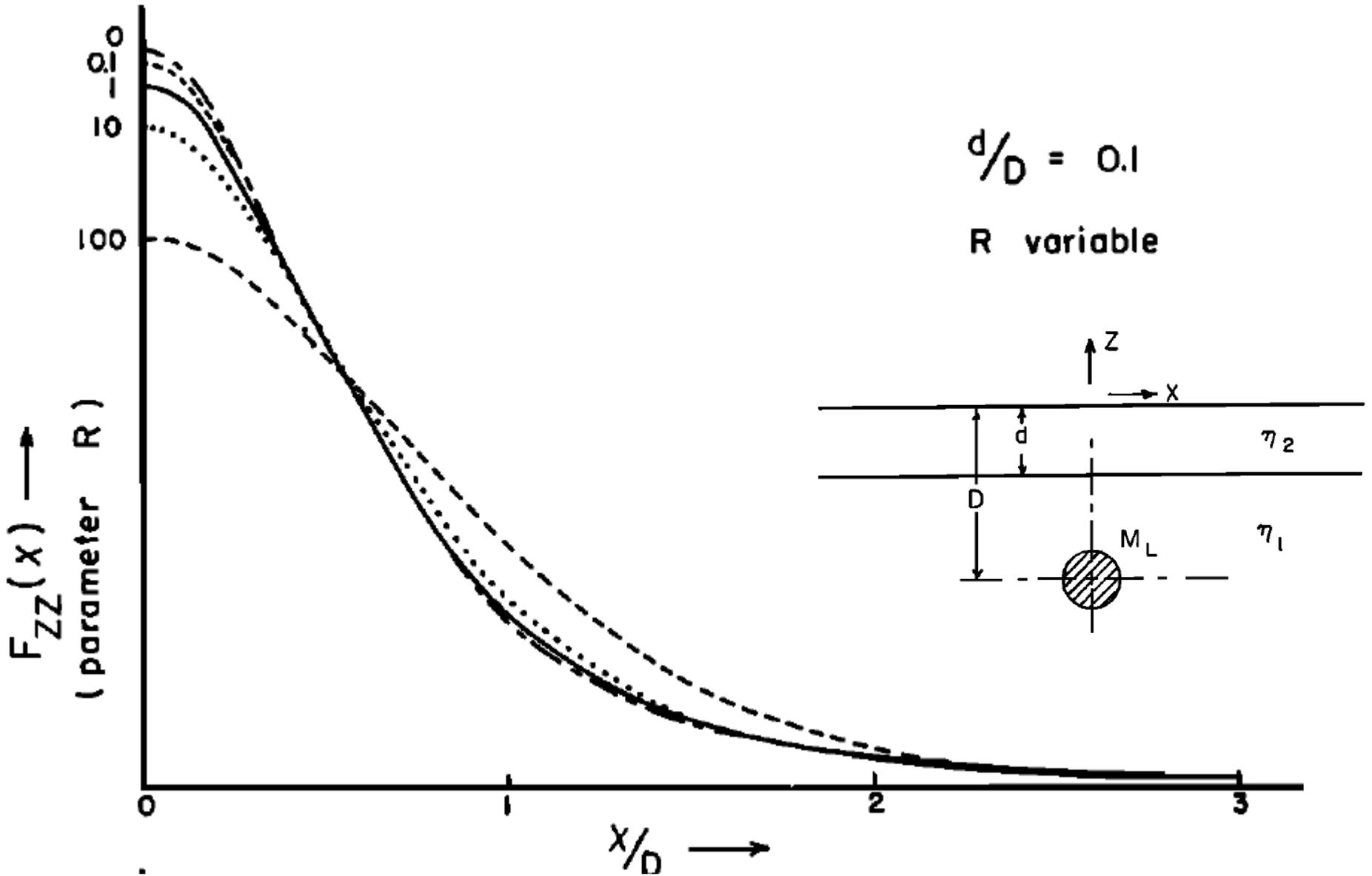


tomography @ 1200 km

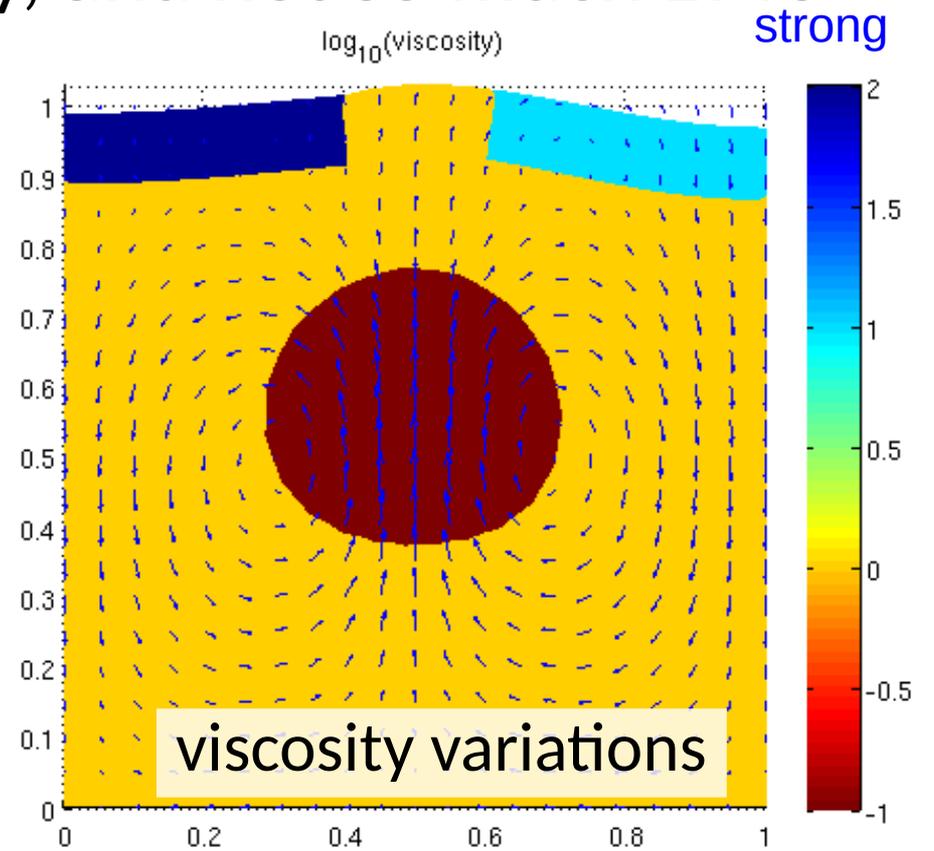
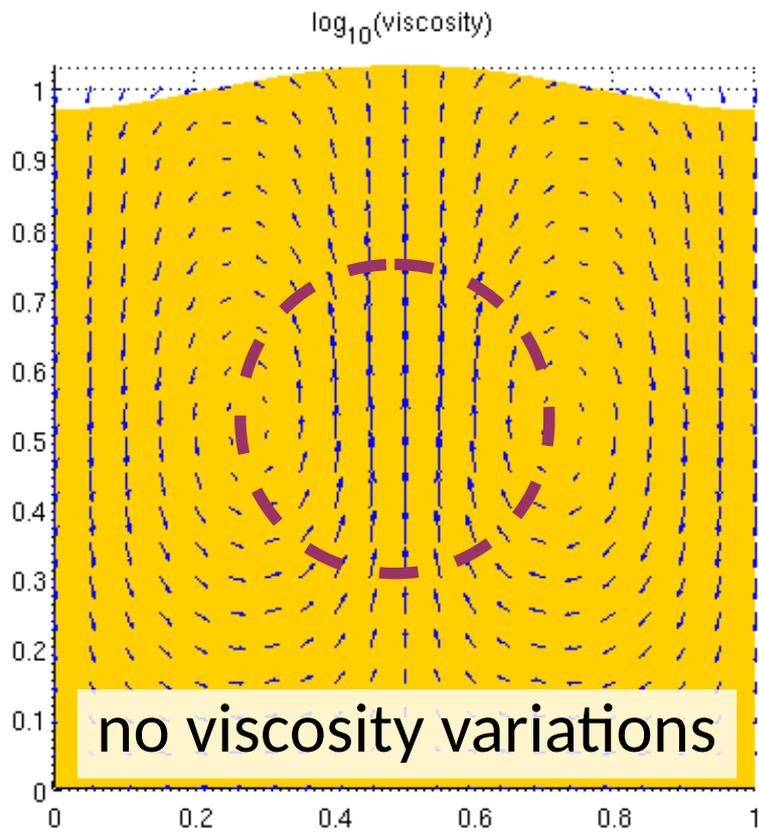
$r_{1-20} = 0.58$



$$R = \eta_1 / \eta_2$$

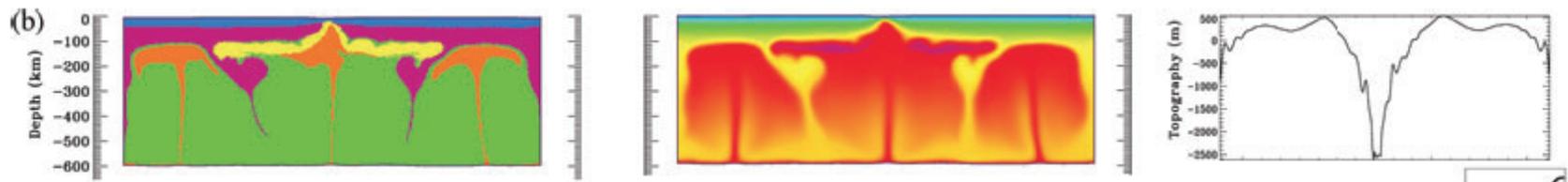


# Topography is about knowing the structure and the history, and not so much LVVs



strong

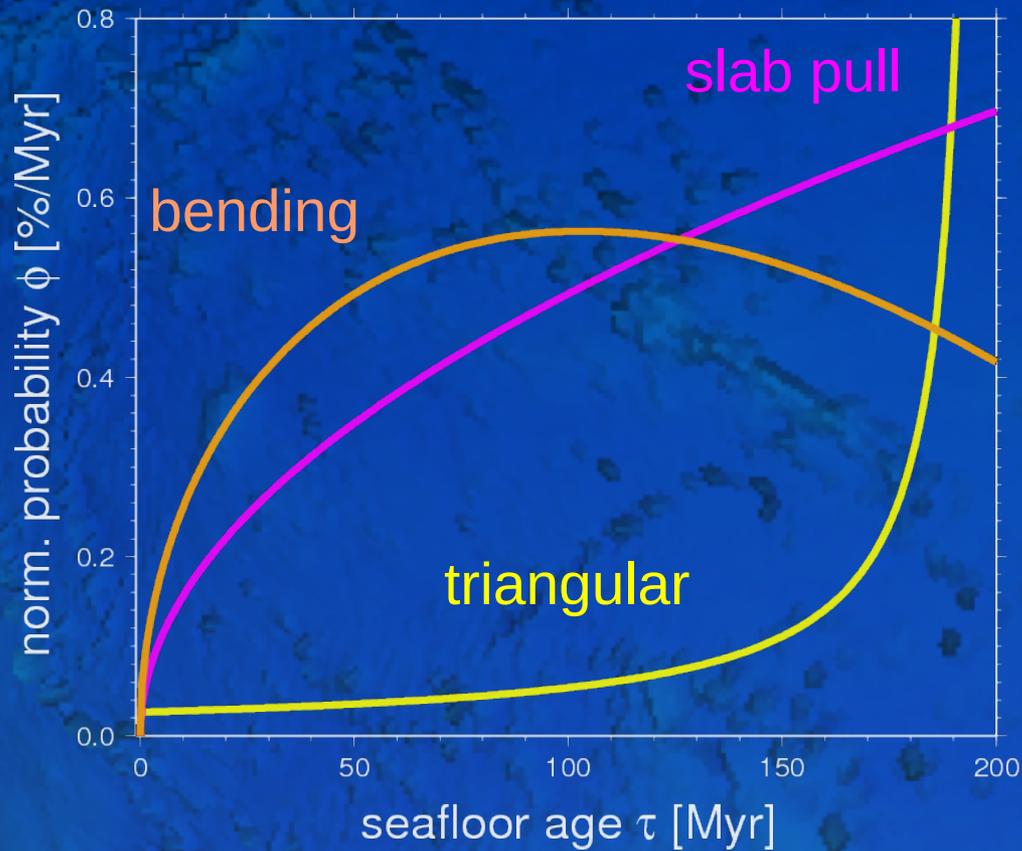
squishy



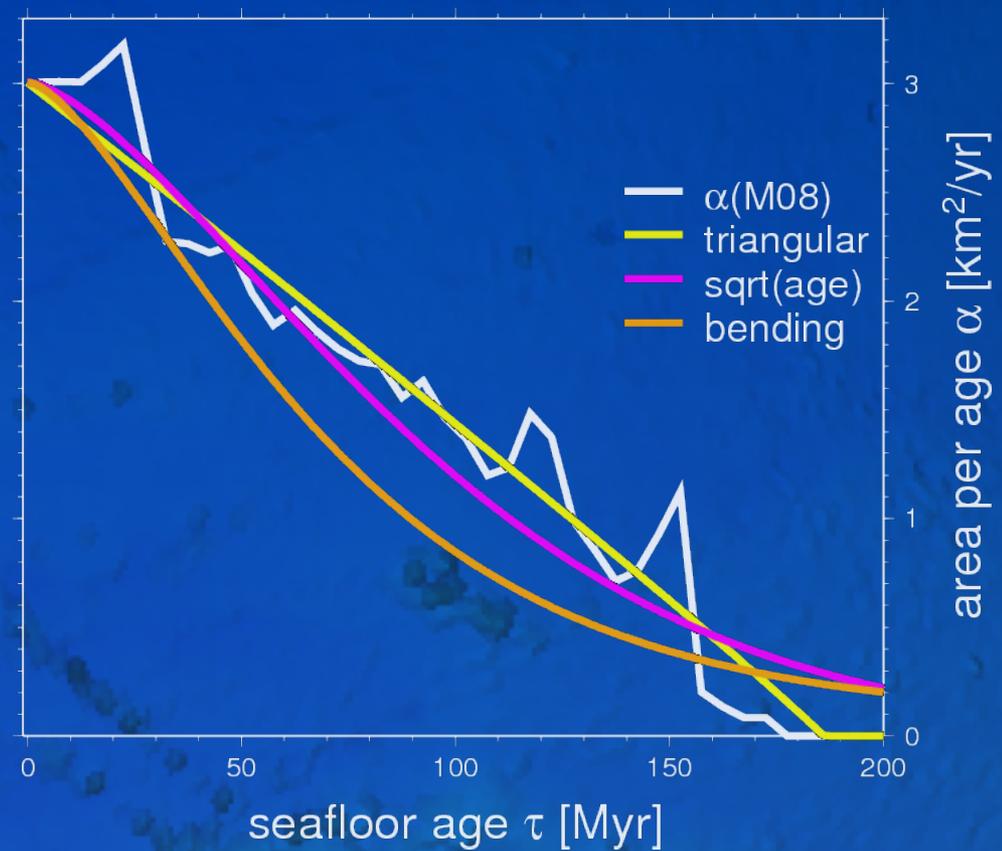
Burov et al.

# Alternative age distributions for constant production rates

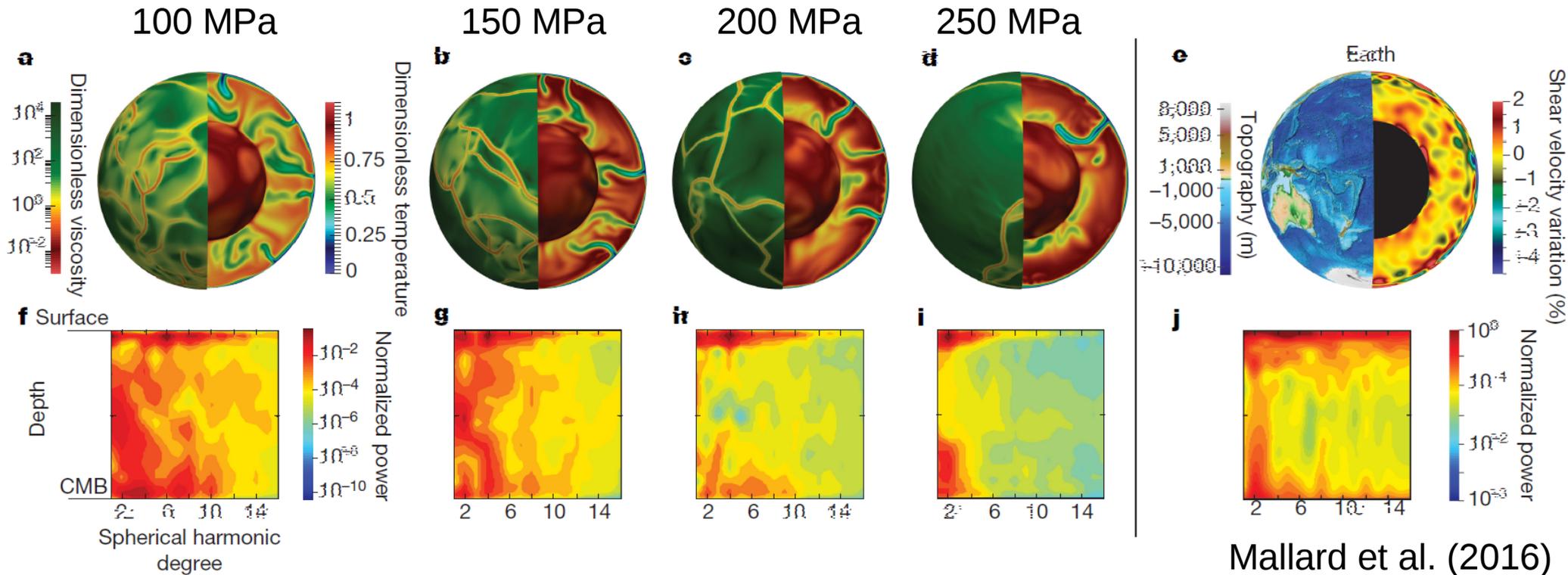
*Subduction probability*



*Age distribution*



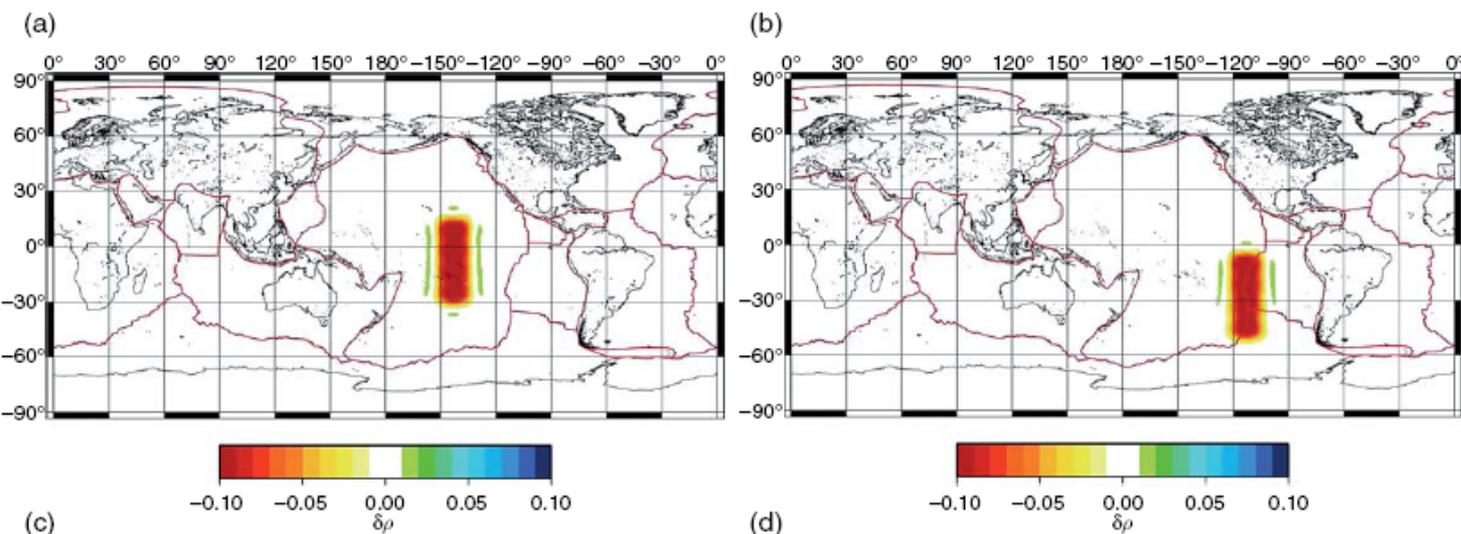
# Yield stress control



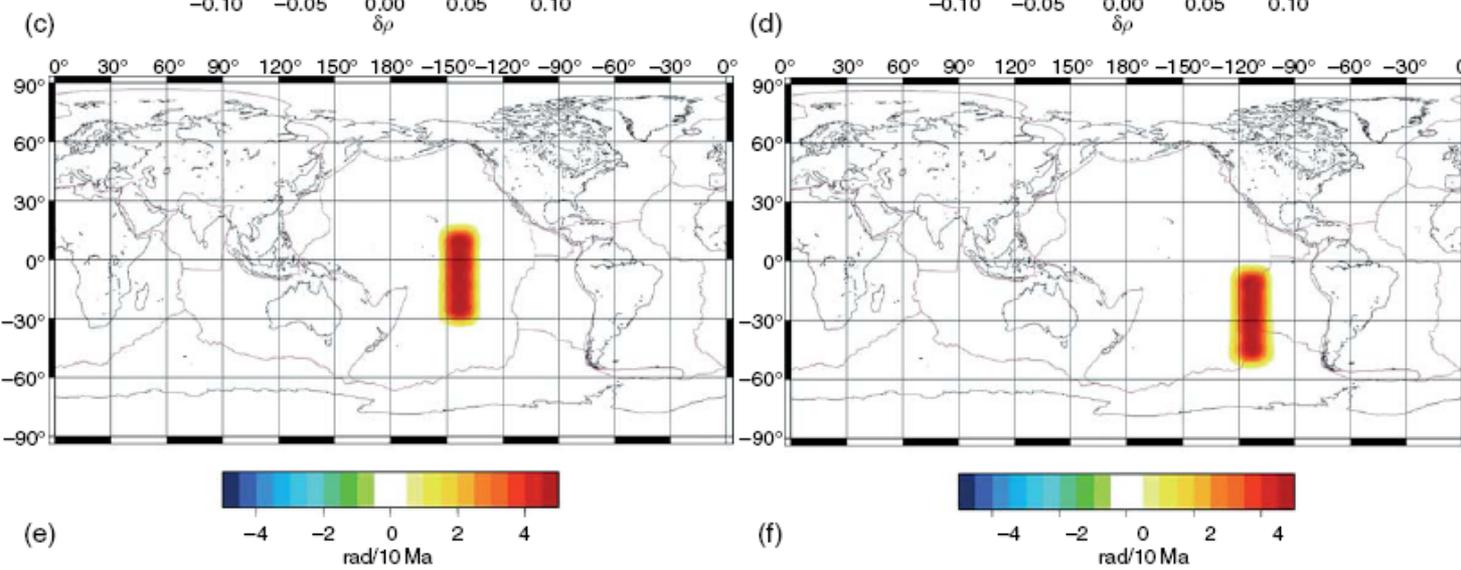
Mallard et al. (2016)

cf. Tackley (2000a,2000), Richards et al. (2001),  
van Heck and Tackley (2008), Foley and Becker (2009)

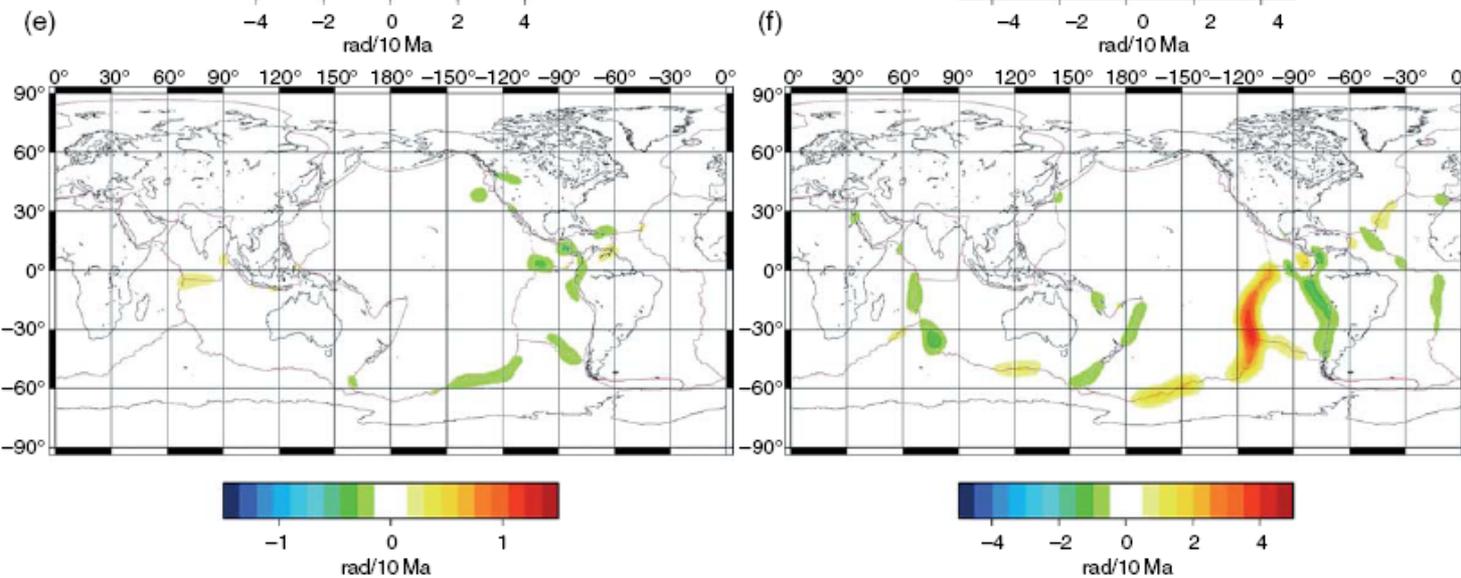
*But:* effect of asthenosphere, internal vs. bottom heating, Ra #,  
continents, and damage/memory



Two density sources



Free-slip solution for poloidal flow

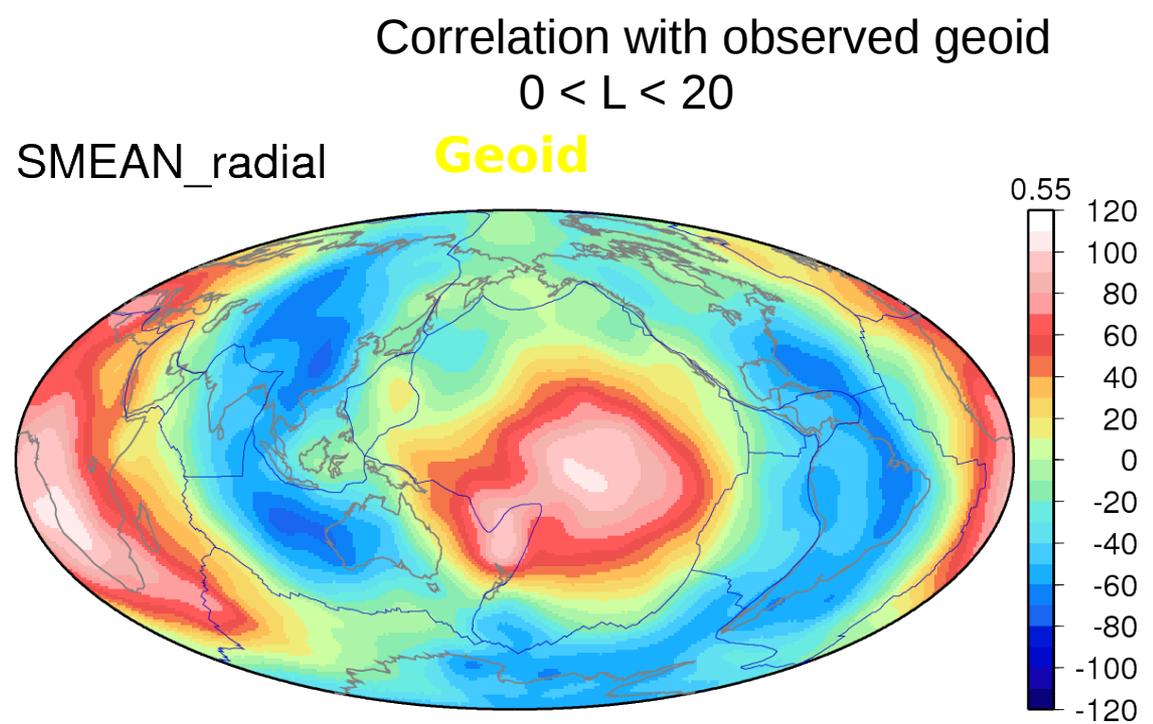
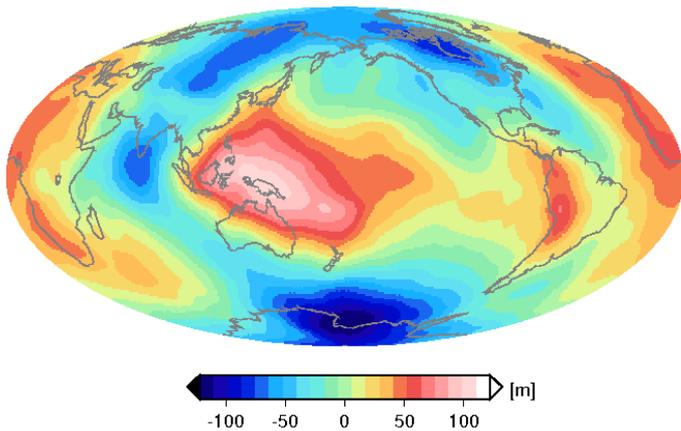


"Plate" solution for poloidal flow

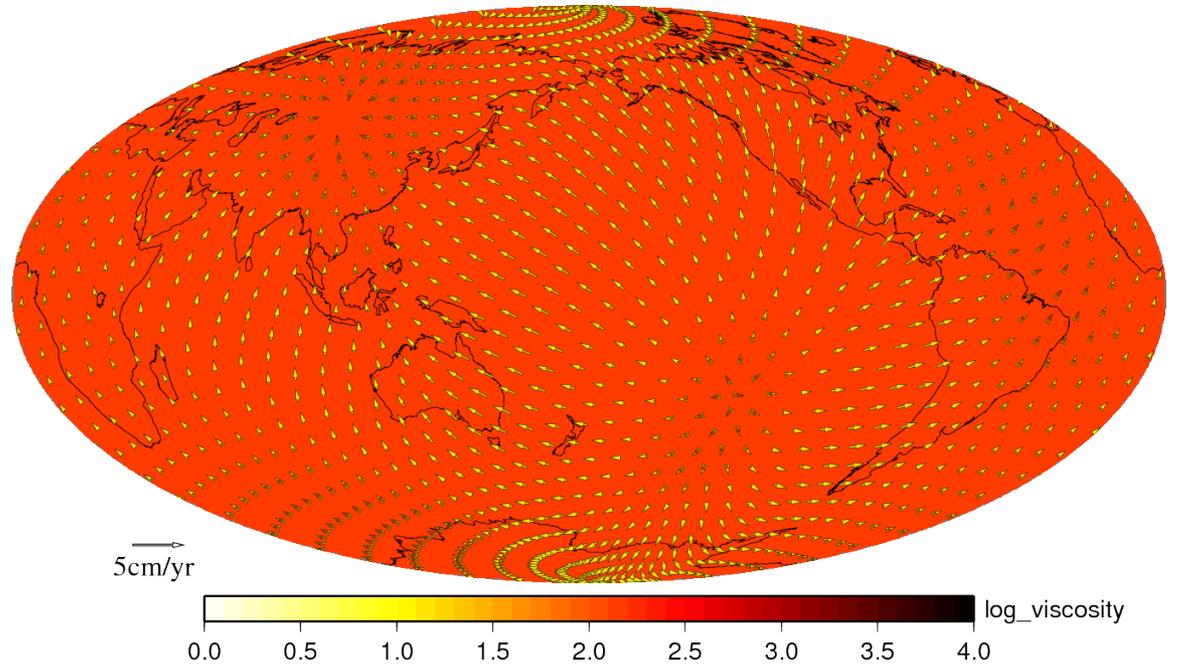


# Geoid and LVVs

- Free slip surface
- Four layer viscosity (not optimized)
- SMEAN tomography



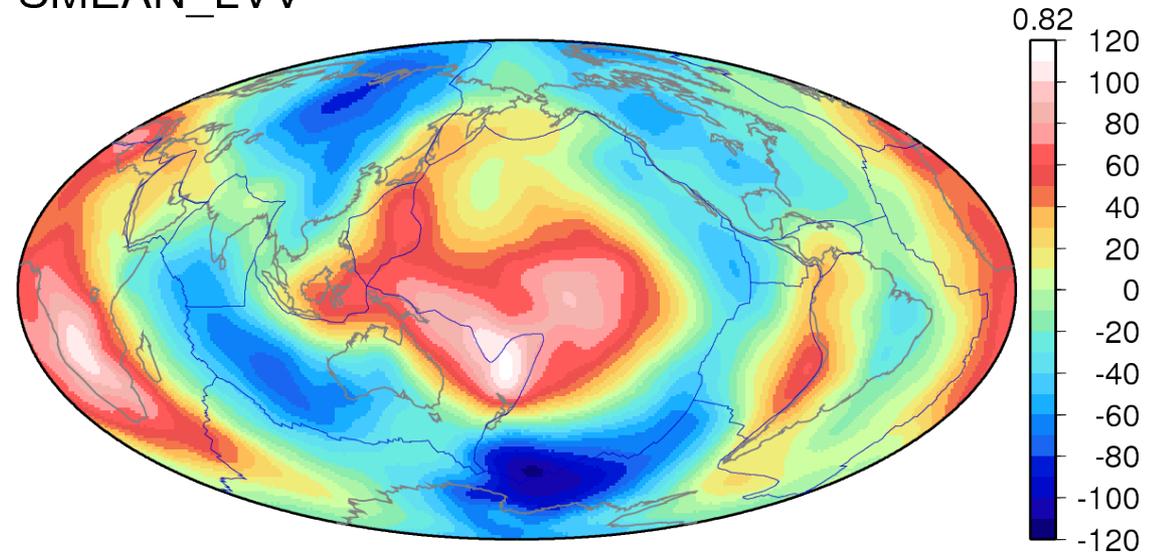
**Surface velocities with viscosity in background**



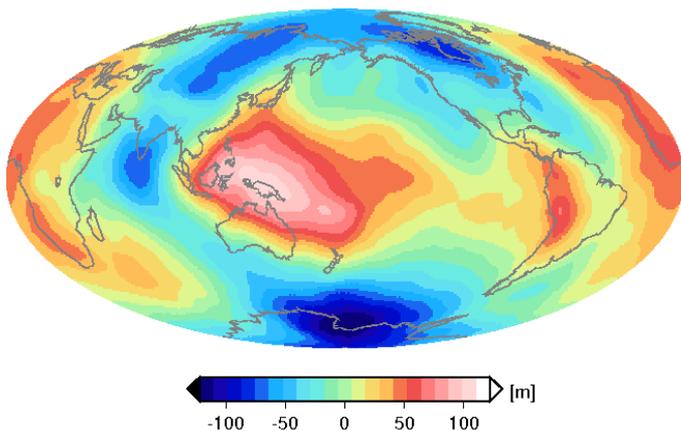
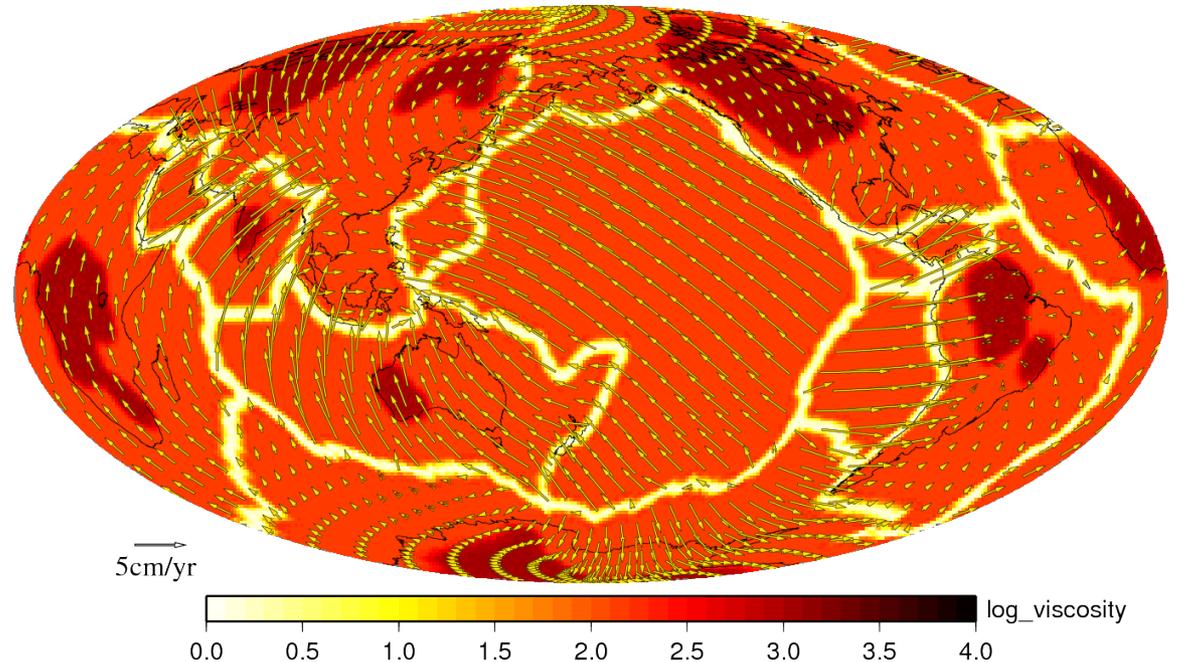
# Geoid and LVVs

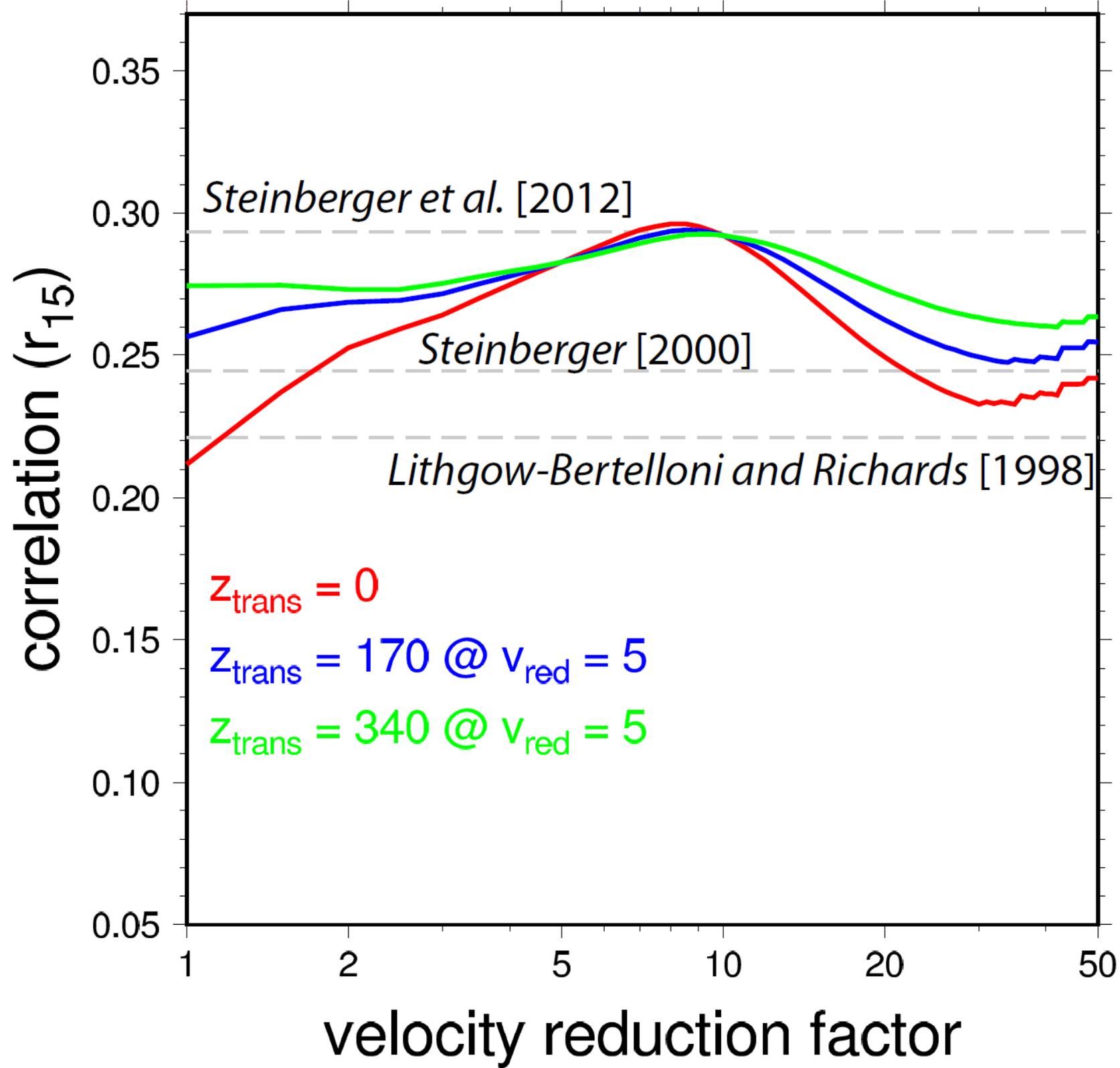
- Free slip surface
- Weak zones, stiff keels

SMEAN\_LVV **Geoid**



**Surface velocities with viscosity in background**





# Subduction velocity scaling

***slab  
bending***

***mantle  
drag***

***slab  
pull***

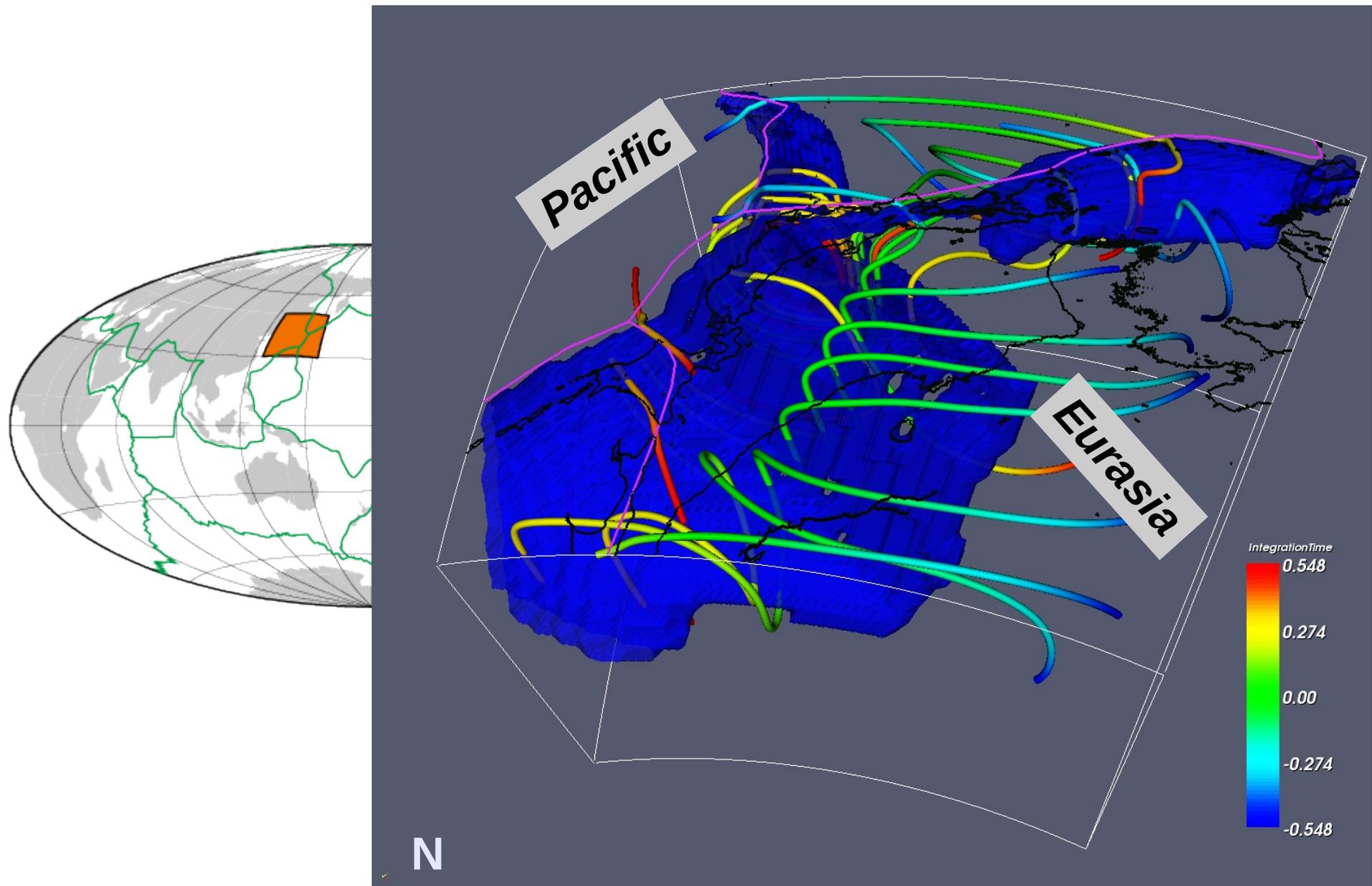
$$V' \approx \Delta\rho ghL / [2\eta_l (h/r)^3 + 3\eta_m A]$$

$$V^\circ = (\Delta\rho^\circ g H^\circ h^\circ) / \eta_m^\circ$$

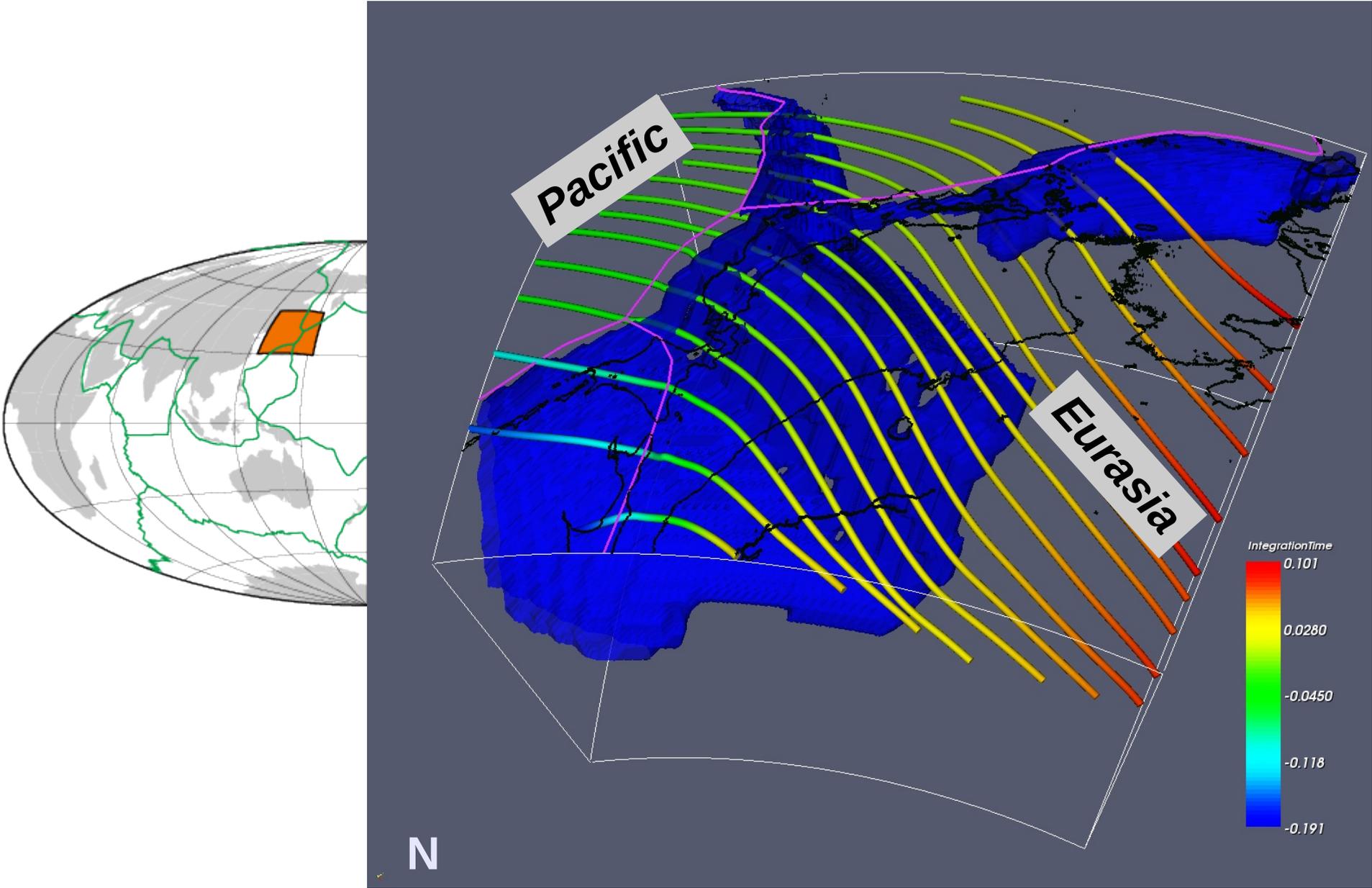
$$V = V' / V^\circ$$

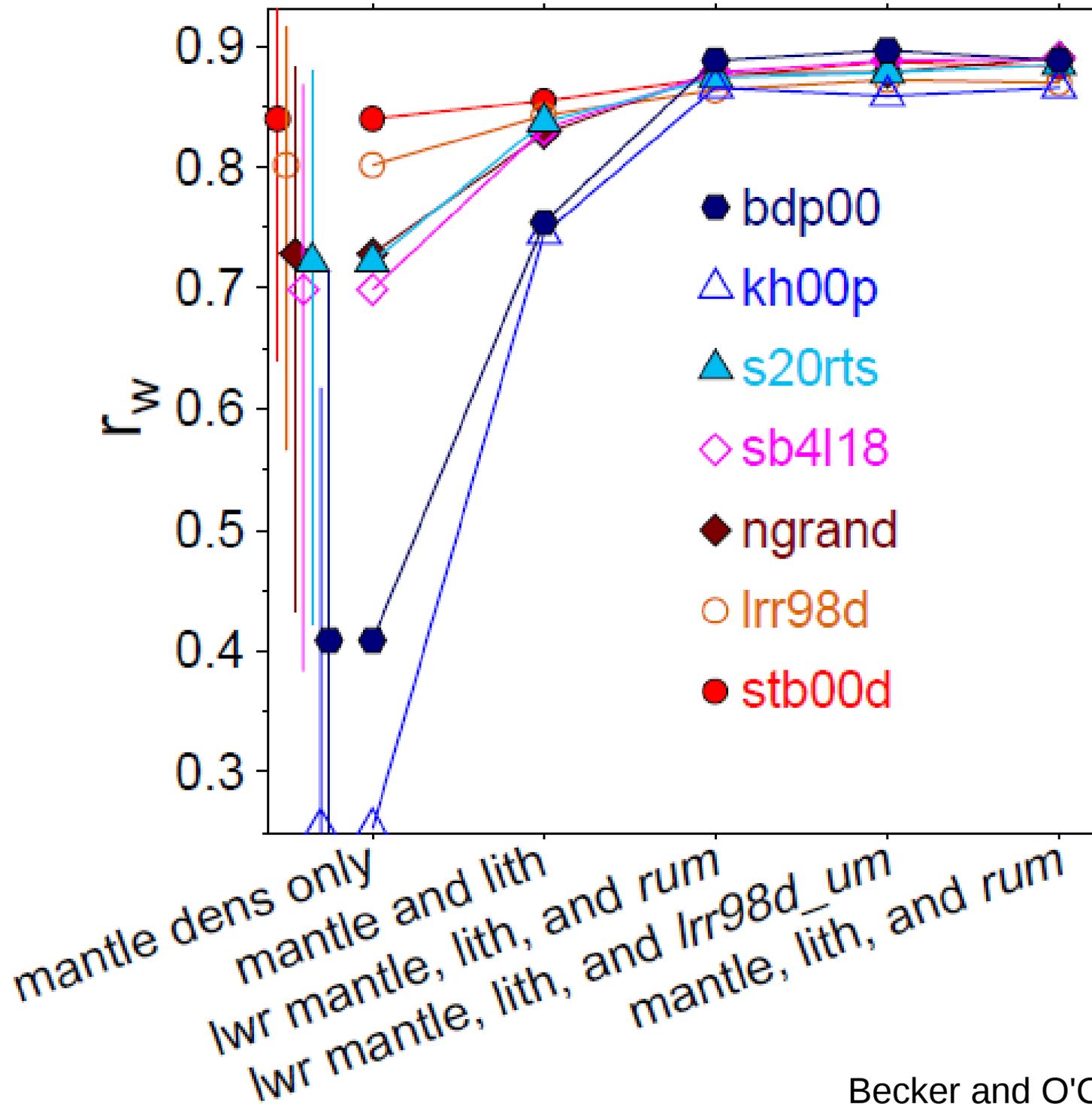
- Subduction velocity = modified Stokes velocity accounting for bending

# Regional models



# Global models





# Gravitational potential energy

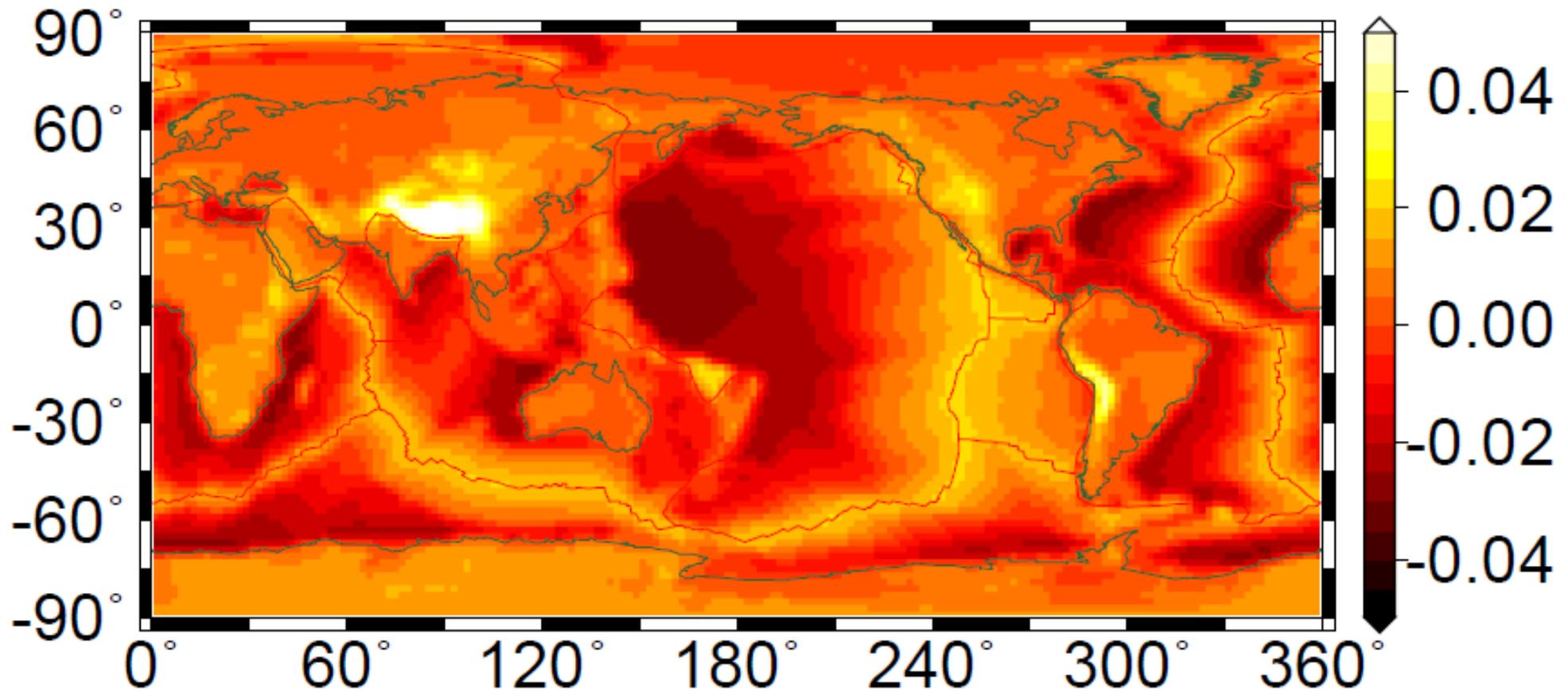


Figure 3. Differential potential energy  $\Delta U = U - \bar{U}$  ( $\bar{U} = 2.616 \times 10^{14} \text{ Jm}^{-2}$ ) of our  $2^\circ \times 2^\circ$  model in units of  $10^{14} \text{ Jm}^{-2}$ . Min/mean/max values of  $\Delta U$  for oceanic and continental lithosphere are  $-0.03/-0.004/0.023$  and  $-0.013/0.007/0.078$ , respectively.

# GPE torques

$\text{lith\_thick} \times 2.0$  ( $r = 0.63, r_w = 0.71$ ) ●  
 $\text{lith\_thick\_cobl} \times 2.0$  ( $r = 0.67, r_w = 0.74$ ) ○  
 $\text{topo\_lith\_thick} \times 2.0$  ( $r = 0.69, r_w = 0.76$ ) ◆  
 $\text{topo\_lith\_thick\_cobl} \times 2.0$  ( $r = 0.49, r_w = 0.73$ ) ◇  
 $\text{rp\_edge\_based} \times 1.0$  ( $r = 0.75, r_w = 0.76$ ) ▲

